Rotational Motion

Moment of inertia ⇒

$\boxed{I = mr^2}$

Ring → 

$I = MR^2$

Disc ⇒ 

$I = \frac{MR^2}{2}$

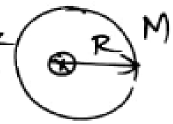
Rod ⇒ 

$M = \frac{Ml^2}{3}$

Rod


$I = \frac{Ml^2}{12}$

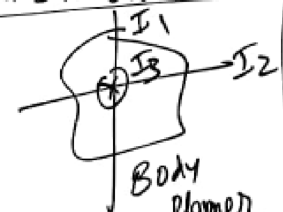
Hollow sphere ⇒

$I = \frac{2}{3}MR^2$ 

Solid sphere ⇒

$I = \frac{2}{5}MR^2$ 

⊥r axis theorem



$\boxed{I_3 = I_1 + I_2}$

∥ axis theorem ⇒

$\boxed{I = I_{com} + Md^2}$

Torque ⇒

* $\vec{\tau} = \vec{r} \times \vec{F}$ dir^n ⇒ right hand

$|\tau| = rF\sin\theta$
$= r_{\perp r}F = rF_{\perp r}$

$\odot = \tau$

* $\tau_{axis} = (\vec{r} \times \vec{F})$ component along the axis of rotation

* Eq^m ⇒ $\boxed{\vec{F}_{net} = 0 \;\&\; \vec{\tau}_{net} = 0}$

$\boxed{\tau = I\alpha}$

Angular Momentum ⇒

$\vec{L} = m(\vec{r} \times \vec{v})$

$\vec{L} = I\omega$

$\vec{\tau}_{net} = 0 \Rightarrow \boxed{\vec{L}_i = \vec{L}_f}$ → conservation of angular momentum.

Angular impulse ⇒

$\boxed{\int \tau \, dt = L_f - L_i}$

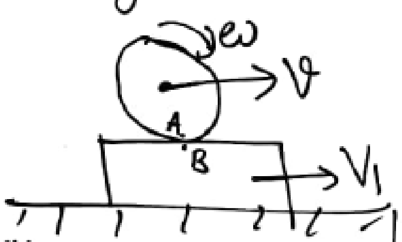
Rotational Motion

Pure Rolling ⇒

Static Friction → Khush

→ When two surfaces which are in contact stays in contact.

#



Condition of pure rolling } ⇒ friction → Happy

$$\boxed{V_A = V_B}$$

$v_A = v_B$

$\omega R \leftarrow \overset{A}{\bullet} \rightarrow v$

$\overset{}{B} \bullet \rightarrow V_1$

$v_A = v_B$

$$\boxed{v - \omega R = V_1}$$

Condition for pure rolling.

if surface (platform) is fixed.

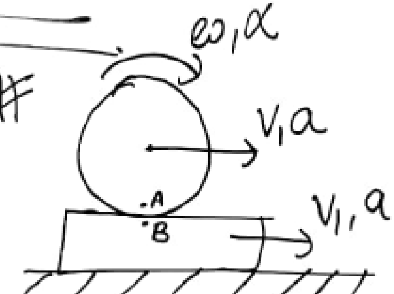
$V_1 = 0$

$v - \omega R = 0$

$$\boxed{v = \omega R}$$
$$\boxed{\omega = \frac{v}{R}}$$

→ condition of pure rolling for fixed surface

#



Condition for pure rolling ⇒ Friction happy.

$\boxed{a_A = a_B}$ along the contact surface

$\uparrow \omega^2 R$

$\alpha R \leftarrow \overset{A}{\bullet} \rightarrow a$

$B \rightarrow a_1$

$$\boxed{a - \alpha R = a_1}$$

if the surface is fixed ⇒ $a_1 = 0$

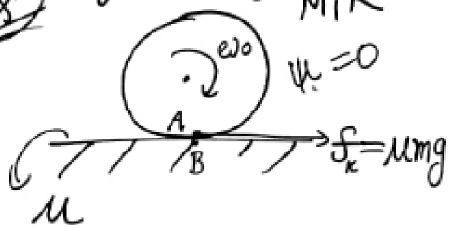
$$\boxed{a = \alpha R}$$
$$\boxed{\alpha = \frac{a}{R}}$$

⇒ Condition for pure rolling for fixed surface

Rotational Motion

Pure Rolling ⇒

Q. $t=0$ Disc M, R



After time t pure rolling start. Find v & ω when it happens?

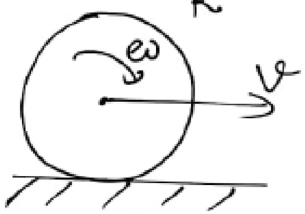
Sol^n ⇒ $f = \mu mg$

Method-1 ⇒ $a = \mu g$

$\tau = \odot \Rightarrow \tau = I\alpha$

$\mu \cancel{M} g \cancel{R} = \frac{\cancel{M}R^{\cancel{2}}}{2}\alpha \Rightarrow \boxed{\alpha = \frac{2\mu g}{R}}$

$t=t$ pure rolling ⇒ $\omega = \frac{v}{R}$



Translation eq^n ⇒

$v = \cancel{u}^{0} + at$

$v = \mu g t \Rightarrow t = \frac{v}{\mu g}$ — (1)

Rotational eq^n ⇒

$\omega = \omega_0 + \alpha t$

$\omega = \omega_0 - \frac{2\mu g}{R} t$ — (2)

$\omega = \frac{v}{R}$ & $t = \frac{v}{\mu g}$ put in (2)

$\frac{v}{R} = \omega_0 - 2\frac{\cancel{\mu g}}{R}\frac{v}{\cancel{\mu g}}$

$\frac{3v}{R} = \omega_0$

$\boxed{v = \frac{\omega_0 R}{3} \text{ m/s}}$

$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \frac{\omega_0}{3} \text{ rad/s}}$

M-II.

$\boxed{N = Mg}$

$\boxed{\tau_A = 0} \Rightarrow$ Angular Momentum conserve.

$\boxed{L_i = L_f}_A \Rightarrow$

$\boxed{L_i = L_f}$ → about A

$\frac{MR^2}{2}\omega_0 = \frac{MR^2}{2}\omega + MRv$ — (1)

Pure rolling ⇒ $\omega = \frac{v}{R}$ put in (1)

$\frac{\cancel{M}R^{\cancel{2}}}{2}\omega_0 = \frac{\cancel{M}R^{\cancel{2}}}{2}\frac{v}{\cancel{R}} + \cancel{MR}v$

$\frac{\omega_0 R}{\cancel{2}} = \frac{3}{\cancel{2}}v$

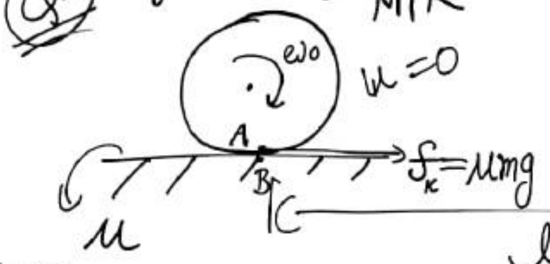
$\boxed{v = \frac{\omega_0 R}{3}}$

$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \frac{\omega_0}{3}}$

Rotational Motion

Pure Rolling ⇒

Q. $t=0$ Disc M, R



from $t=0 \longrightarrow t = \frac{\omega_0 R}{3\mu g}$

Find work done by friction?

Sol^n ⇒ $\boxed{W.D._f = K_f - K_i}$

$K_i = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{MR^2}{2}\omega_0^2 = \frac{MR^2\omega_0^2}{4}$

$K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M\frac{\omega_0^2R^2}{9} + \frac{1}{2}\frac{MR^2}{2}\frac{\omega_0^2}{9} = \frac{MR^2\omega_0^2}{12}$

K.E $\downarrow_{ses}$

$t=t$ $\quad \omega = \frac{v}{R}$ Pure Rolling

$v = \frac{\omega_0 R}{3}, \ \omega = \frac{\omega_0}{3}$

$t = \frac{v}{\mu g} = \frac{\omega_0 R}{3\mu g}$

$W.D._f = K_f - K_i$

$\frac{MR^2\omega_0^2}{12} - \frac{MR^2\omega_0^2}{4}$

$\boxed{W.D_f = -\frac{MR^2\omega_0^2}{6}}$

M-II Translation work done

$W.D_f = f \cdot l$

$v^2 - u^2 = 2aS$

$\frac{\omega_0^2R^2}{9} - 0 = 2\mu g\, l$

$l = \frac{\omega_0^2R^2}{18\mu g}$

$W.D_f = \mu mg \cdot \frac{\omega_0^2R^2}{18\mu g}$

Translational

$\boxed{W.D_f = \frac{M\omega_0^2R^2}{18}}$

Rotational work done

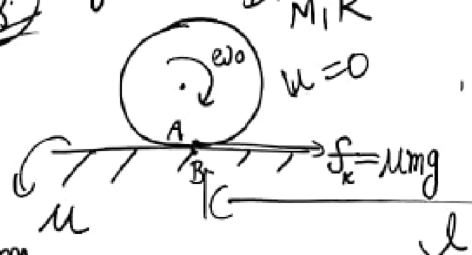
$\boxed{W.D. = \int \tau \cdot d\theta}$

Rotational Motion

Pure Rolling ⇒

Q. $t=0$ Disc M, R

$u=0$



from $t=0 \longrightarrow t=\frac{\omega_0 R}{3\mu g}$

Find work done by friction ?

Soln $\xRightarrow{M\text{-}I}$ $W.D_f = K_f - K_i \Rightarrow \boxed{W.D_f = -\frac{MR^2\omega_0^2}{6}}$

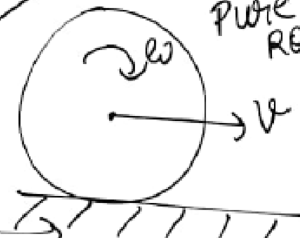
M-II Rotational W.D. ⇒ $W.D = \int \tau \, d\theta$

$W.D = \tau\theta\cos 180^\circ = -\tau\theta$

$W.D = -\mu mgR\theta = -\cancel{\mu} mgR \frac{2\omega_0^2 R}{9\,\cancel{\mu g}}$

$\boxed{W.D_R = -\frac{2}{9}M\omega_0^2R^2}$

$t=t$ $\omega = \frac{v}{R}$ Pure Rolling



$v = \frac{\omega_0 R}{3}$, $\omega = \frac{\omega_0}{3}$

$t = \frac{v}{\mu g} = \frac{\omega_0 R}{3\mu g}$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\frac{\omega_0^2}{9} - \omega_0^2 = 2\alpha\theta$

$-\frac{8}{9}\omega_0^2 = -\frac{4\mu g}{R}\theta$

$\boxed{\theta = \frac{2}{9}\frac{\omega_0^2 R}{\mu g}}$

M-II Translation work done

$W.D_f = f \cdot l$

$v^2 - u^2 = 2as$

$\frac{\omega_0^2R^2}{9} - 0 = 2\mu g\, l$

$l = \frac{\omega_0^2R^2}{18\mu g}$

$W.D_f = \cancel{\mu} mg \cdot \frac{\omega_0^2R^2}{18\,\cancel{\mu g}}$

Translational

$\boxed{W.D_t = \frac{M\omega_0^2R^2}{18}}$

Rotational work done

$\boxed{W.D. = \int \tau \cdot d\theta}$

Total work done by friction ⇒

$W.D_f = W.D_R + W.D_t$

$= -\frac{2}{9}M\omega_0^2R^2 + \frac{M\omega_0^2R^2}{18}$

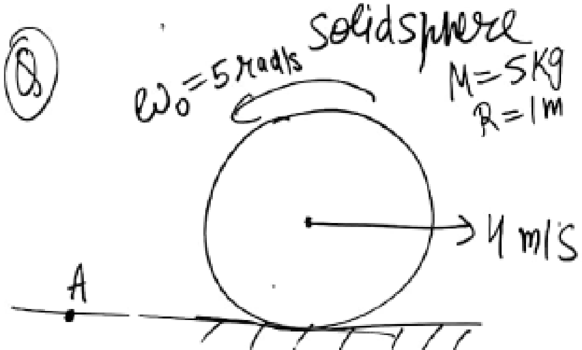
$= M\omega_0^2R^2\left[\frac{-4+1}{18}\right]$

$= M\omega_0^2R^2\left(\frac{-3}{18}\right)$

$\boxed{W.D_f = -\frac{M\omega_0^2R^2}{6}}$

Rotational Motion

Q



(a) Find ω & v when pure rolling starts?

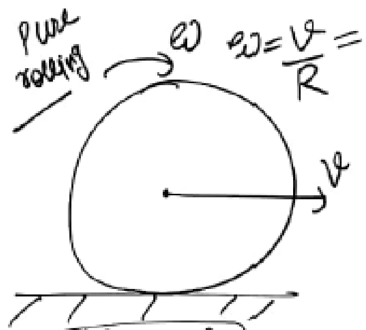
(b) Find work done by friction?

$\underline{\underline{Sol^n}} \Rightarrow \tau_A = 0 \Rightarrow L_i = L_f$

$I = \frac{2}{5}MR^2 = \frac{2}{5}(5)(1)^2 = 2$

$L_i = 2(5) - 5(1)(4) = -10$

$L_f = -2\omega - 5(1)v$



$\omega = \frac{v}{R} = \frac{v}{(1)}$

$\boxed{\omega = v}$ — 1

$-2\omega - 5v = -10$

$-7\omega = -10$

$\boxed{\omega = \frac{10}{7} \text{ rad/s}}$

$\boxed{v = \frac{10}{7} \text{ m/s}}$

$W \cdot D_f = K_f - K_i$

$K_i = \frac{1}{2}(5)(4)^2 + \frac{1}{2}(2)(5)^2$

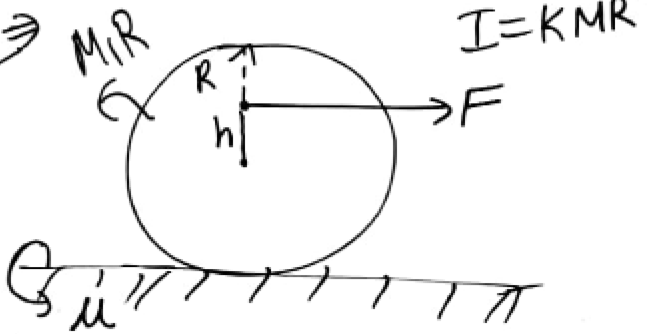
$K_f = \frac{1}{2}(5)\left(\frac{10}{7}\right)^2 + \frac{1}{2}(2)\left(\frac{10}{7}\right)^2$

$W \cdot D_f = K_f - K_i$ ✓

Rotational Motion

Force Equation ⇒

Body ⇒

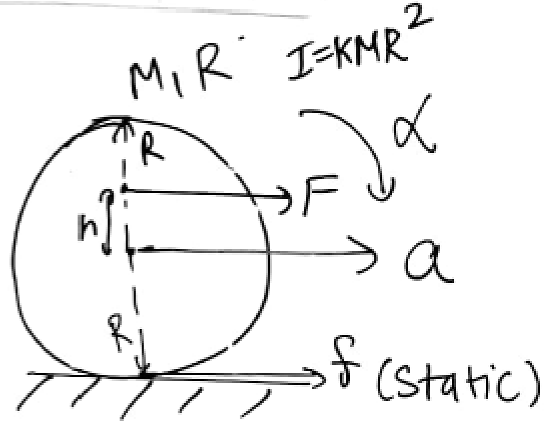


\# if $\mu = 0$

$F = Ma$ | $\tau = I\alpha$

$a = \frac{F}{M}$ | $\alpha = \frac{hF}{I}$

\# when friction is present & sufficient for pure rolling } ⇒ if $\alpha \neq \frac{a}{R}$ then friction tries to make $\alpha = \frac{a}{R}$ (pure rolling)



pure rolling ⇒ $\alpha = \frac{a}{R}$ —(1)

(1) $F_{net} = Ma$

$F + f = Ma$ —(2)

(2) $\tau = I\alpha$ (COM)

$-Fh + fR = -KMR^2\alpha$ —(3)

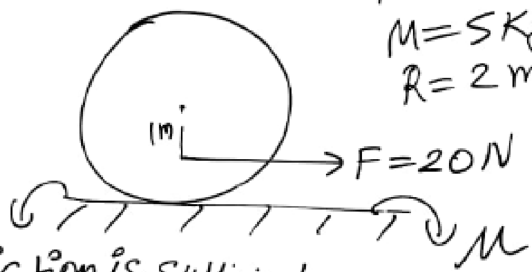
Rotational Motion

Q

Solid sphere

$M = 5\,kg$

$R = 2\,m$

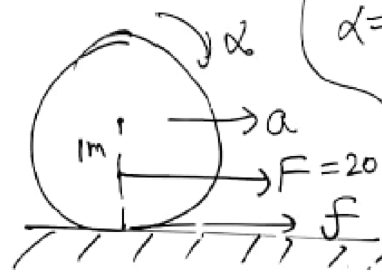


friction is sufficient for pure rolling

(a) Find f, α, a when pure rolling starts?

Solid sphere $I = \frac{2}{5}MR^2$

$= \frac{2}{\cancel{8}}(8)(2)^2 = 8$



(1) Pure rolling $\Rightarrow \alpha = \frac{a}{R}$

$\alpha = \frac{a}{2}$ —(1)

(2) $F_{net} = Ma \Rightarrow 20 + f = 5a$ —(2)

(3) $\tau_{COM} = I_{COM}\alpha \Rightarrow 20(1) + f(2) = -8\alpha$ —(3)

$20 + 2f = 4a$ —(4)

(2) $\times 2$ − (4) $\Rightarrow 40 + 2f = 10a$

$-20 + \cancel{2f} = 4a$

$20 = 14a \Rightarrow \boxed{a = \frac{10}{7}\,m/s^2}$

$\alpha = \frac{a}{2}$

$= \frac{10}{2 \cdot 7}$

$\alpha = \frac{10}{14}$

$\boxed{\alpha = \frac{5}{7}\,rad/s^2}$

$f \Rightarrow$

$20 + f = 5\left(\frac{10}{7}\right)$

$f = \frac{50}{7} - 20$

$\boxed{f = \frac{-90}{7}\,N}$

(b) Find the minimum value of μ required for pure rolling?

$f \leq f_{max}$

$\frac{90}{7} \leq \mu mg$

$\frac{90}{7} \leq \mu\, 5\cancel{0}$

$\boxed{\mu \geq \frac{9}{35}}$

$\boxed{\mu_{min} = \frac{9}{35}} \Rightarrow$ for pure rolling

$\boxed{\mu < \frac{9}{35}} \Rightarrow$ then no rolling slipping occurs