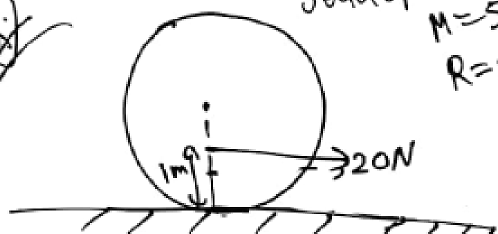
Rotational Motion

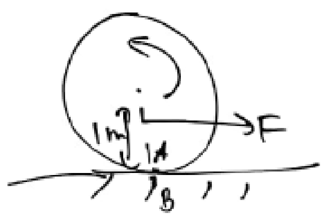
Q

Solid sphere
$M = 5\,kg$
$R = 2\,m$

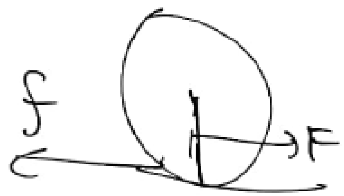


Find direction of friction?

Sol^n ⇒



A ⟶



Dir^n of friction ⇒

1. assume there is no friction
2. Notice that point of contact is running in which dir^n.
3. opposite ~~to~~ the dir^n of point of contact friction will act.

#

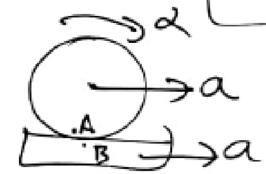
$v_A = v_B$

$\boxed{v - \omega R = v_1}$ → condition of pure rolling

surface is fixed ⇒ $v_1 = 0$

$v = \omega R \Rightarrow \boxed{\omega = \frac{v}{R}}$ #

#



#

$a_A = a_B$

$\boxed{a - \alpha R = a_1}$ → condition for pure rolling

Surface is fixed ⇒ $a_1 = 0$

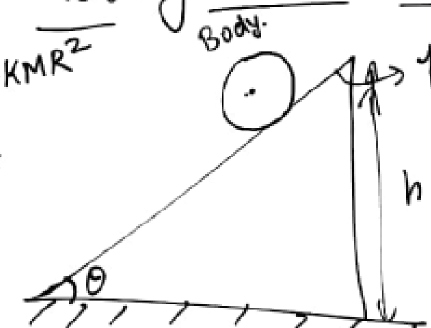
$a = \alpha R \Rightarrow \boxed{\alpha = \frac{a}{R}}$ #

Rotational Motion

Rolling on an inclined plane ⇒

$I = KMR^2$

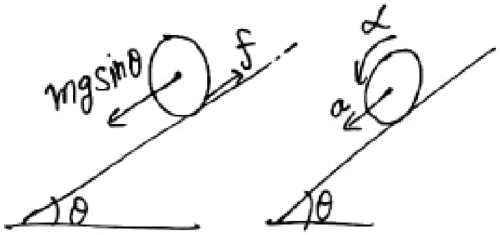
Q.



friction is sufficient for pure rolling.

Find a, α, f for pure rolling?

Soln. ⇒



$a = \alpha R$ —(1)

$mg\sin\theta - f = ma$ —(2)

$\tau_{com} = I_{com}\alpha$

$f\cancel{R} = KMR^{\cancel{2}}\left(\frac{a}{\cancel{R}}\right)$ —(3)

$mg\sin\theta = ma + Kma$

$$\boxed{a = \frac{g\sin\theta}{(K+1)}}$$

$$\boxed{\alpha = \frac{g\sin\theta}{R(K+1)}}$$

$$\boxed{f = \frac{Kmg\sin\theta}{(K+1)}}$$

(1) Ring = Hollow cylinder ⇒ $K_1 = 1$

(2) Disc = Solid cylinder ⇒ $K_2 = \frac{1}{2}$

(3) Hollow Sphere ⇒ $K_3 = \frac{2}{3}$

(4) Solid sphere = $K_4 = \frac{2}{5}$

$K_1 > K_3 > K_2 > K_4 \Rightarrow I_1 > I_3 > I_2 > I_4$

Q. If Ring, disc, Hollow sphere & solid sphere of same Mass M & Radius R is released from top of an incline plane Find the relation b/w

(i) acceleration.

(ii) time.

Rotational Motion

Soln ⇒ Relation of accn ⇒

$$a = \left(\frac{g\sin\theta}{K+1}\right)$$

$K\uparrow \Rightarrow a\downarrow$

$K_1 > K_3 > K_2 > K_4$

$a_1 < a_3 < a_2 < a_4$

Relation of time.

$a\uparrow t\downarrow$

$a_1 < a_3 < a_2 < a_4$

$t_1 > t_3 > t_2 > t_4$

(D) Find Relation b/w Speed when reached at bottom of an incline?

Soln ⇒ $v^2 - u^2 = 2al \Rightarrow v^2 = u^2 + 2al$

$u \to$ same
$l \to$ same

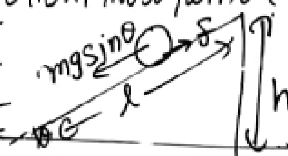
$a\uparrow\ v\uparrow \Rightarrow a_1 < a_3 < a_2 < a_4$

$$\boxed{v_1 < v_3 < v_2 < v_4}$$

(E) Find Relation b/w K.E at bottom most point?

$I = KMR^2$

$\sin\theta = \frac{h}{l}$



$\underset{0}{\cancel{W_g + W_f}} = KE_f - \cancel{KE_i}^{\,0}$

$W_g = KE_f$

$mgh = KE_f$

$$\boxed{K.E_1 = K.E_2 = K.E_3 = K.E_4}$$

OR: $u = 0$

$v = \sqrt{2al} \Rightarrow v^2 = 2al$

$$v^2 = 2\left(\frac{g\sin\theta}{K+1}\right)l$$

$$\omega = \frac{V}{R} = \frac{\sqrt{2al}}{R}$$

$K.E_i = 0$

$KE_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$K.E_f = \frac{1}{2}M\frac{2g\sin\theta}{(K+1)}l + \frac{1}{2}KM\cancel{R^2}\,\frac{2}{\cancel{R^2}}\left(\frac{g\sin\theta}{(K+1)}\right)l$$

$$K.E = \frac{1}{\cancel{2}}M\frac{\cancel{2}g\sin\theta\, l}{\cancel{(K+1)}}\left[\cancel{1+K}\right]$$

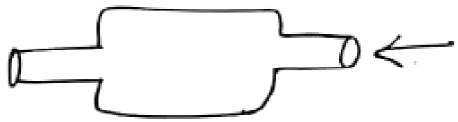
$K.E. = Mg\,l\sin\theta$

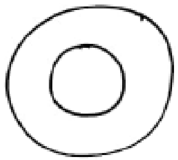
$$\boxed{K.E_f = Mgh}$$

$$\boxed{K.E_1 = K.E_2 = K.E_3 = K.E_4}$$

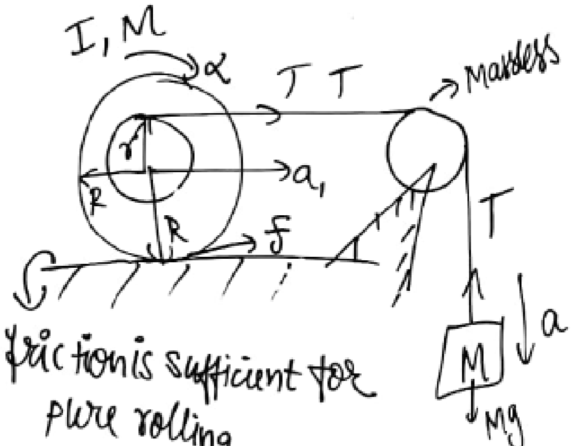
Rotational Motion

Bobine ⇒





Q.



friction is sufficient for pure rolling

Find Tension, friction, acceleration of block, acceleration of bobine, angular acc^n of bobine?

Sol^n ⇒

Block

$$Mg - T = Ma \quad \text{—} (1)$$

bobine

$$f + T = Ma_1 \quad \text{—} (2)$$

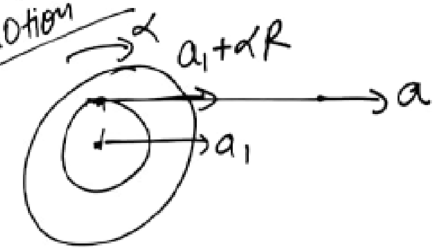
pure rolling ⇒ $\alpha = \frac{a_1}{R}$ — (3)

Bobine ⇒ $\tau_{COM} = I_{COM}\alpha$ ⇒

$$fR - Tr = I(-\alpha)$$

$$fR - Tr = -\frac{Ia_1}{R} \quad \text{—} (4)$$

Constrained Motion



$$a = a_1 + \alpha r \quad \text{—} (5)$$