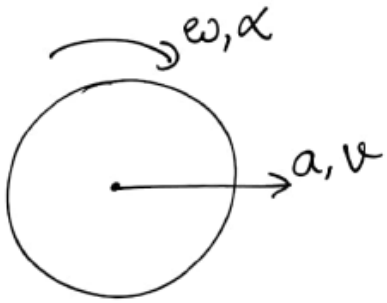
Rotational Motion

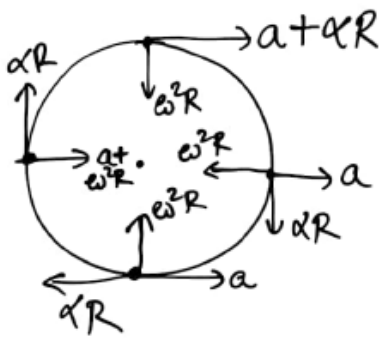
(V) Acceleration ⇒



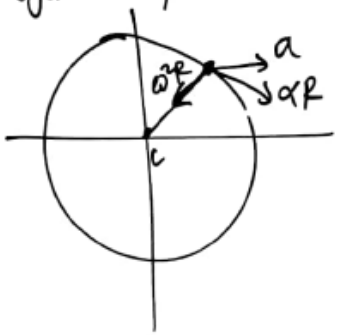
$\vec{a}_{PC} = \vec{a}_P - \vec{a}_C$

$\boxed{\vec{a}_P = \vec{a}_C + \vec{a}_{PC}}$

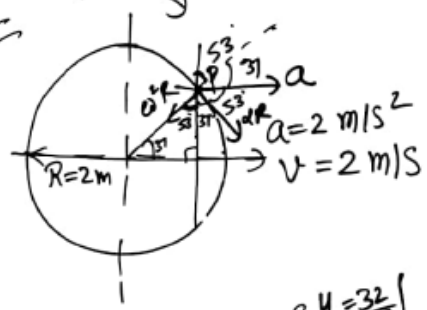
$\vec{a}_P = \vec{a}_C + (\vec{a}_r + \vec{a}_t)$



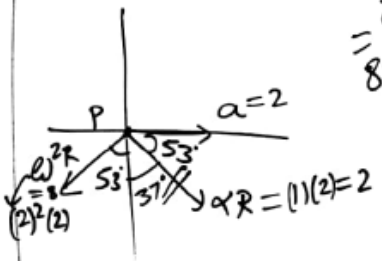
at a general point



Q. $\omega = 2$ rad/s, $\alpha = 1$ rad/s²



Find acceleration at point P ?



$8 \cdot \frac{4}{5} = \frac{32}{5}$

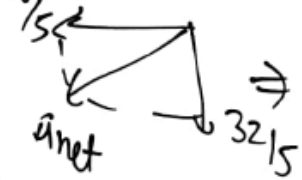
$= 8\sin 53°$

$2 + 2\sin 37 = 2 + 2\left(\frac{3}{5}\right) = 2 + \frac{6}{5} = \frac{16}{5}$

$2\cos 37 + 8\cos 53$

$= 2\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right)$

$= \frac{8}{5} + \frac{24}{5} = \frac{32}{5}$

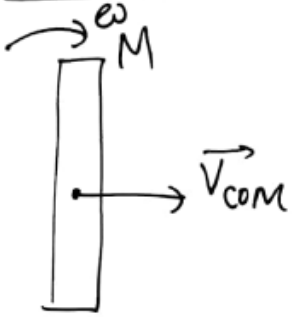


$\Rightarrow a_{net} = \sqrt{\left(\frac{16}{5}\right)^2 + \left(\frac{32}{5}\right)^2} = \frac{16\sqrt{5}}{5}$

$\boxed{a_{net} = \frac{16}{\sqrt{5}}\ m/s^2}$

Rotational Motion

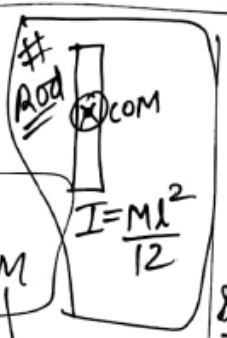
(5) Linear Momentum ($\vec{P}$) ⇒



$$\vec{P} = M\vec{V}_{COM}$$

M → Total mass of body

$\vec{V}_{COM}$ → velocity of COM

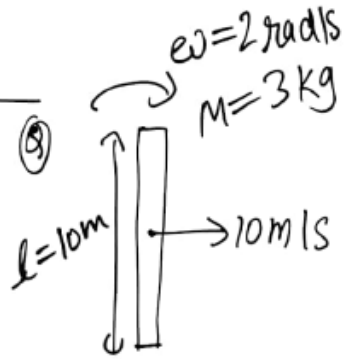


(6) Kinetic Energy (K.E.) ⇒

$$K.E. = \frac{1}{2}MV_{COM}^2 + \frac{1}{2}I\omega^2$$

$$K.E_{system} = \left(\begin{matrix}K.E \text{ of} \\ COM\end{matrix}\right) + \left(\begin{matrix}K.E. \text{ of particle} \\ w.r.t. \text{ COM}\end{matrix}\right)$$

Q.

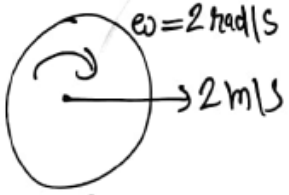


Find linear momentum & Kinetic energy of the system?

Sol ⇒ $\vec{P} = 3(10) = 30$ Kg-m/s

$$K.E. = \frac{1}{2}(3)(10)^2 + \frac{1}{2}\left(\frac{3(10)^2}{12}\right)(2)^2 = 150 + 50 = 200\ J$$

Q.



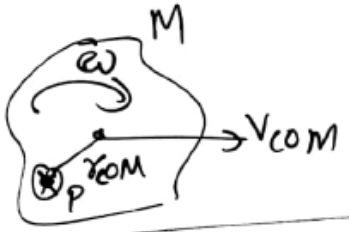
A Solid sphere of Mass 5Kg & radius R = 2 m Find Kinetic energy?

$$I = \frac{2}{5}MR^2$$

$$K.E. = \frac{1}{2}(5)(2)^2 + \frac{1}{2}\cdot\frac{2}{5}(5)(2)^2(2)^2 = 26\ J$$

Rotational Motion

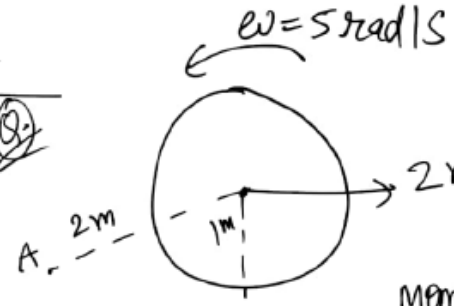
Angular Momentum ⇒



$$\vec{L} = m(\vec{r}_{COM} \times \vec{v}_{COM}) + I\omega$$

$$\vec{L} = \left(\begin{array}{c}\text{Angular Momentum}\\ \text{of COM}\end{array}\right) + \left(\begin{array}{c}\text{Angular momentum}\\ \text{of body w.r.t.}\\ \text{COM}\end{array}\right)$$

Q.



Solid Sphere
$M = 5\,kg$
$R = 1\,m$

Find angular momentum about A ?

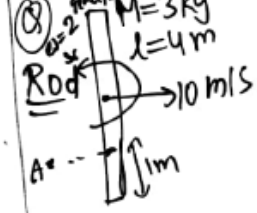
$I = \frac{2}{5}MR^2 = \frac{2}{5}(5)(2)^2 = 8\ kg\text{-}m^2$ ⊙ = +ve

$\vec{L} = m(r_{COM} \times V_{COM}) + I\omega$

$\vec{L} = (mr_{\perp}V) + I\omega = 5(1)(2)\otimes + 8(5)\odot$

$\vec{L} = -10 + 40 = 30\ \odot$

Q.



Find $\vec{L}$ about A ?

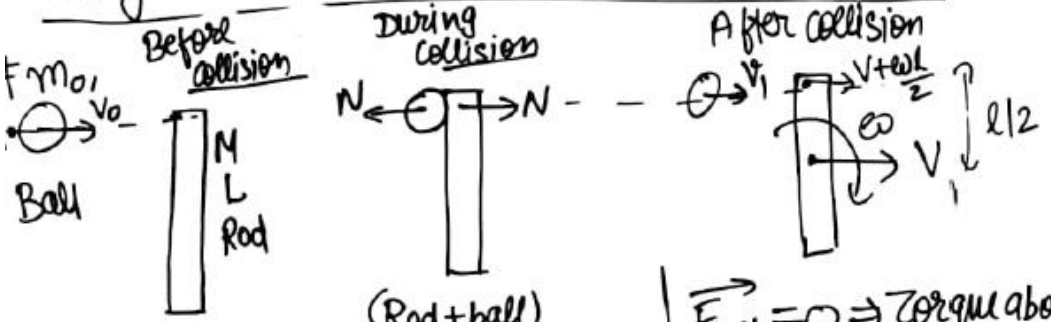
$I = 3(1)(10)\otimes + \frac{3(4)^2}{12}(2)\odot$

$L = -30 + 8 = -22$

$\vec{L} = 22\otimes$

Rotational Motion

Angular Momentum Conservation ⇒



Horizontal frictionless

(Rod + ball) combine system

Linear momentum conserve.

$\vec{F}_{net}=0 \Rightarrow$ Torque about each & every point is same.

$\vec{\tau}_{net}=0$

* Angular Momentum conserve

① Linear Momentum conserve ⇒

$P_i = P_f \Rightarrow m_0 v_0 = m_0 v_1 + MV$

② Angular Momentum conserve ⇒ $L_i = L_f$ about A

$$0 + 0 = 0 + \underbrace{\frac{Ml}{2}V - \frac{Ml^2}{12}\omega}_{\text{Rod}}$$

ball Rod ball

③ Coefficient of restitution ⇒

$e = \frac{V_s}{V_a} \Rightarrow$

$$e = \frac{\left(V + \frac{\omega l}{2}\right) - V_1}{V_0}$$