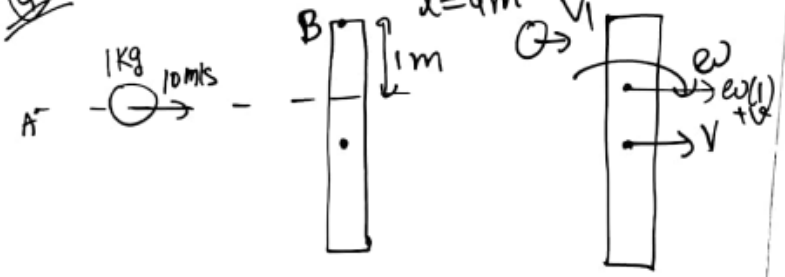
Rotational Motion

Q.

Before collision

$e = \frac{1}{2}$



Find (i) velocity of ball
(ii) velocity of Rod
(iii) Angular velocity of Rod after collision?

Soln ⇒ (Ball + Rod) combined system

$p_i = p_f$

$(1)(10) = V_1(1) + 3V$ — (1)

Angular Momentum

$L_i = L_f$

$0 + 0 = 0 + 3(1)V - \frac{3(4)^2}{12}\omega$

$3V = 4\omega$ — (2)

Coefficient of restitution

$e = \frac{V_s}{V_a} = \frac{(\omega + v) - V_1}{10}$

$\frac{1}{2} = \frac{\omega + v - V_1}{10}$

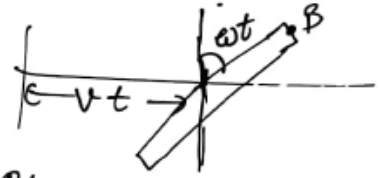
$5 = \omega + v - V_1$ — (3)

$v = \frac{60}{19}$ m/s

$V_1 = \frac{10}{19}$ m/s

$\omega = \frac{45}{19}$ rad/s

(iv) Find co-ordinate of B after time t?



$B\left(vt + \frac{l}{2}\sin\omega t, \frac{l}{2}\cos\omega t\right)$

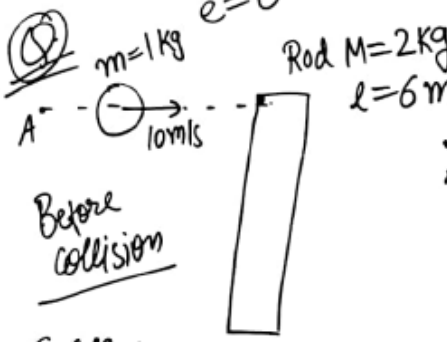
(v) Find the distance moved by the COM when the rod rotates 90°?

Soln ⇒ $\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$

distance $= vt = \frac{60}{19} \times \frac{\pi}{2 \times 45} = \frac{19}{19}$

Rotational Motion

(Q) $e=0$

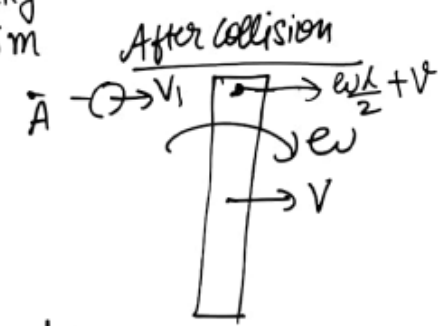


Collision → perfectly inelastic collision?

Find
(i) velocity of ball
(ii) Velocity of Rod
(iii) Angular velocity of Rod After collision?

Soln ⇒ (a) Linear Momentum conserve.

$P_i = P_f \Rightarrow (1)(10) = 1(V_1) + 2V$ — (1)



(b) Angular Momentum conserve.

$0+0 = 0 + M\left(\frac{\ell}{2}\right)V - \frac{M\ell^2}{12}\omega$

$(2)\left(\frac{6}{2}\right)V = \frac{2(6)^2}{12}\omega$

$V = \omega$ — (2)

(c) $e = \frac{V_s}{V_a} = \frac{\left(\frac{\omega\ell}{2}+V\right)-V_1}{10}$

$3\omega + V = V_1$ — (3)

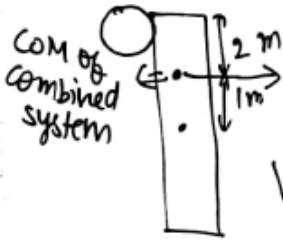
$V = \frac{5}{3}$ m/s (velocity of COM of Rod)

$V_1 = \frac{20}{3}$ m/s

$\omega = \frac{5}{3}$ rad/s

COM of combined system ⇒

$\gamma_{COM} = \frac{m_1\gamma_1 + m_2\gamma_2}{m_1+m_2} \Rightarrow \frac{(1)(0)+2(3)}{3} = 2$ m



$V' = \omega R + V_{COM\ of\ rod}$

$V' = \frac{5}{3}(1) + \frac{5}{3} = \frac{10}{3}$ m/s

Velocity of combined system $= \frac{10}{3}$ m/s

Rotational Motion

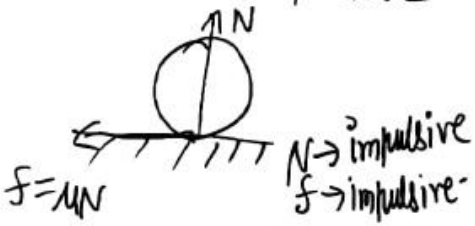
Ball ⇒

off spin (forward)

N → impulsive

f → impulsive

leg spin



N → impulsive

f → impulsive

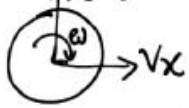
(backward)

(8) I

e

Before collision

After collision



During collision

Find velocity & angular velocity of ball after collision?

Soln ⇒ Impulse = $P_f - P_i$

Y-dirn ⇒ $\int N dt = m(eV_0) - m(-V_0)$

$\int N dt = mV_0(e+1)$

X-dirn ⇒ $\int \mu N dt = mV_x - 0$

$\mu m V_0(e+1) = mV_x$

$$V_x = \mu V_0(1+e)$$

Angular impulse ⇒

$\int \mu N R dt = L_f - L_i$

$\int \mu N R dt = -I\omega - (-I\omega_0)$

$R\mu m V_0(e+1) = -I\omega + I\omega_0$

$$\omega = \frac{I\omega_0 - R\mu m V_0(e+1)}{I}$$