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E:/DATA-13/NOTES/ENGLISH/PHYSICS/-XI/KINEMATICS

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1. KINEMATICS

1.1 INTRODUCTION

MAIN CONTRIBUTORS TO MECHANICS

(A) ARISTOTLE **:** Born in 384 B.C., he was the first man to work in the field of mechanics. He held the view that if a body is moving, something external must be required to keep it in that state and prevent it from coming to a stop.

(B) GALILEO **:** He was born in Pisa, Italy in 1564 A.D. He invented the concept of acceleration. From experiments on motion of bodies on inclined planes or falling freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall down faster under gravity.

(C) NEWTON **:** He was born in 1642 (the same year that Galileo died). He formulated the well-known laws of motion. He worked on theories of light and colour. He designed an astronomical telescope to carry out astronomical observations.

1.2 REST AND MOTION

PARTICLE

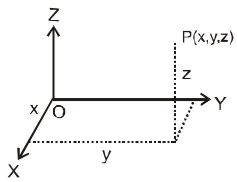
(A) DEFINITION OF PARTICLE

(i) A body of finite size of splitted parts may be considered as a particle only if all parts of the body undergo same displacement and have same velocity and acceleration.

(ii) When every part of an object undergoes same displacement and has same velocity and acceleration, we can describe its motion by the motion of any point of it.

(B) DEFINITION OF FRAME OF REFERENCE

To locate the position of a particle we need a frame of reference. A convenient way to do it, is to take three mutually perpendicular lines intersecting at a point called origin. The three lines are x-axis, y-axis, z-axis i.e. (x,y,z) are taken as the position co-ordinates of the particle.



(C) DEFINITION OF REST

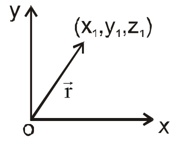
If the position of an object does not change in space with respect to time (relative to an observer), it is said to be at rest.

(D) DEFINITION OF MOTION

If the position of an object in space changes with time (relative to an observer), it is said to be in motion. i.e. If all the three coordinates x,y and z of the particle remain unchanged as time passes, the particle is said to be at rest w.r.t. the frame, otherwise it will be in motion. Motion, therefore is a relative term i.e. it depends on *frame of reference* of observer.

2. BASIC MOTION DEFINING PARAMETERS

1.1 POSITION OF AN OBJECT



POSITION VECTOR

It is a vector from origin to the object which represents the position of object with respect to origin.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

2.2 DISTANCE AND DISPLACEMENT

DEFINITIONS

(A) DISTANCE (Denoted by x or s)

It is the scalar quantity giving *actual length of the path* (irrespective of direction) of motion for moving object.

Dimension $[M^0 L^1 T^0]$, units : SI/MKS $\Rightarrow$ meter(m), cgs $\rightarrow$ centimeter (cm)

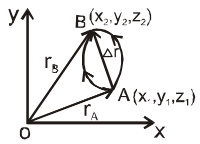
(B) DISPLACEMENT

It is the vector quantity whose magnitude is the *shortest distance between initial and final position of the object* (whatever be the path) and direction is specified by the ray from initial position to final position.

1. It is the change in position vector (Δr) or vector joining initial and final position (r_{AB})

i.e. $\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \vec{r}_B - \vec{r}_A = \vec{r}_{BA}$

$$= (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$$



2. There can be two types of vector notations :

(i) $\Delta\vec{r} = \vec{r}_{AB}$ (where AB means vector from A $\rightarrow$ B)

(ii) $\vec{r}_{BA}$ (here also direction of vector is from A to B but writing B before A means we are giving position of B wrt A, as only r_B will mean position of A wrt origin, we will stick to this)

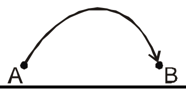
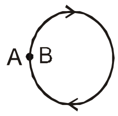
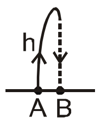
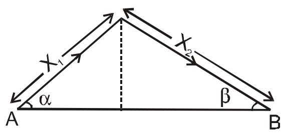
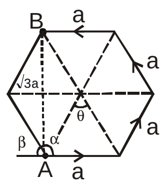
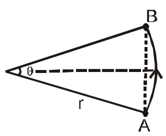
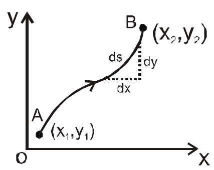
3. Dimension $[M^0 L^1 T^0]$, unit : MKS - meter (m), cgs - centimeter (cm).

4. Displacement of an object remains unchanged by shifting the origin of the position vector. (not every time)

DIFFERENCE BETWEEN DISTANCE AND DISPLACEMENT

Distance	Displacement
(1) It is scalar quantity.	(1) It is vector quantity
(2) It can never be negative.	(2) It can be positive or negative.
(3) For a moving object, it always increases with time.	(3) The magnitude of displacement can decrease with time if object is moving towards initial position.
(4) It is path dependent.	(4) It is path independent.
(5) For two points A and B, it can have many values depending on the path chosen.	(5) For two fixed points A and B, it is single valued.
Ex. *Bus route between two stations twostations gives displacement vector betweenthem*	***Ex.*** *Direct Aeroplane route gives distance between between them.*

ILLUSTRATIONS
(INITIAL POSITION A, FINAL POSITION B, PATH SHOWN BY ARROW)

(i)	A straight path		Distance = | displacement | = x
(ii)	Half circle (radius r)		Distance = ..., | displacement | = ...
(iii)	Full circle (radius r)		Distance = ..., displacement = ...
(iv)	Stone thrown up to height h, back to ground.		Distance = ..., displacement = ...
(v)	Inclined up and down.		Distance = ... |displacement| = ...
(vi)	Regular polygon of n sides, out of which m sides travelled. eg. Hexagon		Distance = ... | displacement | = ... (Use formula $\theta = \beta = \frac{2\pi}{n}$, $\alpha = \pi - \beta = \frac{(n-2)\pi}{n}$)
(vii)	Circular Arc		distance = arc = ... | displacement | = ...
(viii)	Curve y = f(x) $(ds)^2 = (dx)^2 + (dy)^2$ $ds = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$		Distance, S = $\int_{x_1}^{x_2} \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx$ Displacement = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

<u>CONCEPT</u> |Displacement | $\leq$ Distance

<u>MISCONCEPT</u> Modulus of displacement vector gives distance.

<u>CLARIFICATION</u> Displacement equals minimum possible distance and not any distance of any path.

GRAPHICAL INTERPRETATION

(a) Displacement - time graph does not give trajectory
(trajectory = actual path followed by particle which is given by x - y graph)

(b) Whenever we talk of displacement-time graph, it means |displacement |-time graph because vector portion cannot be so easily represented in simple graph. Though, positive and negative signs are used with this magnitude to represent two opposite directions.

2.3 SPEED

DEFINITION

1. It is the distance covered by particle in one sec.
2. It's a scalar quantity.
3. It can never be negative
4. Dimension = $[M^0 L^1 T^{-1}]$, Unit : (MKS) $\Rightarrow$ m/s, (CGS) $\Rightarrow$ cm/sec.

Unit conversion formula $\Rightarrow$ x km/hr = $\left(\frac{5}{18}\right)$ x m/sec.

IMPORTANT NOTE :

To show change in any physical quantity (say distance) we use the symbol Δx. To show changes in time we use the symbol Δt.

First let's define few technical terms :

(a) Independent Variable : Ex. time; doesn't depend on anything

(b) Dependent variable : example displacement (depends on time).

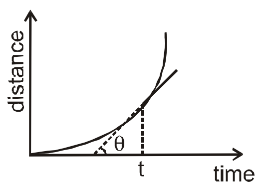
Then we can define Average speed = $\frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

Instantaneous speed, $v = \lim\limits_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

TYPES OF SPEED

(A) <u>INSTANTANEOUS SPEED (V_{INS})</u>

It's the speed of a particle at a particular instant of time or position.

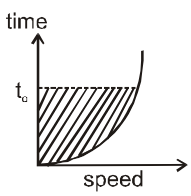


$V_{ins} = \lim\limits_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

$\therefore$ Slope of the tangent at any point in distance time graph gives speed at that point $\therefore v_t = \frac{dx}{dt} = \tan\theta$.

From the above formula, it can also be predicted that

Total distance covered, $x = \int dx = \int v\,dt$.

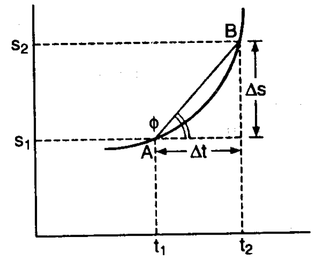


As vdt represents area

$\therefore$ Area under speed time graph over time axis gives distance covered.

(B) AVERAGE SPEED (v_{av}) or (< v >)

If a body covers total distance ΔS over a certain time period Δt, then



$$V_{av} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{\Delta s}{\Delta t}$$

Slope of chord between two different times t_1 (point A) and t_2 (point B) in distance time graph gives average speed.

METHOD OF FINDING AVG. SPEED WHEN MOTION IS BROKEN INTO SEVERAL PARTS

If a particle travels $x_1, x_2, \ldots\ldots x_n$ at speeds $v_1, v_2, \ldots\ldots v_n$ taking time $t_1, t_2 \ldots\ldots t_n$ respectively then $\Delta S = x_1 + x_2 \ldots\ldots + x_n$ or $\Delta S = v_1 t_1 + v_2 t_2 + v_3 t_3 \ldots\ldots + v_n t_n$. (any one will be known)

$\Delta t = t_1 + t_2 + \ldots\ldots t_n$ or $\Delta t = \frac{x_1}{v_1} + \frac{x_2}{v_2} + \ldots\ldots \frac{x_n}{v_n}$.

(any one will be known)

then, substituting above Δs and Δt in equation, $v_{av} = \frac{\Delta s}{\Delta t}$. It will give average speed.

LET'S TAKE THE FOLLOWING POSSIBLE CASES

CASE - I

values of distance and time interval for different parts of motion given individually. WHAT TO DO **:** Simply add all x, all t between two specified positions and find $V_{AV} = \frac{\Delta s}{\Delta t}$.

CASE - II

Velocity is given alongwith either x or t. Find $\Delta s = \sum vt$ and $\Delta t = \sum \frac{x}{v}$

CASE - III

Instantaneous speed and time are given. Find $\Delta s = \int v\,dt$ and then $V_{av} = \frac{\Delta s}{\Delta t}$.

CASE - IV

If $x_1 = x_2 = \ldots\ldots = x_n$ in above case then prove that Average speed is harmonic mean (HM) of individual speed and for n = 2, $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$.

CASE - V

If $t_1 = t_2 \ldots\ldots = t_n$ in above case, then prove that $v_{av} = \frac{\Sigma v_i}{n}$ i.e. Average speed is arithmetic mean (AM) of individual speeds and for n = 2, $v_{av} = \frac{v_1 + v_2}{2}$.

DIFFERENTIATION

General formula : $\frac{dt^n}{dt} = nt^{n-1}$

SOME FORMULAS

1. $\frac{d}{dt}(t^n) = nt^{n-1}$

2. $\frac{d}{dt}(t) = 1$

3. $\frac{d}{dx}(c) = 0$

4. $\frac{d}{dt}(\sin t) = \cos t$

5. $\frac{d}{dt}(\cos t) = -\sin t$

6. $\frac{d}{dt}(e^t) = e^t$

7. $\frac{d}{dt}\ln t = \frac{1}{t}$ $(t^1\ 0)$

8. $\frac{d}{dt}(at+b)^n = na(at+b)^{n-1}$

Rules for differentiation

1. $\frac{d}{dt}(cu) = c\frac{du}{dt}$

2. $\frac{d}{dt}(u \pm v) = \frac{du}{dt} \pm \frac{dv}{dt}$

3. $\frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt}$

Q. Find dx / dt, if x =	YOUR ANSWER	CORRECT ANSWER
1. t^2		
2. t		
3. 1/t		
4. t		
5. 1/ t		
6. $t^{3/2}$		
7. $t^{5/2}$		
8. t^4		
9. t^5		
10. $2t^2$		
11. $\sqrt{2}\, t^2$		
12. 5000		
13. $t^2 + t + 5$		
14. $3t^2 + 2t + 1$		
15. $4t^3 + 4$		
16. t + 1/t		
17. 3t + 2/t		
18. t sint		
19. t^2sint		
20. t sint		

Application in physics:

Average acceleration = slope of secant in v-t graph

Instantaneous acceleration = dv/dt.

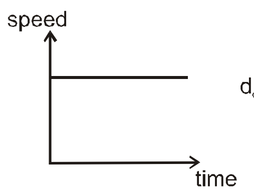
$$v = \frac{dx}{dt},\ a = \frac{dv}{dt};\ F = \frac{dp}{dt};\ i = \frac{dq}{dt};\ ? = \frac{d?}{dt};\ a = \frac{d?}{dt}$$

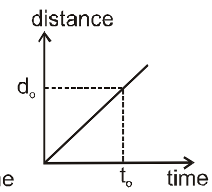
Q. If a body is rotating such that its angle from a fixed location is given by . Find its angular velocity, angular acceleration, and the time at which its angular velocity is zero.

Q. The charge flowing through a conductor beginning with time t = 0 is given by the formula $q = 2t^2 + 3t + 1$ (coulombs) . Find the current i = dq/dt at the end of the 5th second.

UNIFORM SPEED

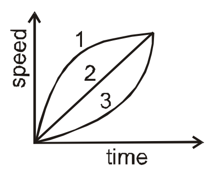
1. Speed, which is not changing with time is called uniform speed.
2. Uniform speed is possible even in 1-D, 2-D or 3-D motion.
3. Slope at every point in x-t graph is same so it's a straight line in x-t graph.





NON-UNIFORM SPEED

1. When speed of particle changes with time, then motion is called non-uniform motion.
2. Curve 1, 2, 3 all represent non-uniform speed.



Ex.1 If $t = \sqrt{x} + 3$ then find the displacement of the particle when its velocity is zero.

INTEGRATION OR ANTI DERIVATIVE

So far we studied as how to find the velocity of particle when its position is given. Now how to do back calculation, i.e. velocity is given, and we have to find the position of the particle. This reverse process is integration.

There are two kinds of integration :

(a) Indefinite integration

(b) Definite integration

General Formula :

$$\int t^n \, dt = \frac{t^{n+1}}{n+1} + c$$

By putting the integration constant we avoid mistake :

	YOUR ANSWER	CORRECT ANSWER
1. $\int dt$	________	________
2. $\int t\,dt$	________	________
3. $\int \frac{1}{t^2} dt$	________	________
4. $\int \sqrt[3]{t^2}\, dt$	________	________
5. $\int (t^2) dt$	________	________
6. $\int (t^{-4}) dt$	________	________
7. $\int (2t) dt$	________	________
8. $\int (2t^{-3}) dt$	________	________
9. $\int \left(\frac{5}{t^2}\right) dt$	________	________
10. $\int \left(-\frac{2}{t^3}\right) dt$	________	________
11. $\int \left(\frac{1}{2t^3}\right) dt$	________	________

2.4 VELOCITY

DEFINITION

1. Displacement in unit time is called velocity.
2. It is vector quantity.
3. Dimension $[M^0 L^1 T^{-1}]$, Unit : (MKS) $\Rightarrow$ m/s, (CGS) $\Rightarrow$ cm/s.

TYPES OF VELOCITY

(A) INSTANTANEOUS VELOCITY

1. It is velocity of a particle at a particular instant.
2. $V_{ins} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt}$

$\therefore$ Slope of the tangent at any point gives value of instantaneous velocity at that point. i.e. $v = \tan\theta$.

EXAMPLE :

The displacement of a particle is given by $\vec{r} = (t^2 + 2t + 1)m$ (where t is the time period). Calculate the velocity of the particle as a function of time.

4. Magnitude of instantaneous velocity is called instantaneous speed i.e. $v = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$ but speed $\neq \frac{d|\vec{r}|}{dt}$.

because $d|\vec{r}|$ is change in magnitude of position vector (and not the magnitude of change in position vector which is $|d\vec{r}|$).

Ex. *In uniform circular motion $d|\vec{r}| = 0$ but speed $\neq 0$.*

5. If direction of velocity changes, then value of | displacement | will be different from value of distance covered.

2.5 ACCELERATION

DEFINITION

1. Rate of change of velocity is called acceleration.
2. It is a vector quantity, dimension $[M^0 L^1 T^{-2}]$, unit $\Rightarrow$ m/s^2.
3. $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
4. If constant force F is acting on mass m, then $\vec{a} = \frac{\vec{F}}{m}$.
5. There is no definite relation between direction of velocity vector and direction of acceleration vector.
6. If only direction of velocity changes, then $\vec{a}$ is perpendicular to $\vec{v}$.
7. **3 ways of change in velocity are**

TYPES OF ACCELERATION

(A) INSTANTANEOUS ACCELERATION

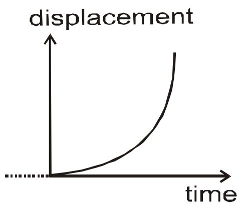
1. Acceleration of a particle at a particular instant is known as instantaneous acceleration.
2. $\vec{a} = \lim_{\Delta t \to 0} \left|\frac{d\vec{v}}{dt}\right| = \frac{d\vec{v}}{dt} = \frac{d^2s}{dt^2}$
3. $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{ds} \cdot \frac{d\vec{s}}{dt} = \frac{\vec{v}\,d\vec{v}}{ds}$

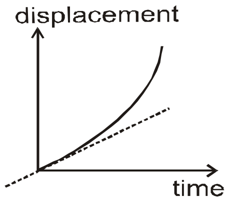
(B) AVERAGE ACCELERATION

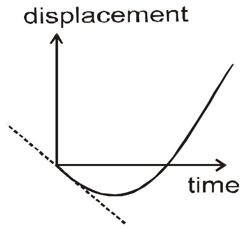
1. If change in velocity is $\Delta\vec{v}$ in time Δt, then $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$.
2. The direction of angular acceleration vector is the direction of the change in velocity vector.

(C) UNIFORM ACCELERATION

1. If the magnitude and direction of acceleration is not changing with time then acceleration is uniform.
   eg. projectile motion.
2. Uniform acceleration does not mean uniform motion because velocity is still changing.
3. Uniform acceleration does not necessarily imply that particle is moving in one direction only. eg. projectile motion.
4. Displacement - time graph for positively accelerated motion is upward opening parabola

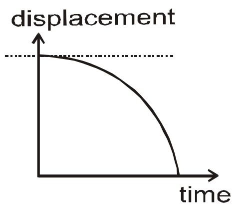


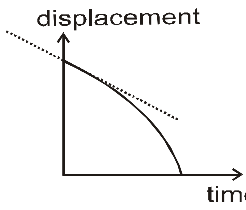


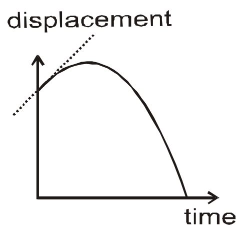


$x_0 = 0$, $v_0 = 0$, $a > 0$ $x_0 = 0$, $v_0 > 0$, $a > 0$ $x_0 = 0$, $v_0 < 0$, $a > 0$

while for negatively accelerated motion, it is downward opening parabola.







$x_0 \neq 0$, $v_0 = 0$, $a < 0$ $x_0 \neq 0$, $v_0 < 0$, $a < 0$ $x_0 \neq 0$, $v_0 > 0$, $a < 0$

(D) <u>NON-UNIFORM ACCELERATION</u>

1. When acceleration (either magnitude or direction or both) changes with time then it's called non-uniform acceleration.

SUMMARY

	speed	**velocity**	**acceleration**
(1) Instant.	$v = \frac{dx}{dt} = \lvert \vec{v} \rvert$	$\vec{v} = \left(\frac{d\vec{r}}{dt}\right)_{\text{at time t}}$	$\vec{a} = \left(\frac{d\vec{v}}{dt}\right)_{\text{at time t}}$
(2) average	$v_{av} = \frac{\text{total distance}}{\text{total time}}$	$\vec{v}_{av} = \frac{\text{total displacement}}{\text{total time}}$	$\vec{a} = \frac{\text{Net change in velocity}}{\text{total time}}$
	$v_{av} \neq \lvert \vec{v}_{av} \rvert$	$= \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$	$= \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$
(3) uniform changing	$\lvert v \rvert$ is not changing but $\vec{v}$ may change with time.	neither direction nor magnitude of velocity is changing so necessarily one - D motion.	acceleration is not but velocity (direction and/or magnitude) is changing. (v is function of first power of t).
(4) non-uniform function	speed is increasing or decreasing with time so acceleration is there.	If any or both of direction and magnitude of velocity is changing then it is non-uniform. So acc. is there.	Rate of change of velocity is changing i.e. (v is of t^n. where n>1)

3. GRAPHICAL INTERPRETATION OF MOTION

3.1 POSITION TIME GRAPH

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

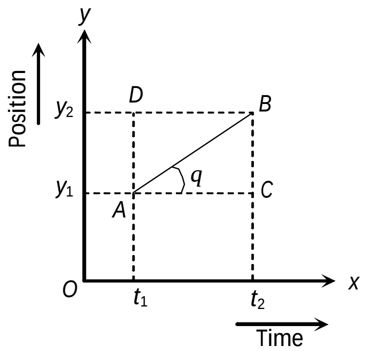
Let AB is a position-time graph for any moving particle

As Velocity $= \dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

From triangle ABC $\tan q = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

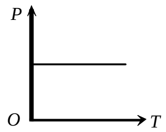
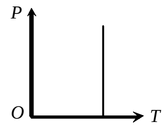
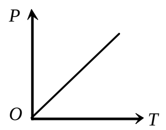
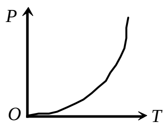
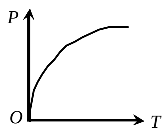
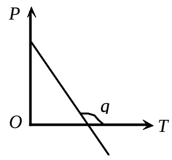
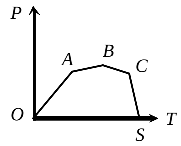
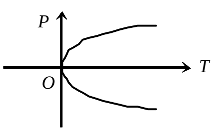
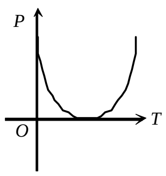
By comparing (i) and (ii) Velocity = $\tan\theta$

$v = \tan\theta$



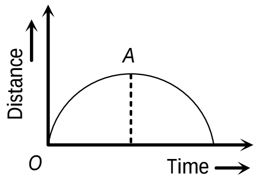
It is clear that slope of position-time graph represents the velocity of the particle.

Various position - time graphs and their interpretation

	$q = 0^o$ so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
 	$q = 90^o$ so $v = \infty$ *i.e.*, line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible. q = constant so v = constant, $a = 0$ *i.e.*, line with constant slope represents uniform velocity of the particle.
 	q is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration. q is decreasing so v is decreasing, a is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	q constant but > 90^o so v will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin

3.2 IMPORTANT POINTS :-

* If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.



* For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{v_1}{v_2} = \frac{\tan\theta_1}{\tan\theta_2}$.

3.3 VELOCITY TIME GRAPH

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

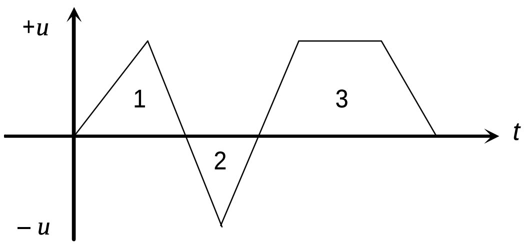
Then Total distance $= |A_1| + |A_2| + |A_3|$

= Addition of modulus of different area. i.e. $s = \int |v|\,dt$

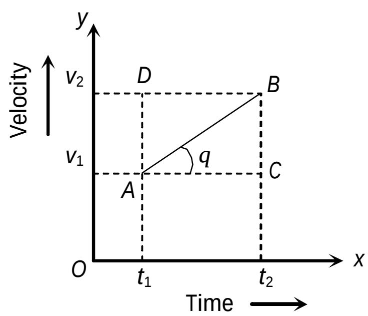
Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. i.e. $r = \int v\,dt$

Here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium.



Acceleration : Let AB is a velocity-time graph for any moving particle



As Acceleration $= \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$ ...(i)

From triangle ABC, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii)

Acceleration (a) $= \tan\theta$

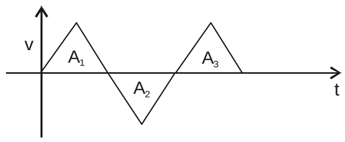
It is clear that slope of velocity-time graph represents the acceleration of the particle.

3. It can also be defined as rate of change of position vector.
4. For the curved path, instantaneous velocity is always tangential to the path followed.
5. Total displacement of particle in time t is $\Delta\vec{r} = \int_0^t \vec{v}\, dt$

$\therefore$ Area under velocity vs time graph and time axis with proper algebraic sign gives displacement, while without sign gives distance.

In graph, distance = $A_1 + A_2 + A_3$

|displacement| = $A_1 - A_2 + A_3$



(B) AVERAGE VELOCITY ($\vec{v}_{av}$ or < $\vec{v}$ >)
1. It is the ratio of displacement with time.
2. $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$
3. Direction of any velocity is same as that of displacement vector.
4. Method of finding avg. velocity when velocity is given as a function of time

$$\vec{v}_{av} = \frac{\int_0^t \vec{v}\, dt}{\int_0^t dt}$$

EXAMPLE :

The velocity of a moving car varies as a function of time and the relation is given by v = 2at + b, where a & b are constants. Calculate the average velocity of the car during time t = 0 to t = t.

(C) UNIFORM VELOCITY
1. When velocity of a particle does not change w.r.t. time, then it's velocity is known as uniform velocity.
2. It is possible only when particle moves along a straight line without reversing it's path.

(D) NON-UNIFORM VELOCITY.
1. When velocity of particle changes w.r.t. time, then it is called non uniform velocity.
2. There can be three reasons for this non-uniformity.
(a) When only magnitude changes - eg. motion of a particle in straight line with constant acceleration.
(b) When only direction changes - eg. uniform circular motion.
(c) When both change - eg. projectile motion, vertical circular motion.

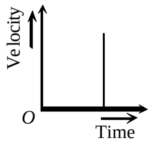
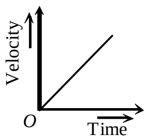
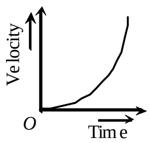
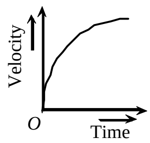
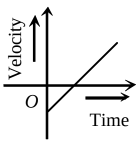
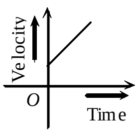
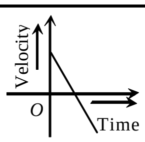
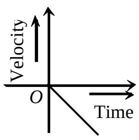
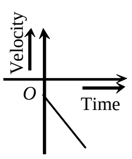
CONCEPTS REGARDING SPEED AND VELOCITY

1. For uniform speed (velocity) motion, average speed (velocity) is equal to instantaneous speed (velocity) but converse may not be true.
i.e. accidentally at one place if $v_{in} = v_{av}$, then it does not necessarily mean that it's a uniform motion.
2. If velocity is constant, then speed is also constant but converse may not be true.

Ex. *Circular motion, where speed is constant but velocity keeps on changing due to change in direction.*

3. All the differences between distance and displacement are applicable between average speed and average velocity.
i.e. $v_{av} \geq |\vec{v}_{av}|$, $v_{av} > 0$ but $\vec{v}_{av} > = < 0$, single valued, path independent.

Various velocity - time graphs and their interpretation

	q = 0, a = 0, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
 	q = 90°, a = ∞, v = increasing *i.e.*, line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible. q =constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	q increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.
  	q decreasing so acceleration decreasing *i.e.* line bending towards time axis represents the decreasing acceleration in the body Positive constant acceleration because q is constant and < 90° but initial velocity of the particle is negative. Positive constant acceleration because q is constant and < 90° but initial velocity of particle is positive.
	Negative constant acceleration because q is constant and > 90° but initial velocity of the particle is positive.
 	Negative constant acceleration because q is constant and > 90° but initial velocity of the particle is zero. Negative constant acceleration because q is constant and > 90° but initial velocity of the particle is negative.

4. EQUATIONS OF KINEMATICS

These are the various relations between u, v, a, t and s for the moving particle where the notations are used as :

u = Initial velocity of the particle at time t = 0 sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

s_n = Distance travelled by the body in nth sec

4.1 When particle moves with zero acceleration (Uniform motion)

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) v = u, s = u t [As a = 0]

4.2 When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar from — Equation of motion in vector from

Scalar form	Vector form
$u = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$u^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})\,t$
$s_n = u + \frac{a}{2}(2n - 1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

4.3 Important points for uniformly accelerated motion

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t sec is proportional to t^2 (i.e. $s \mu\ t^2$).

So we can say that the ratio of distance covered in 1 sec, 2 sec and 3 sec is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in nth sec is proportional to (2n – 1) (i.e. $s_n \propto (2n-1)$)

So we can say that the ratio of distance covered in I sec, II sec and III sec is 1 : 3 : 5.

(iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}$, $s \propto u^2$ [since a is constant]

So we can say that if u becomes n times then s becomes n2 times that of previous value.

(iv) A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$u = \sqrt{\frac{u_1^2 + u_2^2}{2}}$$

4.4 MOTION UNDER GRAVITY

MOTION UNDER GRAVITY

Here we will consider motion of a particle thrown vertically upward (θ = 90º from horizontal) or falling vertically from any height.

<< methods of solving problems on motion under gravity >>

METHOD I

* For stone thrown upward, motion can be divided into two parts :

PART 1 - **stone going upward :** speed decreasing with time due to gravity, which is pulling it downward.

Therefore, this is the case of deceleration, so equations of motion in scalar form will be as $v = u - gt$, $s = ut - 1/2gt^2$, $v^2 = u^2 - 2gh$

PART 2 - **stone falling downward :** speed increasing with time due to gravity, which is pulling it downward. Therefore, this is the case of accelerated motion, so equations of motion in scalar form will be as $v = u + gt$, $s = ut + 1/2\ gt^2$, $v^2 = u^2 + 2gh$

CAUTION : You have to consider upward and downward motion separately.

METHOD II

Here, you can consider complete up and down motion as one motion and applying equations of motion in vector form with proper sign convention will give result with +/- sign, which can be interpreted according to sign convention.

SIGN CONVENTION

(1) Upward positive , downward negative.

(2) Point of projection to be taken as origin. (But not necessary we take any point as origin for simplicity)

(3) All the distances to be measured from point of projection.

$\therefore$ distance above point of projection is positive & vice-versa

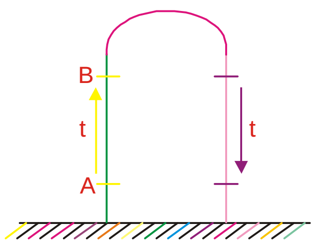
For motion under gravity, acceleration vector due to gravitational force will always be downward, whatever be the direction of velocity, so $\vec{a}_g = -9.8\ m/s^2$ always.

BASIC FIND OUTS OF MUG :

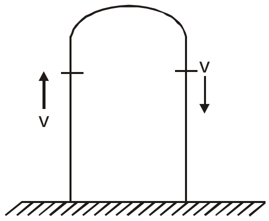
(a) If a particle is projected up with velocity u, then

(i) Maximum height reached by the particle, H = $\dfrac{u^2}{2g}$

(b) A ball thrown vertically up takes the same time to go up and come down and it is true for any part of its motion.



(c) A particle has the same speed at a point on the path while going vertically up and down.



(d) If a particle is dropped from a height H above the ground, then

(i) Velocity of the particle when it reaches the ground i.e. $v = \sqrt{2gH}$

(ii) Time taken to reach the ground i.e. $t = \sqrt{\frac{2H}{g}}$

(e) Whenever a ball is dropped, its initial velocity is equal to the velocity of the body, from where it is being dropped. Just after dropping, acceleration for the ball will be equal to free fall acceleration i.e. gravitational acceleration g.

(f) If we consider constant retarding force due to air resistance, then the ball takes less time to reach the highest position and larger time to reach the ground as compared to that in the absence of air resistance.

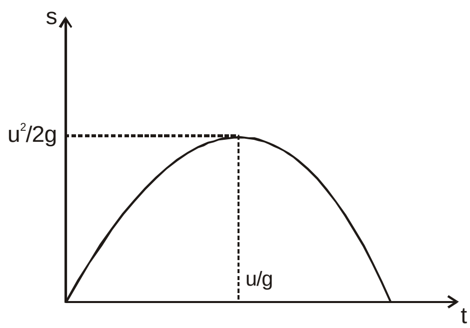
(g) If the body is dropped from a height H, as in time t, it has fallen a distance h from it's initial position, the height of the body from the ground will be h' = H – h, with $h = \frac{1}{2}gt^2$.

(h) As $h = \left(\frac{1}{2}\right)gt^2$, i.e. $h \propto t^2$, distance fallen in time t, 2t, 3t etc. will be in the ratio of $1^2 : 2^2 : 3^2$ .... i.e. square of integers.

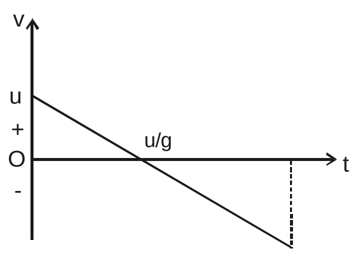
(i) The distance fallen in n^{th} second i.e. $h_n - h_{n-1} = \frac{1}{2}g(n)^2 - \frac{1}{2}g(n-1)^2 = \frac{1}{2}g(2n-1)$. So, distance fallen in I^{st}, II^{nd}, III^{rd} sec. will be in the ratio 1 : 3 : 5 i.e. odd integers only.

(j) Graphs for a body thrown vertically upward :

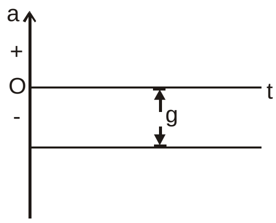
Displacement - time graph :



velocity - time graph :



acceleration - time graph :



Distance travelled in vertical motion = $\left|\frac{u^2}{2a}\right| + \frac{1}{2}\left|a(t - t_o)^2\right|$ for $t > t_o$

4.5 EFFECTS OF MEDIUM ON MOTION UNDER GRAVITY

On a vertically falling body, three forces may act on it at a time

(1) Weight = mg (downward)

(if mass is same for different bodies, then w does not depend on volume V or density ρ).

(2) Thrust force (Th) = mass of medium displaced (upward) by the falling body x g

Th = volume of body x density of medium x g = $V\sigma g$

($\therefore$ Th $\propto \sigma$ i.e. more is the density of medium, more is the thrust force $\therefore$ less is the net acceleration downward).

If $\rho<\sigma$, then thrust force will be greater than the weight of the body and the body will move up eg. Hydrogen balloon. So, net downward acceleration

$$= \frac{\text{Net downward force}}{\text{total mass}} = \frac{W - T_n}{m} = g - \frac{V\sigma g}{V\rho}\ ;\ g' = g\left(1 - \frac{\sigma}{\rho}\right)$$

(Where, $m = V\rho$ & $w = V\rho g$), But here $\sigma > \rho$, so g' is –ve.

(3) Viscous force = $F_v = 6\pi\eta rv$ (upward)

(F_v acts in the direction opposite to motion, so it can act in both upward or downward directions, but in the usual case, in which body is falling downward, F_v is upward)

$6\pi\eta$ are constant, r=radius of body, v is instantaneous velocity of body.

F_v depends on velocity, ie. more v, more F_v which opposes v, so v goes an decreasing

$\therefore$ Fv goes on decreasing

$\therefore$ at an instant, when F_v = w, no net force acts on body, so it falls thereafter with uniform velocity called terminal velocity.

i.e. $mg = 6\pi\eta rv_t$ or $\frac{4}{3}\pi r^3\rho g = 6\pi\eta rv_t$ so $v_t \propto r^2$.

$\therefore$ net downward acc. = $\frac{\text{Net downward force}}{\text{mass}} = \frac{W - F_v}{\text{mass}} = \frac{mg - 6\pi\eta rv}{m}$;

$$g'' = g - \frac{6\pi\eta rv}{m}$$

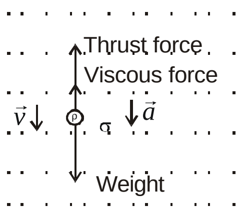
(m can be replaced by $\Rightarrow m = V\rho = \frac{4}{3}\pi r^3\rho$)

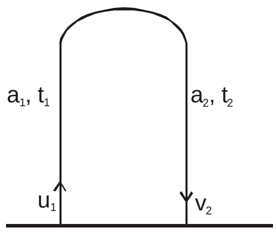
So, here downward acceleration comes out to be a function of v. So it's a case of non uniform acceleration.

If thrust force is also acting along with viscous force then g in above eqn. will be replaced by g' i.e. $g'' = \left(g' - \frac{6\pi\eta rv}{m}\right) = \left(g\left(1 - \frac{\sigma}{\rho}\right) - \frac{6\pi\eta rv}{m}\right)$

CONCEPT If air resistance opposes motion under gravity, then

$a_1 > a_2$, $t_1 < t_2$, $v_2 < u_1$





MOTION OF PARTICLE PROJECTED UPWARD UNDER TWO CONDITIONS

S.N.	motion describing parameters	without air resistance	with air resistance
(1)	acceleration during upward motion	a_1 = g (downward)	a_1' = (g+a) downward
(2)	acceleration during downward motion	a_2 = g (downward)	a_2' = (g-a) downward
(3)	maximum height attained	$H = \frac{u_1^2}{2g}$	$H' = \frac{u_1^2}{2(g+a)}$ $\therefore H'_{max} < H_{max}$
(4)	time to reach H_{max} from ground	$t_1 = \frac{u_1}{g} = \sqrt{\frac{2H}{g}}$	$t_1' = \frac{u_1}{g+a} = \sqrt{\frac{2H'}{g+a}}$ $\therefore t_1' < t_1$
(5)	time to fall to ground from H_{max}	$t_2 = t_1 \left(= \frac{u_1}{g}\right) = \sqrt{\frac{2H}{g}}$	$t_2' = \sqrt{\frac{2H'}{(g-a)}}$ $\therefore t_2' > t_1'$ & $t_2' > t_2$ time to fall > time to rise
(6)	speed with which body falls on ground.	$v_2 = \sqrt{2gH} = u_1$ same as speed with which particle was thrown up	$v_2' = \sqrt{2(g-a)H'}$ $u_1 = \sqrt{2(g+a)H'}$ $\therefore v_2' < u_1$ & $v_2' < v_2$

3.6 MOTION WITH VARIABLE ACCELERATION(REVISITED).

(i) If acceleration is a function of time

a = f(t) then $v = u + \int_0^t f(t)\,dt$ and $s = ut + \int\left(\int f(t)\,dt\right)dt$

(ii) If acceleration is a function of distance

a = f(x) then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)\,dx$

(iii) If acceleration is a function of velocity

a = f(v) then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{v\,dv}{f(v)}$

5. RELATIVE MOTION

5.1 Introduction :

Motion is always a relative term. The motion, so far discussed was relative to a stationary origin. If the reference is now changed to a body, which may / may not be moving, then the motion is termed as *Relative motion*.

We can interpret 4 types of equations for relative motion.

5.2 ORIGIN SHIFTING

Until now, the reference point was a stationary point i.e. origin O. So, for any moving body B, motion defining parameters were

$r, v, a \equiv r_B, v_B, a_B \equiv r_{BO}, v_{BO}, a_{BO}$

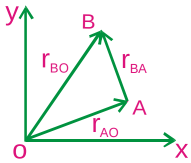
Now, if we change that stationary reference point O with a body A, which is having it's own position, velocity and acceleration wrt origin O, then the new parameters describing motion of B wrt A will be r_{BA}, v_{BA}, a_{BA}.

In vector form

from triangle law of vector addition

$\vec r_{AO} + \vec r_{BA} = \vec r_{BO}$

$\therefore \quad \vec r_{BA} = \vec r_{BO} - \vec r_{AO} = \vec r_B - \vec r_A$



and $\frac{d}{dt}\vec r_{BA} = \frac{d}{dt}(\vec r_B) - \frac{d}{dt}(\vec r_A)$ $\quad\therefore\quad \vec v_{BA} = \vec v_B - \vec v_A$

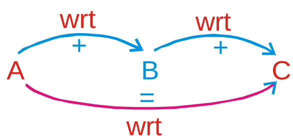
Similarly, $\vec a_{BA} = \vec a_B - \vec a_A$

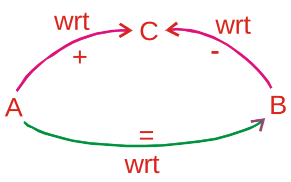
The above treatment can be seen as shifting origin to A and attributing/transferring the velocity and acceleration of A to B in such a manner that A seems to be stationary for B.

5.3 RELATION BETWEEN TWO RELATIVE PARAMETERS.

If v_{AB} is relative velocity of A wrt B and v_{BC} is relative velocity of B wrt C, then

$v_{AB} + v_{BC} = v_{AC}$





or $v_{AC} - v_{BC} = v_{AB}$

where v_{AC} is relative velocity of A wrt C.

(3) NO-NEW FORMULA INTERPRETATION

Equations of motion are equally applicable as

$v_{rel} = u_{rel} + a_{rel}\,t$ or $\vec v_{BA} = \vec u_{BA} + \vec a_{BA}\,t$

$s_{rel} = u_{rel}\,t + \frac{1}{2}a_{rel}\,t^2$ or $\vec s_{BA} = \vec s_{oBA} + \vec u_{BA}\,t + \frac{1}{2}\vec a_{BA}\,t^2$

$v^2_{rel} = u^2_{rel} + 2a_{rel}\,s_{rel}$ or $\vec v^2_{BA} = u^2_{BA} + 2\,\vec a_{BA}\,\vec s_{BA}$

(4) LAW OF INDEPENDENCE OF DIRECTION (RELATIVE MOTION IN TWO DIMENSIONS)

Here also, we can convert one 2-D motion into two 1-D motions. i.e. first divide the motion in two perpendicular directions and then apply relative equations i.e.

$v_{x\,rel} = u_{x\,rel} + a_{x\,rel}\,t$ Similarly, for y direction $v_{y\,rel} = u_{y\,rel} + a_{y\,rel}\,t$

$s_{x\,rel} = u_{xrel}t + \frac{1}{2}a_{x\,rel}\,t^2$ Similarly, for y direction $s_{y\,rel} = u_{yrel}t + \frac{1}{2}a_{y\,rel}\,t^2$

$(v_{xrel})^2 = (u_{xrel})^2 + 2\,a_{x\,rel}\,s_{x\,rel}$ Similarly, for y direction $(v_{yrel})^2 = (u_{yrel})^2 + 2\,a_{y\,rel}\,s_{y\,rel}$

5.4 RELATIVE MOTION IN ONE DIMENSION

Method of solving problems on relative motion

(i) We should adopt a sign convention in the beginning, usually - ⟵⟶ +

(ii) Condition of collision or condition of meeting together

(a) At the time of collision, coordinates of both particles should be same.

i.e. $x_1 = x_2$, and $y_1 = y_2$ (for a 2-D motion)

Similarly, $x_1 = x_2$, $y_1 = y_2$ and $z_1 = z_2$ (for a 3-D motion)

(b) Two particles collide at the same moment. Of course, their time of journeys may be different i.e. they may start at different times (t_1 and t_2 may be different). If they start together, then $t_1 = t_2$.

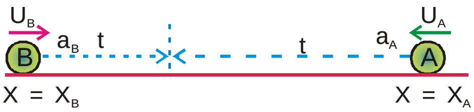
TYPES OF RELATIVE MOTION IN 1-D

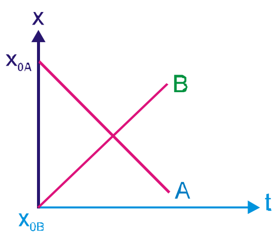
1. When both bodies are moving in opposite direction
2. When both bodies are moving in same direction, both $\vec{v}$ and $\vec{a}$ are in + x direction

1. WHEN BOTH BODIES ARE MOVING IN OPPOSITE DIRECTION

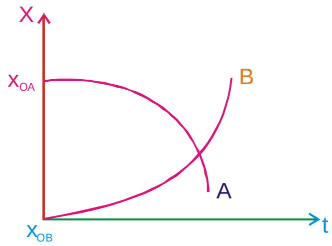
CASE 1 : Two bodies coming towards each other.

(Direction of velocity and acceleration of both bodies are opposite to each other)





$a_A = a_B = 0$



$\vec{u}_{BA} = \vec{u}_B - \vec{u}_A = u_B - (-u_A) = u_B + u_A$ (sum of individual speeds) (in + x direction)

$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = -(u_A + u_B)$ (same magnitude but opp. direction) (in - x direction)

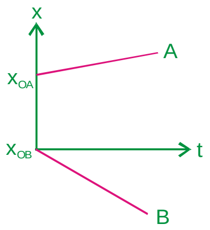
Here collision is sure.

Time of collision can be obtained by solving following equation for t

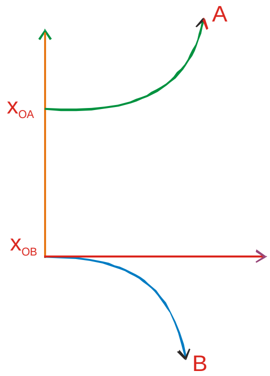
Position of B at the time of collision = position of A at the time of collision

$x_B + u_Bt + \frac{1}{2}a_Bt^2 = x_A - u_At - \frac{1}{2}a_At^2$. (in scalar form)

CASE 2 : Two bodies moving away from each other.



$(a_A = a_B = 0)$

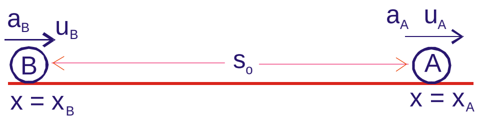


$\vec{u}_{BA} = \vec{u}_B - \vec{u}_A = -u_B - u_A = -(u_B + u_A)$ [sum of individual speeds, in - x direction]

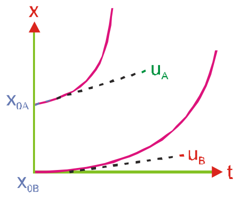
$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = u_A - (-u_B) = u_A + u_B$ [same magnitude, opp. direction, in + x direction] - collision is not possible.

Summary : When two bodies move in opp. direction to each other then magnitude of relative velocity is the sum of individual speeds.

2. WHEN BOTH BODIES ARE MOVING IN SAME DIRECTION, BOTH $\vec{v}$ AND $\vec{a}$ ARE IN + X DIRECTION



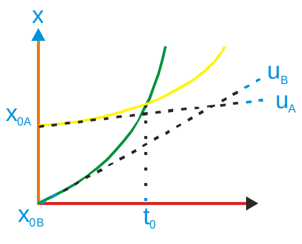
CASE I : $u_A > u_B$ and $a_A > a_B$ then B will never meet A
displacement between them goes on increasing.



$s_{AB} = s_{oAB} + u_{AB}t + \frac{1}{2}a_{AB}t^2 = s_o + (u_A - u_B)\,t + \frac{1}{2}(a_A - a_B)\,t^2$

surely no collision/meeting possible.

CASE II : $u_A < u_B$ and $a_A < a_B$ then collision is sure if $s_{initial} > o$



collision here means $S_{final,rel} = 0$

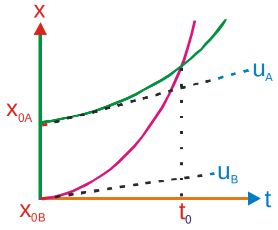
(**Note :** If $S_{initial} = 0$ at $t = 0$, it is the only time when both are together, thereafter B will always be ahead of A.)

Time, at which collision will occur will be obtained by roots of the eqn.

From t, we can calculate v_1 and v_2 at that instant t.

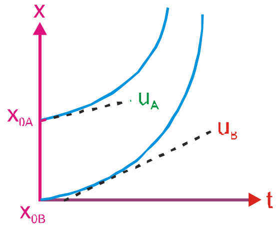
CASE III : $u_A > u_B$, $a_A < a_B$

In this case also, collision is sure because B is gaining velocity.



CASE IV : $u_A < u_B$, $a_A > a_B$

This is again an indefinite case. Here collision depends on the value of $S_{initial}$ because though the initial velocity of A is less than that of B but A is gaining velocity.



Minimum initial distance d_{min} to avoid collision.

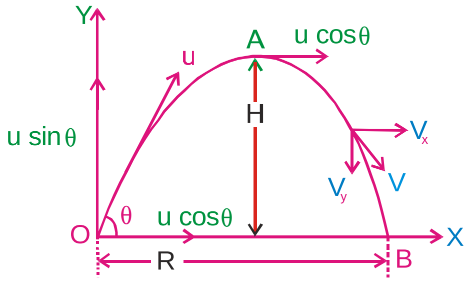
Distance between them initially decreases with time until $v_A < v_B$ but as v_A is increasing at a faster rate, so as soon as v_A becomes greater than v_B then further on distance between them goes on increasing. So, if initially, they are at a separation of d_{min} then at the single instant of meeting together, their speeds will be same & thereafter distance between then will go on increasing.

IMPORTANT POINTS:

Some try to solve the above problem by finding and summing the distances flown by the bird each time it moves from one train to the other. This makes a relatively easy problem quite difficult. It is important to develop a thoughtful, systematic approach to solving problems. Begin by writing an equation for the unknown quantity in terms of other quantities. Then, process by determining the values for each of the other quantities in the equation.

1. PROJECTILE MOTION ON HORIZONTAL PLANE

HORIZONTAL PROJECTILE MOTION



INITIALLY GIVEN PARAMETERS

POINT OF PROJECTION : The point O from where projectile is thrown.

ANGLE OF PROJECTION (θ) : The angle from horizontal, towards which projectile is thrown.

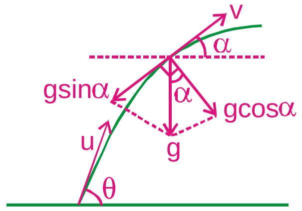
INITIAL SPEED OF PROJECTION (u) : The speed with which projectile is thrown.

The projectile motion near the surface of earth consists of superposition of two simultaneous independent motions:

1. Horizontal motion at *constant* horizontal *speed* u cos$q$ and horizontal acceleration = 0 as no horizontal force is present.
2. Vertical motion with varying vertical speed and *constant acceleration* due to gravity=-g due to downward gravity force.
3. Thus, here also we can apply law of independence of direction and solve the problems by dividing one 2-D motion into two 1-D motions.

	GENERAL EQUATIONS	HORIZONTAL MOTION	VERTICAL MOTION
(1)	component of ini. speed	u_x = u cosθ	u_y = u sinθ
(2)	acceleration	a_x = 0	a_y = - g
(3)	velocity at any instant	$v_x = u_x$ = u cosθ	at any time t, v_y = u sinθ - gt at any height y $v_y^2 = u^2 \sin^2\theta - 2gy$
(4)	position at any instant	x = u cosθ t	$y = u \sin\theta\, t - \frac{1}{2}gt^2$

(2) POSITION OF PARTICLE AT ANY MOMENT = $(x,y) = (u \cos\theta\ t,\ u \sin\theta\, t - \frac{1}{2}gt^2)$



NORMAL AND TANGENTIAL ACCELERATION AT ANY MOMENT

'g' can be split into two parts, "g cos α" normally inward to the curve and "g sin α" tangentially opp. to the direction of motion, where α is the angle tangent makes with horizontal at any instant (note that α is not the angle of projection θ which is a constant entity, α decreases with ascent of projectile).

RESULTANT VELOCITY OF PARTICLE V AT ANY MOMENT

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2} = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

$\therefore$ Resultant velocity goes on decreasing during the first half of motion and becomes minimum at max. height.

$v_{min} = u_x = u\cos\theta$ at max. height.

at an angle α from horizontal i.e. $\tan\alpha = \frac{v_y}{v_x} = (u\sin\theta - gt)/u\cos\theta$

CHANGE IN MOMENTUM (during complete flight)

Initial velocity $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

Change in velocity for complete motion, $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -2u\sin\theta\,\hat{j}$

Change in momentum for complete motion, $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{u}_f - \vec{u}_i)$

$\Delta\vec{p} = m(-2u\sin\theta)\,\hat{j} = -2mu\sin\theta\,\hat{j} = mgT(-\hat{j})$

or $\Delta\vec{p} = -mgT\,\hat{j}$

KINETIC ENERGY AND POTENTIAL ENERGY AT ANY INSTANT T

$KE = 1/2\, mv^2 = 1/2\, m\,[u^2\cos^2\theta + (u\sin\theta - gt)^2]$

Velocity is minimum at highest point, which equals to horizontal component $u\cos\theta$, so

$KE(min) = 1/2\, m\,(u\cos\theta)^2 = 1/2\, m u^2\cos^2\theta = KE_O\cos^2\theta$

Where, KE_O is initial kinetic energy = $1/2 mu^2$

Potential energy at any instant = $mg\,y = mg\,(u\sin\theta t - \frac{1}{2}gt^2)$

$$= mg\,x\tan\theta\left(1 - \frac{x}{R}\right)$$

Potential energy will be max at highest point and equal to

$$(PE)_H = mgH = mg\,\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta$$

so, $(PE)_H + (KE)_H = \frac{1}{2}mu^2(\sin^2\theta + \cos^2\theta) = \frac{1}{2}mu^2$

Which is the ME at the point of projection. So, in projectile motion mechanical energy is conserved.
Furthermore,

$$\left(\frac{PE}{KE}\right)_H = \frac{(1/2)mu^2\sin^2\theta}{(1/2)mu^2\cos^2\theta} = \tan^2\theta$$

So if $\theta = 45^\circ$, PE = KE = $\left(\frac{1}{2}\right)$ ME (at top most point).

i.e., if a body is projected at an angle of 45° to the horizontal then at the highest point, half of its mechanical energy is kinetic and half potential.

RADIUS OF CURVATURE AT ANY POINT ON THE PATH OF A PROJECTILE

Consider a particle moving along any curve (may be parabola or circle or any other). At any instant t, let its velocity vector v is making an angle α with the horizontal. We choose tangential axis and normal axis as shown in fig.

Centripetal acceleration of particle is directed towards normal axis. Component of g towards normal axis provides centripetal acceleration.

As $a_c = \frac{v^2}{R_c}$ (where a_c = centripetal acceleration , R_c = radius of curvature).

$$\therefore R_c = \frac{(\text{instantaneous velocity})^2}{\text{centripetal acceleration}}$$

(i) Radius of curvature in terms of t :

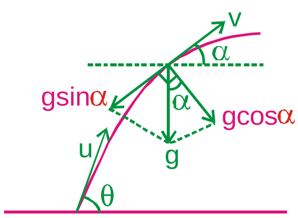
Radius of curvature $R_c = \frac{v^2}{a_c} = \frac{v^2}{g\cos\alpha}$

As $v^2 = v_x^2 + v_y^2 = (u\cos\theta)^2 + (u\sin\theta - gt)^2 = u^2 + g^2t^2 - 2ugt\sin\theta$

and $\tan\alpha = \frac{v_y}{v_x} = \frac{(u\sin\theta - gt)}{u\cos\theta}$

$$\therefore R_c = \frac{u^2 + g^2t^2 - 2ugt\sin\theta}{g\cos\alpha}$$

where, $\alpha = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$



(ii) Radius of curvature in terms of y

v and α can be calculated in terms of θ and y

$v_y^2 = (u\sin\theta)^2 - 2gy$

$v_x = u\cos\theta$

$\therefore$ $v^2 = u^2\cos^2\theta + u^2\sin^2\theta - 2gy = u^2 - 2gy$

$\tan\alpha = \frac{v_y}{v_x} = \frac{\sqrt{(u\sin\theta)^2 - 2gy}}{u\cos\theta}$

$$\therefore R_c = \frac{u^2 - 2gy}{g\cos\alpha}$$

where $\alpha = \tan^{-1}\left(\frac{\sqrt{(u\sin\theta)^2 - 2gy}}{u\cos\theta}\right)$

Equation of Radius of curvature in general = $R_c = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

MEMORY POINTS PROJECTILE MOTION

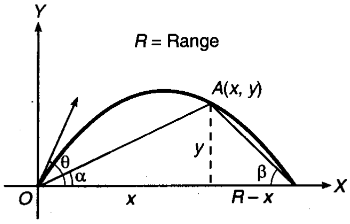
(1) Range is same for two angles, θ and $(90 - \theta)$.

(2) Sum of heights at angles θ and $(90 - \theta)$ is independent of θ, it is $H_\theta + H_{90-\theta} = \frac{U^2}{2g}$ and $\frac{H_\theta}{H_{90^\circ-\theta}} = \tan^2\theta$.

(3) $H = \frac{1}{4} R \tan\theta$ or $R = 4H \cot\theta$. If range is n times of H, then $\tan\theta = \frac{4}{n}$.

(for $\theta = 45^\circ$, R is 4 times H)

(4) $\tan\theta = \tan\alpha + \tan\beta$. If B is the highest point, then $\alpha = \beta$ and then $\tan\alpha_H = \frac{1}{2}\tan\theta$.



(5) The greatest height to which man can throw a stone is H. The greatest distance upto which he can throw the stone is 2H.

$\therefore H = \frac{1}{8} gT^2$ (H = max. height, T = time of flight)

2. PROJECTION FROM A MOVING BODY

Introduction

(If a particle is projected from some moving body then particle gains the instantaneous velocity only of the moving body and not it's acceleration).

projectile motion from a moving body

Consider a boy who throws a ball from a moving trolley. Let the velocity of ball relative to boy is u.

$$\vec{V}_{ball,trolley} = \vec{V}_{ball} - \vec{V}_{trolley}$$

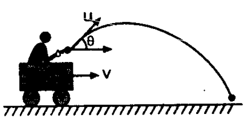
$$\vec{V}_{ball} = \vec{V}_{ball,trolley} + \vec{V}_{trolley}$$

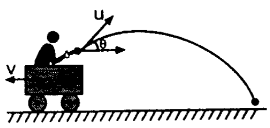
Above equation shows that absolute velocity of ball is vector sum of its velocity with respect to trolley and velocity of trolley. Apply this equation to horizontal as well as vertical motion of the ball. Now consider following cases:

CASE (I) **:** Ball is projected in direction of motion of trolley

Horizontal component of ball's velocity = $u\cos\theta + v$

Vertical component of ball's velocity = $u\sin\theta$





Horizontal component = $u\cos\theta + v$ Horizontal component = $u\cos\theta - v$

Vertical component = $u\sin\theta$ Vertical component = $u\sin\theta$

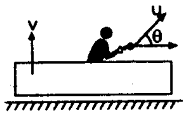
CASE (II) **:** Ball is projected opposite to direction of motion of trolley

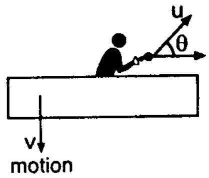
Horizontal component of ball's velocity = u cosθ - v

Vertical component of ball's velocity = u sinθ

CASE (III) : Similarly, for a ball projected upwards from an upward moving platform, horizontal component of ball's velocity = u cosθ

Vertical component of ball's velocity = u sinθ + v





Horizontal component = u cos θ
Vertical component = u sin θ + v

Horizontal component = u cos θ
Vertical component = u sin θ- v

CASE (IV) : For a downward moving platform

Horizontal component of ball's velocity = u cosθ

Vertical component of ball's velocity = u sinθ- v

CONCEPT 1

Initial projection angle so that particle passes through a given point P (x,y): From the equation of trajectory,

$$y = x \tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \quad ....(13)$$

$$y = x \tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$

or $$\tan^2\alpha - \frac{2u^2}{gx}\tan\alpha + \frac{2u^2y}{gx^2} + 1 = 0$$

If point P is within the range of projectile then roots of above equation must be real. Complex roots imply that point P is out of range. By method of completing the square,

Putting discriminant of the above equation = 0 ....(14)

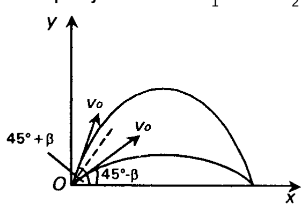
we obtain :

$\tan\alpha = \frac{u^2}{gx}$ (where u^2 / gx is the root of the equation) ....(15)

which is the required angle such that the trajectory just reaches point P on the envelope of possible trajectories for a given u.

For complementary angles of projection if T_1 and T_2 are the respective times of flight, then

$$T_1T_2 = \frac{2R}{g}$$



CONCEPT - 2

COLLISION OF PROJECTILE WITH A WALL.

ASSUMPTION **:** collision is elastic, wall is erect and smooth.

FORMULA USED : speed of approach = speed of separation

Speed of wall does not change after collision too.

NOTATION **:** v_{xb} = horizontal component of instantaneous speed of projectile just before collision

v_{xa} = horizontal component of instantaneous speed of projectile just after collision in opposite direction.

v_w = instantaneous velocity of wall at the time of collision. Let us take + v_w if wall is coming towards projectile before collision (to avoid complexity, we are not using vector notations here, just applying common sense of collision).

x_1 = horizontal distance covered by projectile before collision.

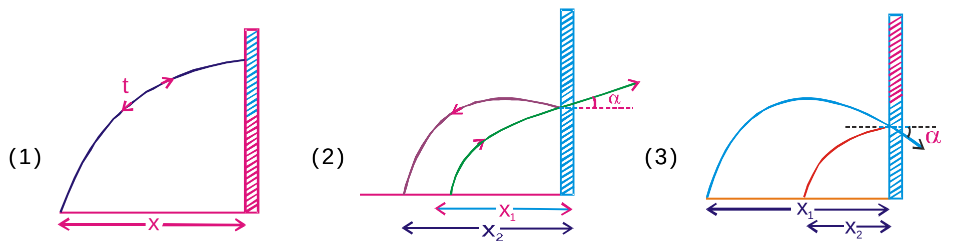
x_2 = horizontal distance covered by projectile after collision.

Range = R if wall/collision were absent.

CASE I Wall is stationary

$v_{xa} = v_{xb}$ so $x_1 + x_2$ = Range (if there were no wall).

i.e. ball rebounds with same horizontal speed.



R = 2x	R = $x_1 + x_2$	R = $x_1 + x_2$
T = 2t	T = $t_1 + t_2$	T = $t_1 + t_2$
ball was at topmost point at the time of collision (α = 0)	ball was moving up at the time of collision (α = + ve)	ball was moving downward at the time of collision (α = - ve)

CASE II Wall is moving with uniform speed v_w towards projectile

speed of approach = speed of separation

$v_{xb} + v_w = v_{xa} - v_w$

$\therefore\ v_{xa} = 2v_w + v_{xb}$

$\therefore$ horizontal component of projectile velocity increases by twice the speed of wall

$\therefore$ $x_1 + x_2$ > Range

CASE III Wall is moving with uniform speed v_w (away from projectile)

speed of approach = speed of separation

$v_{xb} - v_w = v_{xa} + v_w$

$\therefore\ v_{xa} = v_{xb} - 2v_w$

CASE IV Wall is accelerating towards projectile

$\Rightarrow$ same treatment as case II, here we need to find out speed of wall at the time of collision to be used as v_w in formula $v_{xb} + v_w = v_{xa} - v_w$

Hence, on elastic collision from vertical smooth wall (whether at rest or uniform or non-uniform motion, only horizontal component of velocity changes. There is no change in vertical component, so time of flight and maximum height attained remain unchanged.

3. PROJECTION FROM A HEIGHT

HORIZONTAL PROJECTION (PROJECTILE THROWN PARALLEL TO THE HORIZONTAL)

$u_x = u$

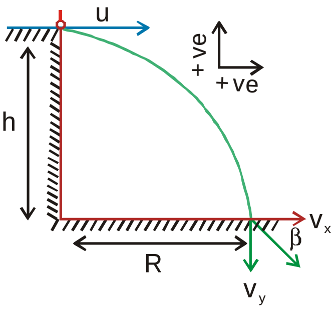
$u_y = 0$

$a_y = -g$

Horizontal motion $x = ut$ .... (1)

Vertical motion $-h = 0\,(t) - \frac{1}{2}gt^2$ .... (2)

From Eqn. (1) and (2)



$\Rightarrow$ $t = \sqrt{\frac{2h}{g}}$

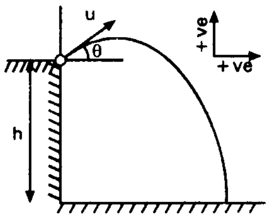
Horizontal range (R) $= u_x t = u\sqrt{\frac{2h}{g}}$

PROJECTION AT AN ANGLE qABOVE HORIZONTAL

$u_x = u\cos\theta$

$u_y = u\sin\theta$

$a_y = -g$



Eqn. of horizontal motion :

$x = u\cos\theta\, t$ .... (1)

Eqn. of vertical motion :

$-h = u\sin\theta\, t - \frac{1}{2}gt^2$ .... (2)

From Eqn. (1) and (2),

$gt^2 - 2u\sin\theta t - 2h = 0$

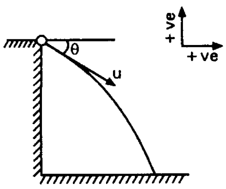
or $t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$

PROJECTION AT AN ANGLE θ BELOW HORIZONTAL

$u_x = u\cos\theta$

$u_y = -u\sin\theta$

$a_y = -g$



Similarly, for projection at an angle θ downwards with horizontal, the eqns. are

Eqn. of horizontal motion :

$x = u\cos\theta\, t$ ....(1)

Eqn. of vertical motion :

$$-h = -u \sin\theta\, t - \frac{1}{2} g t^2 \qquad ....(2)$$

from equation (2),

or $\qquad gt^2 + 2u \sin\theta\, t - 2h = 0$

or $\qquad t = \dfrac{-2u\sin\theta \pm \sqrt{4u^2 \sin^2\theta + 8gh}}{2g}$ Neglect -ve root of t, as

negative value of t has no meaning.

NOTE : In all the above three cases, we can calculate the velocity of projectile at the instant of striking the ground by using

$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan\theta = \frac{V_y}{V_x}$$ [angle at which projectile strikes ground]

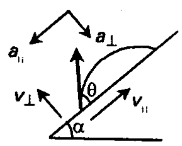
4. PROJECTILE ON AN INCLINED PLANE

Now, we are considering the motion of a projectile on an inclined plane which makes an angle α with the horizontal.

Projectile makes angle θ with the inclined plane.

The various parameters can be represented as :

TIME OF FLIGHT



Here $\qquad v_\perp = v_o \sin\theta$

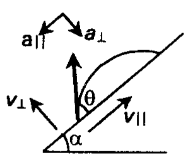
and $\qquad a_\perp = g\cos\alpha$

Thus, $\qquad T = \dfrac{2v_\perp}{a_\perp} = \dfrac{2v_o \sin\theta}{g\cos\alpha}$

RANGE ALONG THE INCLINED PLANE

The range of the projectile along the inclined plane is given by

$$R' = v_{||}T - \frac{1}{2} a_{||} T^2$$



Since $\qquad T = \dfrac{2v_\perp}{a_\perp} = \dfrac{2v_o \sin\theta}{g\cos\alpha}$

On solving, we get :

$\therefore \qquad R' = \dfrac{2v_o^2}{g}\,\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$ [Putting $v_{11} = v_0 \cos\theta$ and $a_{11} = g \sin\alpha$]

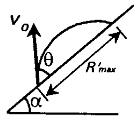
IMPORTANT POINTS

(a) The maximum range occurs when

$$\theta = \frac{\pi}{4} - \frac{\alpha}{2}$$

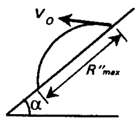
(b) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R'_{max} = \frac{v_o^2}{g(1+\sin\alpha)}$$



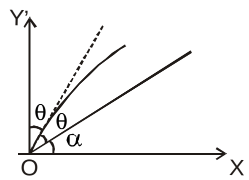
(c) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R''_{max} = \frac{v_o^2}{g(1-\sin\alpha)}$$



(d) For maximum range in inclined projectile, the direction of projection bisects the angle that the inclined plane makes with vertical direction to ground (OY')

i.e. $\frac{\pi}{2} - (\theta+\alpha) = \theta$.



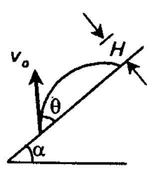
MAXIMUM HEIGHT OF THE PROJECTILE

If a projectile is thrown up an inclined plane, as shown in the **fig.** maximum height attained is given by

$$H = \frac{v_\perp^2}{2a_\perp} \Rightarrow \qquad H = \frac{(v_o\sin\theta)^2}{2g\cos\alpha} = \frac{v_o^2\sin^2\theta}{2g\cos\alpha}$$

For projectile up the inclined plane, we can use these formulae which are applicable to 'same-level-horizontal-projectile', also

$$T = \frac{2V_\perp}{a_\perp}\ ;\ R' = V_{11}T - \frac{1}{2}a_{11}T^2;\ H = \frac{V_\perp^2}{2a_\perp}$$



maximum range occurs at $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$ i.e. (θ < 45°)

$$R'_{max}\text{ (up the plane)} = \frac{v_0^2}{g(1+\sin\alpha)},\ R'_{max}\text{ (down the plane)} = \frac{v_0^2}{g(1-\sin\alpha)}\ ;\ H = \frac{v_0^2\sin^2\theta}{2g\cos\alpha}$$

NOTE : The angle of projectile θ is measured from inclined plane, not ground.

PROJECTILE WITH VARIABLE ACCELERATION

Suppose a projectile moves in the two dimensional plane with velocity $v = a\hat{i} + bx\hat{j}$ where a and b are constant. Initially, consider the particle to be situated at origin i.e. at x = 0 & y = 0. Now, let us first find out the equation of trajectory of the projectile. So.

$v = a\hat{i} + bx\hat{j}$

$\therefore$ $v_x = a$ and $v_y = bx$

But $v = \frac{dr}{dt}$

i.e. $\frac{dx}{dt} = a$ and $\frac{dy}{dt} = bx$

$x = at$ and $dy = bxdt$

On substituting value of x we have

$dy = b.\,at\,dt$

On integrating

$$y = \frac{abt^2}{2}$$

$$y = \frac{ab}{2}\left(\frac{x}{a}\right)^2$$

$$y = \frac{bx^2}{2a}$$

This is the equation of trajectory of the projectile.

Now, radius of curvature of trajectory is given by,

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Since $y = \frac{b}{2a}.x^2$; $\frac{dy}{dx} = \frac{b}{2a}.2x = \frac{bx}{a}$

$$\frac{d^2y}{dx^2} = \frac{b}{a}$$

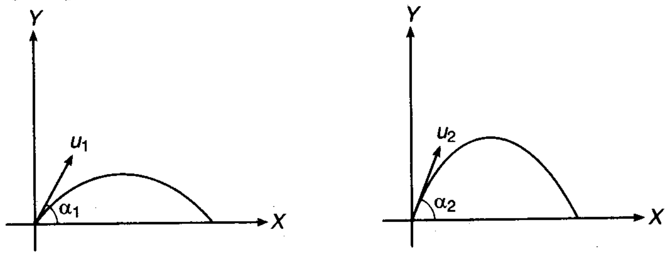
$$\therefore \quad R = \frac{\left[1+\left(\frac{bx}{a}\right)^2\right]^{3/2}}{\frac{b}{a}} = \frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]^{3/2}$$

5. RELATIVE MOTION

Relative motion Between two projectiles :

Let us now discuss the relative motion between two projectiles or the path of one projectile observed by the other. Suppose that two particles are projected from the ground with speed u_1 and u_2 at angles α_1 and α_2 as shown in fig. Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

$a_{12} = a_1 - a_2 = g - g = \text{zero}.$



i.e., the relative motion between the two particles is uniform. Now,

$u_{1x} = u_1 \cos\alpha_1, \quad u_{2x} = u_2 \cos\alpha_2$

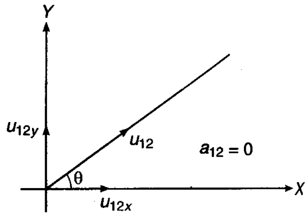
$u_{1y} = u_1 \sin\alpha_1$ and $u_{2y} = u_2 \sin\alpha_2$

Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1 \cos\alpha_1 - u_2 \cos\alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

u_{12x} and u_{12y} are the x and y components of relative velocity of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight line at an angle $\theta = \tan^{-1}\left(\frac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.



Now, if $u_{12x} = 0$ or $u_1 \cos\alpha_1 = u_2 \cos\alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90°$). Similarly, if $u_{12y} = 0$ or $u_1 \sin\alpha_1 = u_2 \sin\alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

CONDITION OF COLLISION OF TWO PROJECTILES

Now, let the particles are projected simultaneously from two different heights h_1 and h_2 with speeds u_1 and u_2 in the directions shown in fig. Then the particles collide in air if relative velocity of 1 with respect to 2($\vec{u}_{12}$) is along line AB or the relative velocity of 2 with respect to 1 ($\vec{u}_{21}$) is along the line BA. Thus,

$\vec{u}_1 = u_1 \cos\alpha_1\,\hat{i} + u_1 \sin\alpha_1\,\hat{j}$ $\qquad$ $\vec{u}_2 = -u_2 \cos\alpha_2\,\hat{i} + u_2 \sin\alpha_2\,\hat{j}$

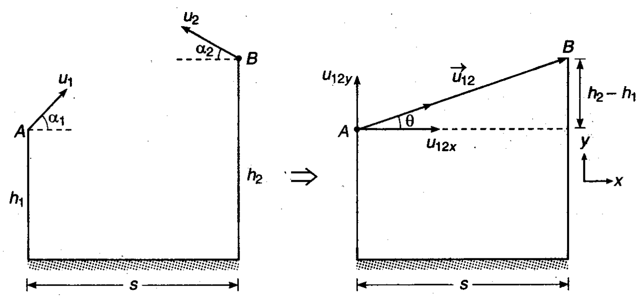
$$\tan\theta = \frac{u_{12y}}{u_{12x}} = \left(\frac{h_2 - h_1}{s}\right)$$

Here, $u_{12y} = u_{1y} - u_{2y} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

and $u_{12x} = u_{1x} - u_{2x} = (u_1 \cos\alpha_1) - (-u_2 \cos\alpha_2) = u_1 \cos\alpha_1 + u_2 \cos\alpha_2$

If both the particles are initially at the same level ($h_1 = h_2$), then for collision

$u_{12y} = 0$ or $u_1 \sin\alpha_1 = u_2 \sin\alpha_2$



The time of collision of the two particles will be

$$t = \frac{AB}{|\vec{u}_{12}|} = \frac{AB}{\sqrt{(u_{12x})^2 + (u_{12y})^2}}$$

Further, the above conditions are not merely sufficient for collision to take place. For example, the time of collision discussed above should be less than the time of collision of either of the particles with the ground.

EXERCISE#1

Q.1 **A body moves over one fourth of a circular arc in a circle of radius r. The magnitude of distance travelled and displacement will be respectively-**

(A) $\frac{pr}{2}, r\sqrt{2}$ (B) $\frac{pr}{4}, r$

(C) $pr, \frac{r}{\sqrt{2}}$ (D) πr, r

TOPIC SKILLS **:** **To calculate shortest distance using change in position with time.**

Q.2 **The displacement of the point of the wheel initially in contact with the ground, when the wheel roles forward half a revolution will be (radius of the wheel is R)**

(A) $\frac{R}{\sqrt{p^2+4}}$ (B) $R\sqrt{p^2+4}$

(C) $2\pi R$ (D) πR

TOPIC SKILLS **:** **To calculate the change in position of the point considered on the circle.**

Q.3 **A bus goes from station A to D through B, C. The distance and time are given in fig.** $A \xrightarrow[2t]{x} B \xrightarrow[t]{x} C \xrightarrow[t]{2x} D$**. Find the average speed between (a) AC (b) BD (c) AD.**

(A) $\frac{2x}{3t}, \frac{3x}{2t}, \frac{x}{t}$ (B) $\frac{x}{3t}, \frac{3x}{2t}, \frac{x}{t}$

(C) $\frac{2x}{3t}, \frac{3x}{t}, \frac{x}{t}$ (D) $\frac{2x}{3t}, \frac{3x}{2t}, \frac{2x}{t}$

TOPIC SKILLS **:** **To calculate the average velocity in given splits of time**

Q.4 **If $V(t) = 6t^2 + 2t + 1$, then find the avg speed during period t = 0 to t = 3 sec.**

(A) 16 m/s (B) 8 m/s

(C) 10 m/s (D) 22 m/s

TOPIC SKILLS **:** **To calculate average velocity by applying calculus.**

Q.5 **A man travels first half distance with speed 6 km/hr and second half distance with speed 2 km/hr. Find Average speed.**

(A) 4 km/hr. (B) 3 km/hr.

(C) 1.5 km/hr. (D) 4.5 km/hr.

TOPIC SKILLS **:** **To calculate average velocity by distance speed concept.**

Q.6 **A point travelling along a straight line traverses one third the distance with a velocity v_o. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the point averaged over the whole time of motion will be-**

(A) $\frac{v_o(v_1+v_2)}{3(v_1+v_2+v_o)}$ (B) $\frac{3v_o(v_1+v_2)}{v_1+v_2+v_o}$

(C) $\frac{v_o(v_1+v_2)}{v_1+v_2+4v_o}$ (D) $\frac{3v_o(v_1+v_2)}{v_1+v_2+4v_o}$

TOPIC SKILLS **:** **To calculate average velocity in given splits of distance.**

Q.7 **The velocity of a car is given by $\vec{v}$ = (4t + 2) m/s, where t is the time period. Find the total displacement of the car in time t. = 0.255.**

(A) 2m (B) 1.6m

(C) 0.62m (D) 0.2 m

TOPIC SKILLS **:** **Calculation of displacement applying calculus**

Q.8 **If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is**

(A) $\frac{1}{2}\sqrt{v_1 v_2}$ (B) $\frac{v_1 + v_2}{2}$

(C) $\frac{2v_1v_2}{v_1 + v_2}$ (D) $\frac{5v_1v_2}{3v_1 + 2v_2}$

TOPIC SKILLS : **Calculation of average speed using distance split concept.**

Q.9 **The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where is in metres and t in sec. The displacement, when velocity is zero, is**

(A) 24 metres (B) 12 metres

(C) 5 metres (D) Zero

TOPIC SKILLS : **Calculation of velocity & displacement by calculus applications.**

Q.10 **The motion of a particle is described by the equation $x = a + bt^2$ where a = 15 cm and b = 3 cm. Its instantaneous velocity at time 3 sec will be**

(A) 36 cm/sec (B) 18 cm/sec

(C) 16 cm/sec (D) 32 cm/sec

TOPIC SKILLS : **To calculate Instantaneous velocity by calculus application.**

Q.11 **A person completes half of its his journey with speed n_1 and rest half with speed n_2 . The average speed of the person is**

(A) $u = \frac{u_1 + u_2}{2}$ (B) $u = \frac{2u_1 u_2}{u_1 + u_2}$

(C) $u = \frac{u_1 u_2}{u_1 + u_2}$ (D) $u = \sqrt{u_1 u_2}$

TOPIC SKILLS : **To calculate average speed using time split**

Q.12 **The motion of a body is given by the equation**

$$\frac{dv(t)}{dt} = 6.0 - 3v(t)$$

when v (t) is the speed in m/s and t in sec. If the body was at rest at t = 0 ;

Then, test the correctness of the following results.

(A) the terminal speed is 2.0 m/s.

(B) the magnitude of initial acceleration is 6.0 m/s^2.

(C) the speed varies with time as $v(t) = 2(1 - e^{-3t})$ m/s.

(D) the speed is 1.0 m/s when the acceleration is half the initial value.

TOPIC SKILLS : **Applying calculus & finding X, a etc.**

Q.13 **The displacement of a particle, moving in a straight line, is given by $s = 2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is**

(A) 2 m/s^2 (B) 4 m/s^2

(C) 6 m/s^2 (D) 8 m/s^2

TOPIC SKILLS : **Applying calculus for finding 'a'**

Q.14 **The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to**

(A) $\frac{a}{b}$ (B) $\frac{2a}{3b}$

(C) $\frac{a}{3b}$ (D) Zero

TOPIC SKILLS : **Applying calculus for finding 'a' at given time 't'**

Q.15 **The displacement of the particle is given by $y = a + by + ct^2 - dt^4$. The initial velocity and acceleration are respectively**

(A) b,- 4d (B) - b,2c

(C) b,2c (D) 2c,- 4d

TOPIC SKILLS : Calculation of velocity & acceleration by applying calculus

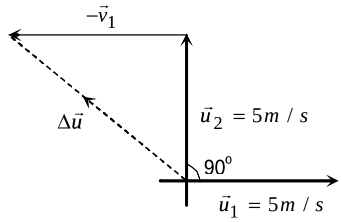
Q.16 **The relation between time t and distance x is $t = bx^2 + bx$, where a and b are constants. The retardation is (v is the velocity)**

(A) $2av^3$ (B) $2bv^3$

(C) $2abv^3$ (D) $2b^2v^3$

TOPIC SKILLS : Graphical approch of acceleration & time

Q.17 **A body starts from the origin and moves along the x-axis such that velocity at any instant is given by $(4t^3 - 2t)$, where t is in second and velocity is in m/s. What is the acceleration of the particle, when it is 2m from the origin?**

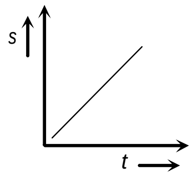


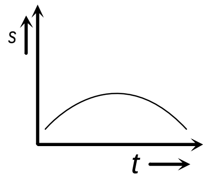
(A) $28\ m/s^2$ (B) $22\ m/s^2$

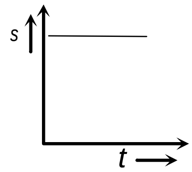
(C) $12\ m/s^2$ (D) $10\ m/s^2$

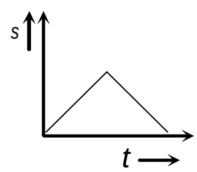
TOPIC SKILLS : Acceleration is calculated by application of calculus

Q.18 **Which of the following graph represents uniform motion**

(A) 

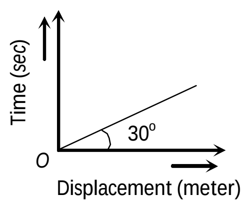
(B) 

(C) 

(D) 

TOPIC SKILLS : To find the required graph using S-t graph uniformity

Q.19 **From the following displacement time graph find out the velocity of a moving body**



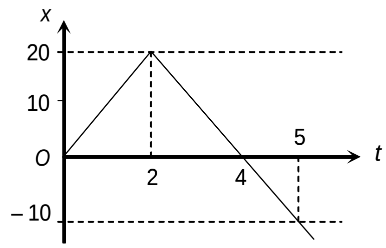
TOPIC SKILLS : Inverse slope of S-t graph will gives velocity.

(A) $\frac{1}{\sqrt{3}}$ m/s (B) 3 m/s

(C) $\sqrt{3}$ m/s (D) $\frac{1}{3}$

Q.20 **The diagram shows the displacement-time graph for a particle moving in a straight line. The average velocity for the interval t = 0, t = 5 is**

TOPIC SKILLS : **Calculating av. velocity using graph (S-t)**

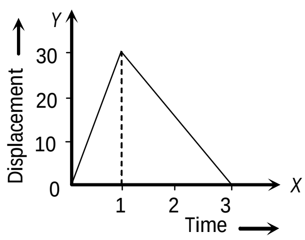


(A) 0 (B) 6 ms^{-1}

(C) – 2 ms^{-1} (D) 2 ms-1

Q.21 **Figure shows the displacement time graph of a body. What is the ratio of the speed in the first second and that in the next two seconds**

TOPIC SKILLS : **Slopes ratio**



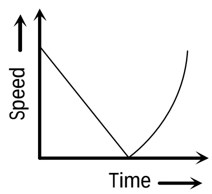
(A) 1 : 2 (B) 1 : 3

(C) 3 : 1 (D) 2 : 1

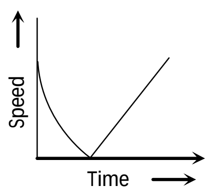
Q.22 **A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored**

TOPIC SKILLS : **Uniform motion of vertically projected body.**

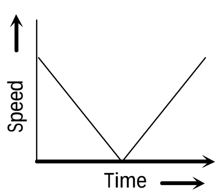
(A)



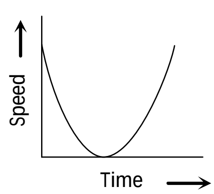
(B)



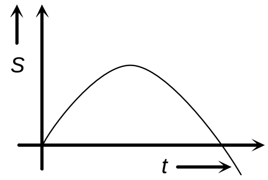
(C)



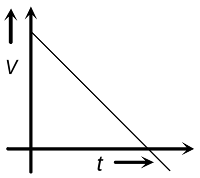
(D)

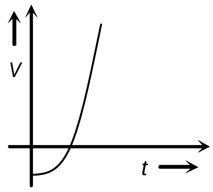


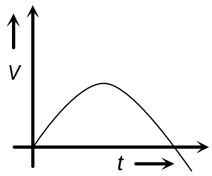
Q.23 **The graph of displacement v/s time is**

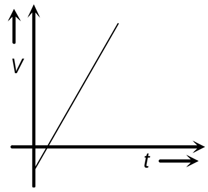


Its corresponding velocity-time graph will be

(A)


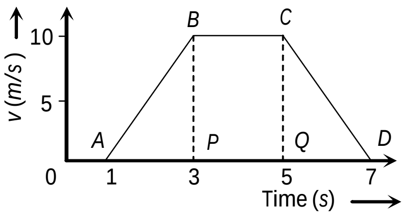
(B)


(C)


(D)


***TOPIC SKILLS* :**
Graphical approach to velocity using S-t graph

Q.24 **For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds**

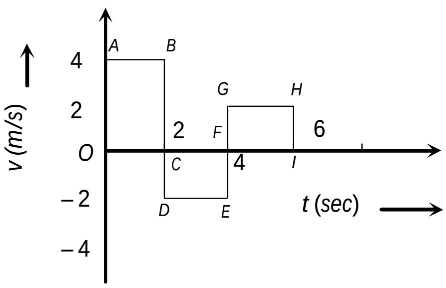


(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

***TOPIC SKILLS* :**
Slope of the graph gives displacement.

Q.25 **The velocity time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 sec are respectively**



(A) 8m, 16m (B) 16m, 8m

(C) 16m, 16m (D) 8m, 8m

***TOPIC SKILLS* :**
Calculation of displacement using V-t graph

Q.51 **The maximum height attained by a projectile is increased by 10%. Keeping the angle of projection constant, what is percentage increase in the time of flight ?**

(A) 5% (B) 10%
(C) 20% (D) 40%

TOPIC SKILLS **:** **Maximum height related with time of hight**

Q.52 **The velocity of projection of a body is increased by 2%. Keeping other factors as constant, what will be percentage change in the maximum height attained ?**

(A) 1% (B) 2%
(C) 4% (D) 8%

TOPIC SKILLS **:** **Concept of velocity & Maximum height**

Q.53 **In the above question, what will be the percentage change in the time of flight ?**

(A) 1% (B) 2%
(C) 4% (D) 8%

TOPIC SKILLS **:** **Concept of velocity & Time of Height.**

Q.54 **In the above question, what will be the percentage change in the range of projectile ?**

(A) 1% (B) 2%
(C) 4% (D) 8%

TOPIC SKILLS **:** **Range of projectile.**

Q.55 **A bomb is dropped from an aeroplane moving horizontally at constant speed. When air resistance is taken into consideration, the bomb-**

(A) Falls to earth exactly below the aeroplane
(B) Falls to earth behind the aeroplane
(C) Falls to earth ahead of the aeroplane.
(D) Flies with the aeroplane.

TOPIC SKILLS **:** **Relative motion & horizontal projectile**

Q.56 **A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following a :**

(A) Straight line path (B) Circular path
(C) Parabolic path (D) Hyperbolic path

TOPIC SKILLS **:** **Relative motion & horizontal projectile**

Q.57 **When a particle is thrown horizontally, the resultant velocity of the projectile at any time t is given by :**

(A) gt (B) $\frac{1}{2}gt^2$
(C) $\sqrt{u^2 + g^2t^2}$ (D) $\sqrt{u^2 - g^2t^2}$

TOPIC SKILLS **:** **Horizontal projectile.**

Q.58 **Two paper screens A and B are separated by 150 m. A bullet pierces A and then B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is : (g = 10 ms^{-2})**

(A) $100\sqrt{3}$ ms^{-1} (B) $200\sqrt{3}$ ms^{-1}
(C) $300\sqrt{3}$ ms^{-1} (D) $500\sqrt{3}$ ms^{-1}

TOPIC SKILLS **:** **Projectile motion velocity at any point**

Notes

EXERCISE#3

ONE OR MORE CORRECT OPTION TYPE :

Q.1 **A particle is moving in XY plane. At t = 0, it is located at the origin O(0, 0) and has the velocity vector $v_0 = a\left(\sqrt{3}\hat{i} + \hat{j}\right)$, where 'a' is a positive constant and $\hat{i}$, $\hat{j}$ are unit vectors in the positive direction of the x and y axes. Its acceleration is a constant and is given by $\vec{a} = -(\hat{i} + \hat{j})$. In its subsequent motion (t > 0) it will cross the x-axis at the instant of time given by : [a > 0]**

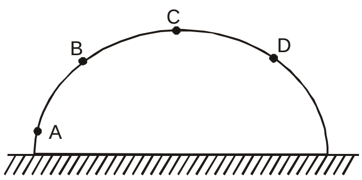
(A) t = a

(B) t = $\sqrt{3}$ a

(C) t = 2a

(D) t = 4a

Q.2 **A stone is projected from ground. Its path is as shown in figure. At which point its speed is decreasing at fastest rate ?**



(A) A

(B) B

(C) C

(D) D

Q.3 **A particle moves with an initial velocity v_0 and retardation bv, where v is its velocity at any time t.**

(a) The particle will cover a total distance of v_0 / β.

(b) The particle will continue for a very long time.

(c) The particle will stop shortly

(d) The velocity of particle will become v_0 / 2 after time 1/β.

(A) a, b

(B) a, b, d

(C) a, c

(D) None of these.

Q.4 **A particle travels along a straight path covers one-third of total distance with velocity v_0 and remaining $\frac{2}{3}$rd with velocity v_1 for half the time and velocity v_2 for other half of the time. The mean velocity averaged over whole of the distance is :**

Notes

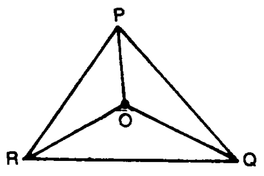
(A) $\frac{3v_0(v_1+v_2)}{(v_1+v_2+4v_0)}$

(B) $\frac{3v_0v_1v_2}{(v_1+v_2+v_0)}$

(C) $\frac{v_0(v_1-v_2)}{(v_1+v_2+v_0)}$

(D) $\frac{v_0(v_1+v_2)}{(v_1+v_2+v_0)}$

Q.5 **Three persons P, Q, R are at the three corners of an equilateral triangle of each side *l*. They start moving simultaneously with velocity v such that P always moves towards Q, Q always moves towards R and R always moves towards P. After what time they would meet each other at O ?**



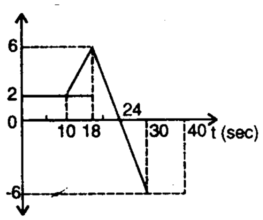
(A) $\frac{\ell}{v}$

(B) $\frac{2\ell}{v}$

(C) $\frac{2\ell}{\sqrt{3}v}$

(D) $\frac{2\ell}{3v}$

Q.6 **A particle moves in a straight line with the velocity as shown in the fig. At t = 0, x = - 16 m-**



(a) The maximum value of the position coordinate of the particle is 54 m.
(b) The maximum value of the position coordinate of the particle is 36 m.
(c) The particle is at the position of 36 m at t = 18 sec.
(d) The particle is at the position of 36 m at 30 sec.

(A) a, c, d
(B) a, b, c
(C) a, b, d
(D) b, c, d

Notes

Q.7 **Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional is :**

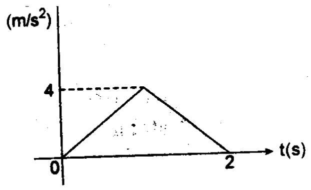
(A) $|\vec{v}|$ must decrease when $\vec{a} < 0$

(B) Speed must increase when $\vec{a} > 0$

(C) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0

(D) Speed will decrease when $\vec{v} < 0$ and $\vec{a}$ are < 0

Q.8 **The acceleration time graph of a particle moving in a straight line is shown in figure. The velocity of the particle at t = 0 is 2 m/s. The velocity after 2 seconds will be.**



(A) 6 m/s
(B) 4 m/s
(C) 2 m/s
(D) 8 m/s

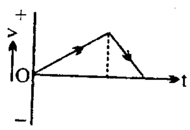
Q.9 **The displacement versus velocity equation of a particle moving in a straight line is given as**

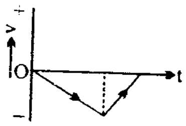
$x = \sqrt{a - b\mathbf{v}^2}$ (a and b are constants)

Choose the incorrect answer.

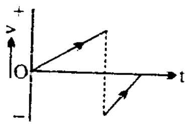
(A) The particle is in simple harmonic motion
(B) Time period of the particle is $2\pi\sqrt{\beta}$
(C) Amplitude of the particle is $\sqrt{\alpha}$
(D) Maximum velocity of the particle is $\sqrt{\frac{\alpha}{\beta}}$

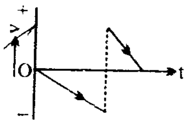
Q.10 **A ball dropped from a height h, hits the ground and rises to height h/2. The variation of the instantaneous velocity v with time t (v - t graph) (assuming the vertically upward direction as positive) for the ball from the instant it is dropped to the instant it comes to instant rest again is correctly shown by-**

(A) 

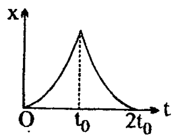
(B) 

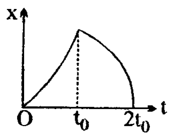
Notes

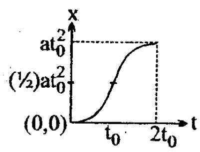
(C) 

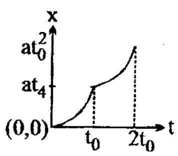
(D) 

Q.11 **A particle starts from origin accelerates for t sec then decelerates with same acceleration till $2t_0$ sec along the x-direction. The graph representing variation of displacement (x) with time (t) is-**

(A) 

(B) 

(C) 

(D) 

Q.12 **A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant-**

(A) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

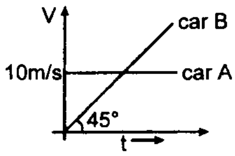
(B) The particle will come to rest at infinity.

(C) The distance traveled by the particle is $\frac{v_0^{3/2}}{\alpha}$

(D) The distance traveled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$

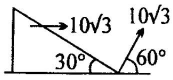
Q.13 **Initially car A is 10.5 m ahead of car B. Both start moving at time t = 0 in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be-**

Notes



(A) t = 21 sec
(B) t = 2√5 sec
(C) 20 sec
(D) None of these

Q.14 A particle is at angle 60° with speed 10Ö3, from the point 'A' as shown in the figure. At the same time the wedge is made to move with speed 10Ö3 towards right as shown in the figure. Then the time after which particle will strike with wedge is -

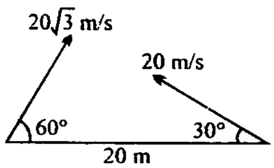


(A) 2 sec
(B) 2√3 sec
(C) $\frac{4}{\sqrt{3}}$ sec
(D) None

Q.15 A body is projected at time (t = 0) from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacement x & y (in meter) respectively vary with time t in second as, $x = 10\sqrt{3}t$ and $y = 10t - t^2$. Then the maximum height attained by the body is-

(A) 200 m
(B) 100 m
(C) 50m
(D) 25 m

Q.16 In the figure shown, the two projectiles are fired simultaneously. The minimum distance between them during their flight is-



(A) 20 m
(B) 10√3 m
(C) 10m
(D) None

Q.17 A projectile moves from the ground such that its horizontal displacement is x = Kt and vertical displacement is y = Kt(1 – at), where K and a are constants and t is time. Find out total time of flight (T) and maximum height attained (Y_{max}) its

Notes

(A) $T = \alpha,\ Y_{max} = \frac{K}{2\alpha}$

(B) $T = \frac{1}{\alpha},\ Y_{max} = \frac{2K}{\alpha}$

(C) $T = \frac{1}{\alpha},\ Y_{max} = \frac{K}{6\alpha}$

(D) $T = \frac{1}{\alpha},\ Y_{max} = \frac{K}{4\alpha}$

Q.18 A particle is projected with velocity V_0 along x-axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e. $a = ax^2$, the distance at which the particle stops is

(A) $\sqrt{\frac{3V_0}{2\alpha}}$

(B) $\left(\frac{3V_0}{2\alpha}\right)^{\frac{1}{3}}$

(C) $\sqrt{\frac{2V_0^2}{3\alpha}}$

(D) $\left(\frac{3V_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Q.19 A body is projected vertically upwards at time t = 0 and it is seen at a height H at instants t_1 and t_2 seconds during its flight. The maximum height attained is (g is acceleration due to gravity)

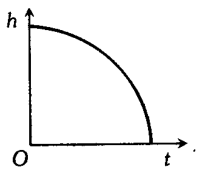
(A) $\frac{g(t_2 - t_1)^2}{8}$

(B) $\frac{g(t_1 + t_2)^2}{4}$

(C) $\frac{g(t_1 + t_2)^2}{8}$

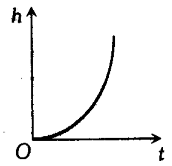
(D) $\frac{g(t_1 - t_1)^2}{4}$

Q.20 Which of the following is the graph between the height (h) of a projectile and time (t), when it is projected from the ground

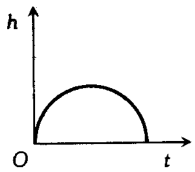
(A) 

Notes

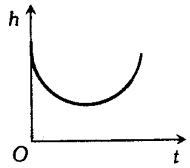
(B)



(C)

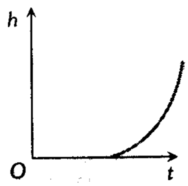


(D)

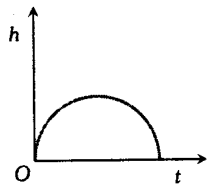


Q.21 Which of the following is the altitude-time graph for a projectile thrown horizontally from the top of the tower

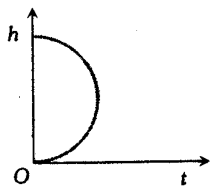
(A)



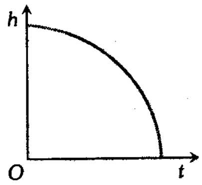
(B)



(C)



(D)



Notes

PASSAGE TYPE :

PASSAGE 1

A particle starts from rest with a time varying acceleration a= (2t – 4). Here t is in seconds and a in m/s.

Q.22 Particle comes to rest after a time t = ...second.

(A) 1
(B) 4
(C) 3
(D) 2

Q.23 Maximum velocity of particle in negative direction is at t = ...seconds.

(A) 3
(B) 4
(C) 2
(D) 1

Q.24 The velocity time graph of the particle is ....

(A) Parabola passing through origin
(B) Straight line not passing through origin
(C) Parabola not passing through origin
(D) Straight line passing through origin.

PASSAGE 2

When you throw a ball in air with some velocity at same angle with horizontal, vertical component of velocity at highest point is zero and horizontal component of velocity remains unchanged.

Question : Velocity of a projectile at height 15 m from ground is $\left(20\hat{i}+10\hat{j}\right)$ m/s. Here, $\hat{i}$ is in horizontal direction and $\hat{j}$ is vertically upwards. Then:

Q.25 Speed with which particle is projected from ground is ....m/s.

(A) 30
(B) $20\sqrt{2}$
(C) $\sqrt{20}$
(D) $3\sqrt{40}$

Q.26 Angle of projectile with ground is ....

(A) 45^0
(B) 30^0
(C) 25^0
(D) 60^0

Q.27 Maximum height from the ground is ....m

(A) 30
(B) 60
(C) 40
(D) 20

Q.28 Horizontal range on the groun is ....m

(A) 60
(B) 50
(C) 80
(D) 70

Notes

ASSERTION - REASON TYPE QUESTIONS

The following questions consist of two statements one labelled Assertion (A) and the another labelled Reason (R). Select the correct answers to these questions from the codes given below :

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not correct explanation of A

(C) A is true but R is false

(D) A is false but R is true.

(E) A and R both are false.

Q.29 **Assertion :** When $t=\sqrt{x}+3$, the motion is constantly accelerated motion.

Reason : The second derivative of x in this case will be a constant with time.

Q.30 **Assertion :** In a free fall, weight of a body becomes effectively zero.

Reason : Acceleration due to gravity acting on a body having free fall is zero.

Q.31 **Assertion :** An object can have constant speed (Instantaneous) but variable velocity (Instantaneous).

Reason : Speed is a scalar but velocity is a vector quantity.

Q.32 **Assertion :** In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180^0.

Reason : At the highest point, velocity of projectile will be in horizontal direction only

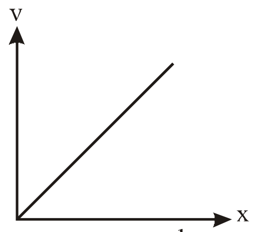
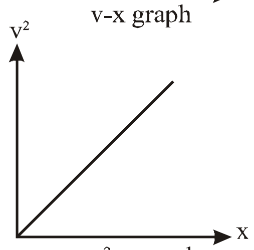
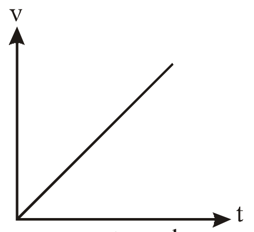
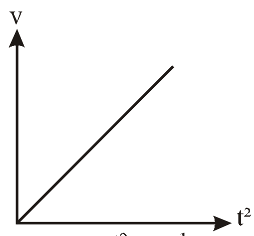
Q.33 **Assertion :** When range of a projectile is maximum, its angle of projection may be 45^0 or 135^0.

Reason : Whether θ is 45^0 135^0, value of range remains the same, only the sign changes.

Notes

MATRIX MATCH TYPE :

Q.34 **Column I gives some graphs for a particle moving along x-axis in positive x-direction. The variables v, x and t represent speed of particle. x-coordinate of particle and time respectively. Column Ii gives certain resulting interpretation. Match the graphs in column I with the statements in column II.**

Column I	Column II
(A)  v-x graph	(p) Acceleration of particle is uniform
(B)  v^2-x graph	(q) Acceleration of particle is nonuniform
(C)  v-t graph	(r) Acceleration of particle is directly proportional to t
(D)  v-t^2 graph	(s) Acceleration of particle is directly proportional to x

Q.35 **A particle moves along a straight line such that its displacement S varies with time t as** $S=\alpha+\beta t+\gamma t^2$

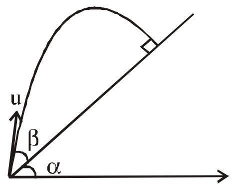
Column I	Column II
(A) Acceleration at t = 2 sec.	(p) $\beta+5\gamma$
(B) Average velocity during 3rd sec.	(q) 2γ
(C) Velocity at t = 1 sec.	(r) α
(D) Initial displacement	(s) $\beta+2\gamma$

Notes

Q.36 **A particle is projected with velocity 20 $\sqrt{2}$ m/s at 45^0 with horizontal. After is (g = 10 m/s^2) match the following table :**

	Column I		Column II
(A)	Average velocity	(p)	$10\sqrt{5}$ m/s
(B)	Change in velocity	(q)	25 m/s
(C)	Instantaneous velocity	(r)	10 m/s
		(s)	None

Q.37 **The projectile collides perpendicularly with the inclined plane. (Refer the figure)**



	Column I		Column II
(A)	Maximum height attained by the projectile from the ground.	(p)	zero
(B)	Maximum height attained by the projectile from inclined plane.	(q)	g
(C)	Acceleration of the projectile before striking the inclined plane.	(r)	$\frac{u^2 \sin^2 \beta}{2g\cos\alpha}$
(D)	Horizontal component of acceleration of the projectile	(s)	$\frac{u^2 \sin^2(\alpha+\beta)}{2g}$

ANSWERKEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
Ans.	C	A	A	A	D	A	C	A	D	D	C	A,C	A	A	D	
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Ans.	C	D	D	C	C	D	B	C	A	B	A	D	C	A	C	
Que.	31	32	33													
Ans.	A	E	A													
Que.	34				35				36				37			
Ans.	A-q,s	B-p	C-p	D-q,r	A-q	B-p	C-s	D-r	A-q	B-r	C-r		A-s	B-r	C-q	D-p

EXERCISE#4

Notes

Q.1 Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distance in which the two cars are stopped from that instant is [AIEEE 2002]

(A) 1 : 1
(B) 1 : 4
(C) 1 : 8
(D) 1 : 16

Q.2 Two balls A and B of same masses are thrown from the top of the building. A, thrown upward with velocity V and B, thrown downward with velocity V, then [AIEEE 2002]

(A) Velocity of A is more than B at the ground
(B) Velocity of B is more than A at the ground
(C) Both A & B strike the ground with same velocity
(D) None of these

Q.3 A ball is projected with kinetic energy E at an angle of 45^0 to the horizontal. At the highest point during its flight, its kinetic energy will be [AIEEE 2002]

(A) Zero
(B) E/2
(C) $E/\sqrt{2}$
(D) E

Q.4 In a projectile motion, velocity at maximum height is [AIEEE 2002]

(A) $\frac{u\cos\theta}{2}$
(B) $u\cos\theta$
(C) $\frac{u\cos\theta}{2}$
(D) None of these

Q.5 A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball in T/3 seconds [AIEEE 2004]

(A) h/9 meters from the ground
(B) 7h/9 meters from the ground
(C) 8h/9 meters from the ground
(D) 17h/18 meters from the ground

Q.6 For a given velocity, a projectile has the same range R for two angles of projection. If t_1 and t_2 are the times of flight in the two cases then [AIEEE 2004]

(A) $t_1t_2 \propto R^2$
(B) $t_1t_2 \propto R$
(C) $t_1t_2 \propto \frac{1}{R}$
(D) $t_1t_2 \propto \frac{1}{R^2}$

Notes

Q.7 **A bullet fired into a fixed target loses half of its velocity after pentrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion.**

[AIEEE 2005]

(A) 1.5 cm
(B) 1.0 cm
(C) 3.0 cm
(D) 2.0 cm

Q.8 **The relation between time and distance is t = $Ax^2 + bx$, where a and b are constants. The retardation is**

[AIEEE 2005]

(A) $2\alpha v^3$
(B) $2\beta v^3$
(C) $2\alpha\beta v^3$
(D) $2\beta^2 v^3$

Q.9 **A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then** **[AIEEE 2005]**

(A) $S = \frac{1}{2}ft^2$
(B) $S = \frac{1}{4}ft^2$
(C) $S = \frac{1}{72}ft^2$
(D) $S = \frac{1}{6}ft^2$

Q.10 **A parachutist after balling out falls 50 m without frinction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out**

[AIEEE 2005]

(A) 293 m
(B) 111 m
(C) 91 m
(D) 182 m

Q.11 **A particle located at x = 0, starts moving along the positive x-direction with a velocity 'v that varies as v = $\alpha\sqrt{x}$. the displacement of the particle varies with time as** **[AIEEE 2006]**

(A) t
(B) $t^{1/2}$
(C) t^3
(D) t^2

Notes

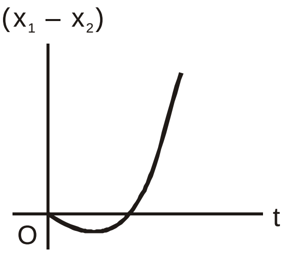
Q.12 **The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is x =0 at t = 0, then its displacement after unit time (t = 1) is**

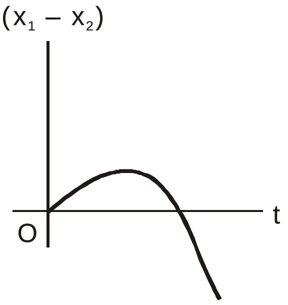
[AIEEE 2007]

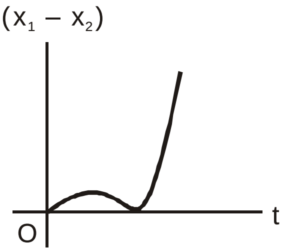
(A) $v^0 + 2g + 3f$
(B) $v_0 + g/2 + f/3$
(C) $v_0 + g + f$
(D) $v_0 + g/2 + f$

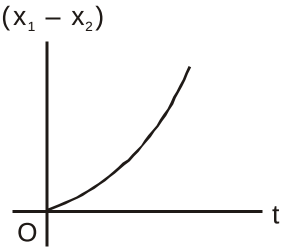
Q.13 **A body is a rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't' and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t.**

[AIEEE 2008]

(A)


(B)


(C)


(D)


Notes

Q.14 A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is [AIEEE 2009]

(A) 10 units

(B) $7\sqrt{2}$ units

(C) 7 units

(D) 8.5 units

Q.15 An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by : $\frac{dv}{dt}=-2.5\sqrt{v}$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be : [AIEEE 2011]

(A) 2s

(B) 4s

(C) 8s

(D) 1s

Q.16 A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : [AIEEE2011]

(A) $\pi\frac{v^4}{g^2}$

(B) $\frac{\pi}{2}\frac{v^4}{g^2}$

(C) $\pi\frac{v^4}{g^2}$

(D) $\pi\frac{v^4}{g}$

Q.17 A small block slides, without frinction, down an inclined plane starting from rest. Let S_n be the distance travelled from t = (n – 1) seconds to t = (n) second. Then $\frac{S_n}{S_{n+1}}$ is : [IIT 2004]

(A) $\frac{2n-1}{2n}$

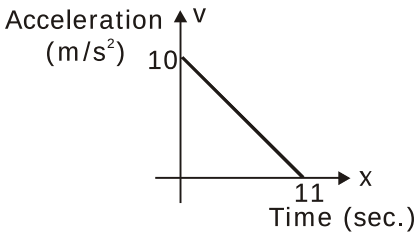
(B) $\frac{2n+1}{2n-1}$

(C) $\frac{2n-1}{2n+1}$

(D) $\frac{2n}{2n+1}$

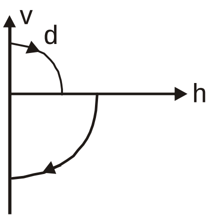
Notes

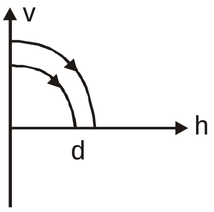
Q.18 **A body starts from rest at time t = 0, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be : [IIT 2004]**

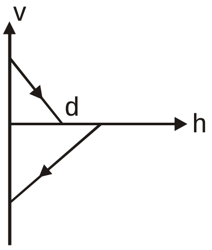


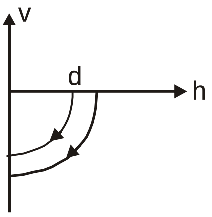
(A) 110 m/s
(B) 55 m/s
(C) 650 m/s
(D) 550 m/s

Q.19 **A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as : [IIT 2004]**

(A)


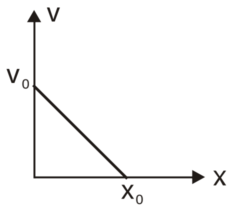
(B)


(C)


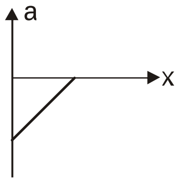
(D)


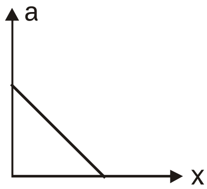
Notes

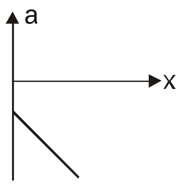
Q.20 **The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be :**

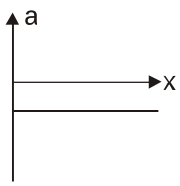


[IIT 2005]

(A)


(B)


(C)


(D)


Q.21 **Column I describes some situations in which a small object moves. Column II describes some characteristics of these motion. Match the situations in Column I with the characteristics in Column II.**

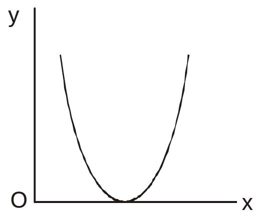
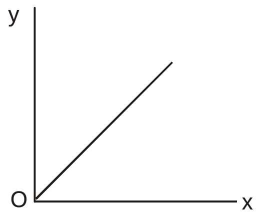
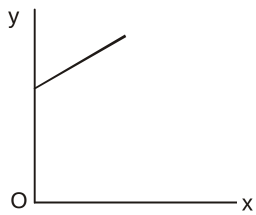
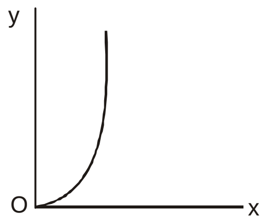
[IIT 2007]

	Column I	**Column II**
(A)	The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$	(p) The object executes a simple harmonic motion.
(B)	The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy v = – kx, where k is a positive constant.	(q) The object does not change its direction.

Notes

(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the objects keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where M_e is the mass of the earth and Re is the radius of the earth, Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

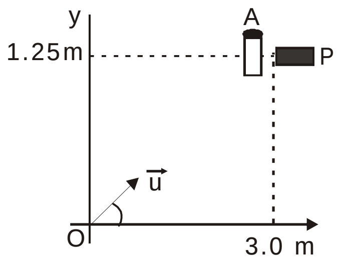
Q.22 Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. [IIT 2008]

Column I	Column II
(A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis).	(p) 
(B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q) 
(C) Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle.	(r) 
(D) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)	(s) 

Notes

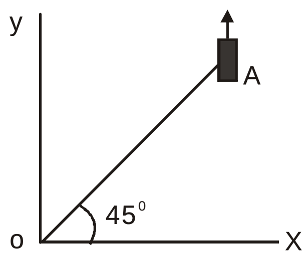
Q.23 **An object A is kept fixed at the point x = 3m and y = 1.25 m on a plank P raised above the ground. At time t = 0, the plank starts moving along the +x direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity $\vec{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45^0 to the horizontal. All the motions are in the x-y plane. Find $\vec{u}$ and the time after which the stone hits the object. Take g = 10 m/s^2.**

[IIT 2000]



Q.24 **On a frinctionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45^0 with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.**

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame. **[IIT 2005]**



(b) Find the speed of the ball with respect to the surface if $\phi = \frac{4\theta}{3}$

ANSWERKEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	B	B	C	B	B	A	C	A	D	B	A	B	A
Que.	16	17	18	19	20	21				22					
Ans.	A	C	B	A	A	A-p	B-q,r	C-p	D-q,r	A-p,s	B-q,s	C-s	D-q		

Q.23 t = 1 sec. $\vec{u} = 3.75\,\hat{i} + 6.25\,\hat{j}$

Q.24 (a) $\theta = 45^0$ (b) 2 m/s