Session – 40th – Modern Physics III – Radioactivity – (Mean life)

Recap :– Rutherford-Soddy theory, Half life, activity

Q.1 What is power o/p of $^{235}_{92}U$ reactor if it takes 30 days to use 2kg fuel. (Q-value – 185 MeV)

Soln → $\text{power} = \frac{\text{Energy gen.}}{\text{time}}$

Q-value ⇒ 185 MeV

$E = 185 \times N_{U^{235}}$

235 gm – N_A

2×10^3 gm ≡ $\frac{2\times10^3 \times 6.023\times10^{23}}{235}$

$E = \frac{2\times10^3\times6.023\times10^{23}}{235} \times 185$ MeV

$\text{Power} = \frac{2\times10^3\times6.023\times10^{23}\times185\times1.6\times10^{-19}}{235\times30\times24\times3600}$ (MJ/s)

$= 58.53$ MW (MJ/s → MW)

Q.2. In an old rock, $N_U : N_{Pb} = 1:2$, $T_{1/2}^U = 4.5\times10^9$ yrs. $N_{Pb}^\circ = 0$. How old is rock?

Q.3 $T_{1/2} = 5$ yrs. What is probability of disintegration of a nucleus in 10 yrs

A). 0.5 B). 0.25

C). 0.75 D). 0.60

Q.5. 2/3rd of a sample disintegrates in 7 days. How much fraction will decay in 21 days?

Q.4. Find relation b/w active nuclei (N) v/s activity (A)?

A). 

B). 

C). 

D). 

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$U \longrightarrow Pb$

@ $t=0$	N_0	0
@ $t=t$	N	$2N$

active nuclei of U → $N = N_0 e^{-\lambda t}$

$(N_0 - N) = 2N$

$\Rightarrow 3N = N_0$

$\Rightarrow N = N_0/3$

$\frac{N_0}{3} = N_0 e^{-\lambda t}$

$1/3 = e^{-\lambda t}$

$\ln 3 = -\lambda t$

$t = \frac{\ln 3}{\lambda}$

$T_{1/2} = \frac{\ln 2}{\lambda}$

$\lambda = \frac{\ln 2}{T_{1/2}}$

$\Rightarrow t = \frac{\ln 3}{\lambda}$

$= \frac{\ln 3}{\ln 2}(T_{1/2})$

$= \frac{\ln 3}{\ln 2} \cdot 4.5 \times 10^9$ yrs.

$t = 7.1 \times 10^9$ yrs.

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Soln S).

$X \longrightarrow Y$

@t=0 N_0

@t=7d $N_0/3$

@t=21d ? (N') → $(N_0/27)$

$N = N_0 e^{-\lambda t}$

$\frac{N_0}{3} = N_0 e^{-\lambda \cdot 7} \Rightarrow \boxed{e^{-7\lambda} = 1/3}$

$N' = N_0 e^{-\lambda \times 21}$

$\frac{3N'}{N_0} = e^{-21\lambda + 7\lambda}$

$= e^{-14\lambda}$

$= (e^{-7\lambda})^2 = (1/3)^2$

$\Rightarrow 3N'/N_0 = 1/9 \Rightarrow \boxed{N' = N_0/27}$

$N_{decayed}^{21\,days} = N_0 - N_0/27$

$= \frac{26}{27} N_0$

Q.3 $T_{1/2} = 5$ yrs

What is probability of disintegration of a nucleus in 10 yrs

A). 0.5 B). 0.25

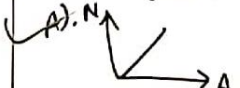
C). 0.75 D). 0.60

Q.5 $2/3^{rd}$ of a sample disintegrates in 7 days. How much fraction will decay in 21 days?

$\frac{26}{27}$

Q4. Find relation b/w active nuclei (N) v/s activity (A)?

A).



B).

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Recap :– Rutherford-Soddy theory, Half life, activity

Mean life of a radioactive substance → Average life of all the nuclei.

$$T_m = \frac{\text{Sum of the lives of all nuclei}}{\text{total no. of all nuclei.}} = \frac{\int (dN)\, t}{N_0}$$

$X \rightarrow Y$

@t=0 N_0 0

@t=∞ 0 N_0

	@t=0	@t=t	@t=t+dt
No. of active nuclei of X	N_0	N	$N - \boxed{dN}$

Q. what is the life of these dN nuclei ?

Ans. → (A) dt (B) t+dt (C) t (D) 0

Ans. All the dN nuclei were present from t=0 to t=t.

Q. what is the life of $(N_0 - N)$ nuclei ?

Ans. All will have different life time.

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@ $t = t$: N ; @ $t = t + dt$: $N - |dN|$

$$T_m = \frac{\int (dN)\,t}{N_0} = \int_0^\infty \frac{\left|\frac{dN}{dt}\right| \cdot t\,dt}{N_0}$$

$N = N_0 e^{-\lambda t}$

$\frac{dN}{dt} = N_0(-\lambda)e^{-\lambda t} = -N_0\lambda e^{-\lambda t}$

$\left|\frac{dN}{dt}\right| = N_0\lambda e^{-\lambda t}$

$$T_m = \int_0^\infty \frac{N_0\lambda e^{-\lambda t} \cdot t\,dt}{N_0} = \lambda \int_0^\infty e^{-\lambda t}\, t\,dt$$

(I = $e^{-\lambda t}$, II = t)

* Integration by parts.

$$\int_a^b I \cdot II = [I \times \int II]_a^b - \int_a^b (I' \times \int II)$$

$$= \lambda\left[\left[t \times \left(\frac{e^{-\lambda t}}{-\lambda}\right)\right]_0^\infty - \int_0^\infty \left(\frac{dt}{dt}\right) \times \left(\frac{e^{-\lambda t}}{-\lambda}\right) dt\right]$$

$$= \lambda\left[\left[\frac{-t}{\lambda e^{\lambda t}}\right]_0^\infty + \int_0^\infty \left(\frac{1}{\lambda}\right) \cdot e^{-\lambda t}\,dt\right]$$

$$= \lambda \times \frac{1}{\lambda} \int_0^\infty e^{-\lambda t}\,dt = \int_0^\infty e^{-\lambda t}\,dt$$

$$T_m = \int_0^\infty e^{-\lambda t}\,dt = \frac{e^{-\lambda t}}{(-\lambda)}\Big|_0^\infty = 0 - \left(\frac{1}{-\lambda}\right) \cdot 1$$

$$\boxed{T_m = \frac{1}{\lambda}}$$

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3 variants of Exponential decay law (from exam p.o.v.)

$N = N_0 e^{-\lambda t}$

$\downarrow \times \lambda$

$\lambda N = \lambda N_0 e^{-\lambda t}$

$-\frac{dN}{dt} = \lambda N$

$A = \lambda N$

A_0 → Activity @ $t = 0$

$A = A_0 e^{-\lambda t}$

A, Activity @ $t = t$

$\times \left(\frac{M_w}{N_A}\right)$

$m = m_0 e^{-\lambda t}$

1st variant

$N = N_0 e^{-\lambda \frac{t}{T_{1/2}} \times T_{1/2}}$

$= N_0 e^{-\left[\lambda\left(\frac{t}{T_{1/2}}\right) \times \frac{\ln 2}{\lambda}\right]}$

$= N_0 e^{-\frac{t}{T_{1/2}} \ln 2}$

$= N_0 e^{\ln 2^{\left(-t/T_{1/2}\right)}}$

$N = N_0 2^{-t/T_{1/2}}$

$A = A_0 2^{-t/T_{1/2}}$

$m = m_0 2^{-t/T_{1/2}}$

2nd variant

$N = N_0 e^{-\lambda t}$

$N = N_0 e^{-t/T_m}$

$A = A_0 e^{-t/T_m}$

$m = m_0 e^{-t/T_m}$

3rd variant