Session 24: Ray Optics – Chromatic Aberration & Optical Instruments

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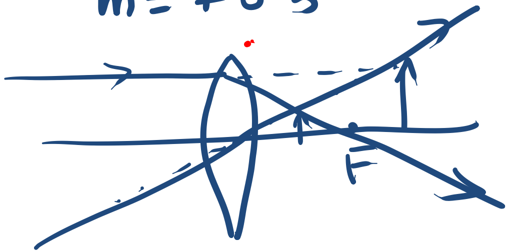
- Recap
- Newton's formula for lenses
- Dispersion through lenses
    - Chromatic aberration
    - Dispersive power
    - Achromatic condition
- Optical Instruments
    - Human Eye
    - Defects of vision
    - Microscopes & Telescopes

Ray Diagram - Recap (Convex lens)

Ex.1. Find region of obj & img.

$m = +0.5$



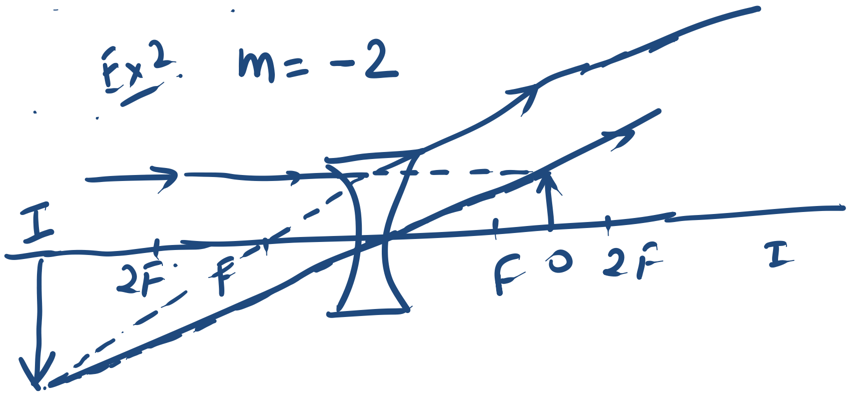
$m = -0.5$

$m = +2$

$m = -2$

Concave lens

Ex.2. $m = -2$

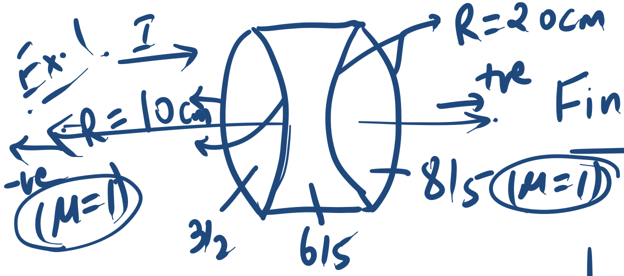


$m = +2$

$m = +0.5$

$m = -0.5$

Equivalent Power → (Re Cap)



Find F_{eq}

$$P_{eq} = P_1 + P_2 + P_3$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{10} + \left(\frac{-3}{100}\right) + \left(\frac{3}{50}\right)$$

$$= \frac{10 - 3 + 6}{100}$$

$$f_{eq} = \frac{100}{13} = \boxed{7.68}$$

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{-10}\right)$$

$$= \frac{1}{2} \times \frac{2}{10}$$

$$f_1 = 10\,cm.$$

$$\frac{1}{f_2} = \left(\frac{6}{5} - 1\right)\left(\frac{1}{-10} - \frac{1}{20}\right)$$

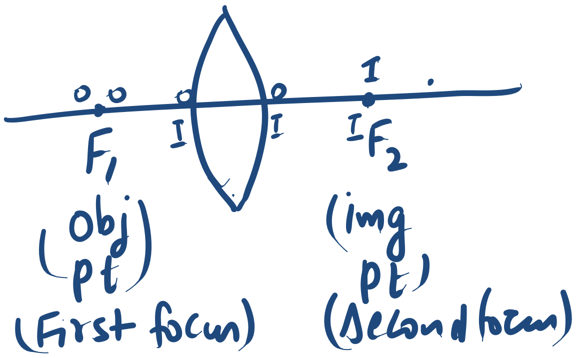
$$= \frac{1}{5} \times \frac{-3}{20}$$

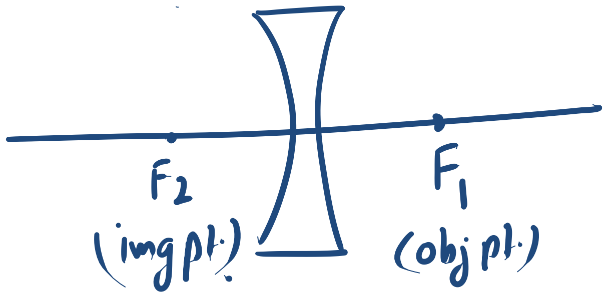
$$f_2 = -100/3$$

$$\frac{1}{f_3} = \left(\frac{8}{5} - 1\right)\left(\frac{1}{20} - \frac{1}{-20}\right)$$

$$= \frac{3}{5} \times \frac{2}{20} \Rightarrow f_3 = 50/3$$

Newton's formula for lenses :-





Both obj & img always lie on opposite sides of their respective focii F_1 & F_2.
(obj pt.) (img pt.)

$$x_1 x_2 = f^2$$

$$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$x_1 \rightarrow$ dist of obj from obj. pt. (first focus)

$x_2 \rightarrow$ " " img " img pt. (second focus)

$f \rightarrow$ focal length of lens.

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Chromatic Aberration

(Dispersion through lenses)

✓ $\mu(\lambda) = a + b/\lambda^2$

$v(\lambda) = c/\mu(\lambda)$.

$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Differentiate →

$-\frac{df}{f^2} = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Divide

$\Rightarrow -\frac{df}{f} = \frac{d\mu}{(\mu-1)}$

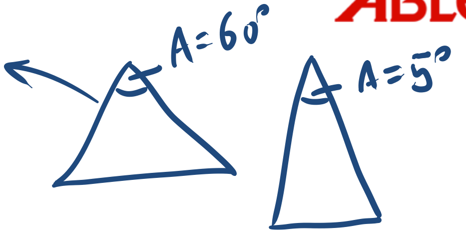
Dispersive power, ω

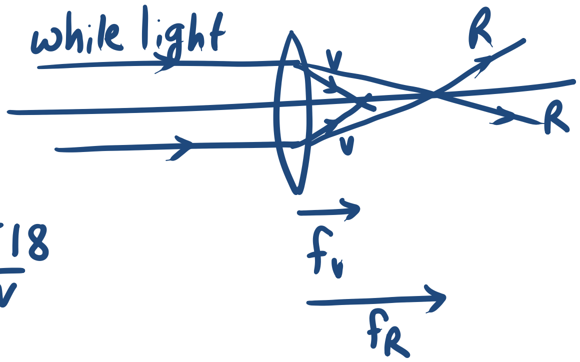
$\omega = \frac{\delta_V - \delta_R}{\delta_Y}$

$= \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)}$

$\boxed{\omega = \frac{d\mu}{\mu - 1}}$

1.512 (R) to 1.518 (V) → y





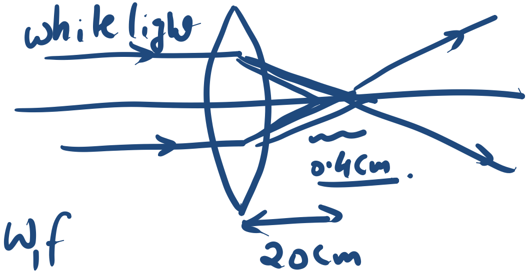
Longitudinal (axial) chromatic aberration (L < A) $= -df = -(f_R - f_V) = \frac{f\,d\mu}{(\mu-1)}$.

$= f\omega$.

$\Rightarrow \boxed{-df = f\omega}$.

Ex. $\omega_1 = 0.02$ for flint glass → made a lens (convex) → $f = 20cm$.

Find LCA?

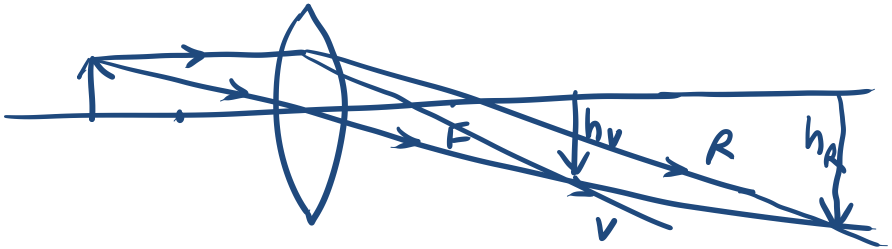


$LCA = \omega_1 f$

$-df = 0.02 \times 20cm$

$= 0.4\ cm$

Lateral chromatic Aberration →



$$\text{Lateral chromatic Aberration} = (h_V - h_R)$$

$$= -(h_R - h_V)$$

$$m = \frac{f}{f+u} = \frac{h_I}{h_o}$$

Variation of different quantities on EM spectrum

deviation ↑ ($\delta_v > \delta_R$) } visible region

μ, Refractive index ↑

freq ↑

$E = h\nu$ ↑

←

γ X UV VIBGYOR IR MM Radio

→

(wavelength) λ ↑

Visible region {

(focal length) f ↑

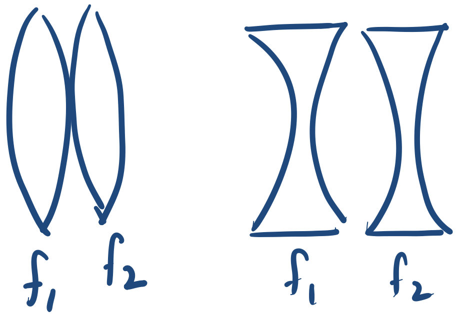
(Image length) h_I ↑

(h_I) → (LCA)

Condition for Achromatism

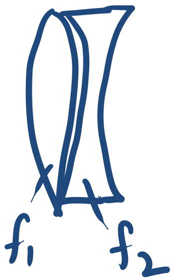
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

eq. focal length of a combination of lenses.



Differentiate :-

$$-\frac{dF}{F^2} = -\frac{df_1}{f_1^2} + -\frac{df_2}{f_2^2} = 0$$



$dF = 0$

$-df = \omega f$

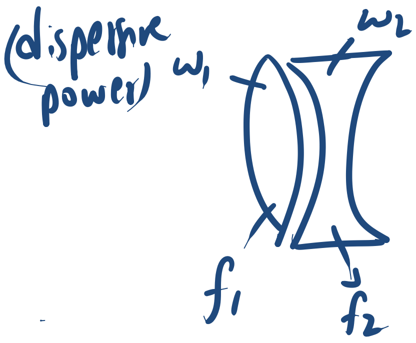
$$\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

$$\Rightarrow \boxed{\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0}$$

Condⁿ for Achromatism

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$



1). ω_1 & $\omega_2 > 0 \Rightarrow f_1$ & $f_2 \rightarrow$ opp signs.

$$\frac{1}{F_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

2). If $\omega_1 = \omega_2 \Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = 0 \Rightarrow \frac{1}{F_{eq}} = 0$

(No more a lens) $F_{eq} = \infty$

3). $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ } What's the condⁿ if we want convergent combination.

$\Rightarrow F > 0$

$|f_C| < |f_D|$

$\omega_C < \omega_D$

4). If $|f_1| = |f_2|$

$\Rightarrow F_{eq} \rightarrow \infty$.

(No more a lens).

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Ex. Flint glass ($\omega_1 = 0.02$) and Crown glass ($\omega_2 = 0.04$) combined to form a pair of convex & concave lens. Achromatic combination.

$F_{eq} = 40$ cm. Find f_1 & f_2.

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad -(1) \qquad \frac{1}{F_{eq}} = \frac{1}{40} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{0.02}{f_1} + \frac{0.04}{f_2} = 0.$$

$$f_1 = \frac{-f_2}{2} \qquad \frac{1}{40} = \frac{-2}{f_2} + \frac{1}{f_2}$$

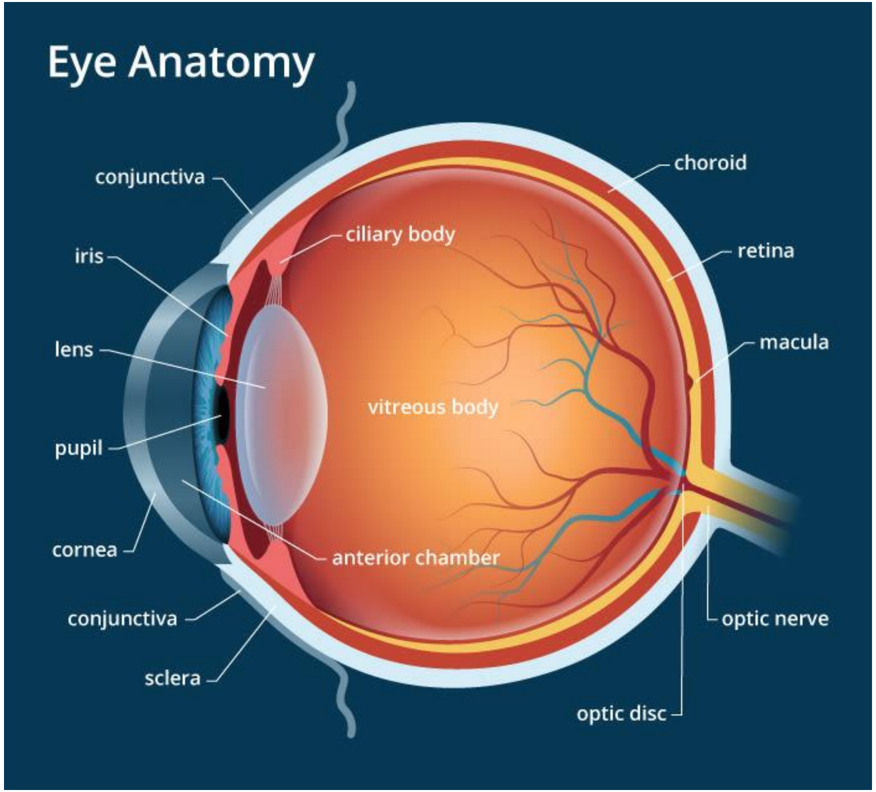
$$\frac{1}{40} = -\frac{1}{f_2}$$

$\Rightarrow$ $f_2 = -40$ cm & $f_1 = 20$ cm

$\lvert f_1 \rvert < \lvert f_2 \rvert$	$\omega_1 < \omega_2$

Optical Instruments

Human Eye



Structure of Eye :-

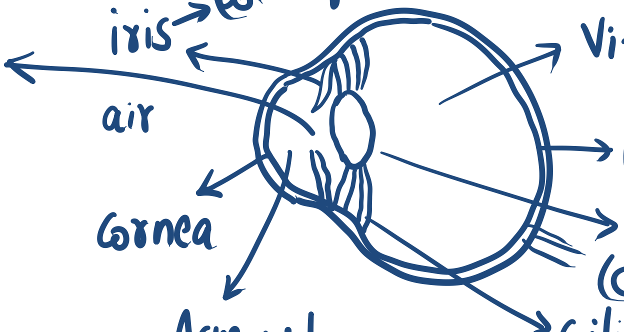
iris → color to our eyes
↳ (amt of melanin)

→ pupil → amt of light that would enter
(diaphragm)
→ always black
(light that goes in doesn't come back)

low light condⁿ → pupil will expand

good light " – → " – contract.

Lens → Curvature is controlled by ciliary muscles.
focal length 2.5cm → 2.2cm

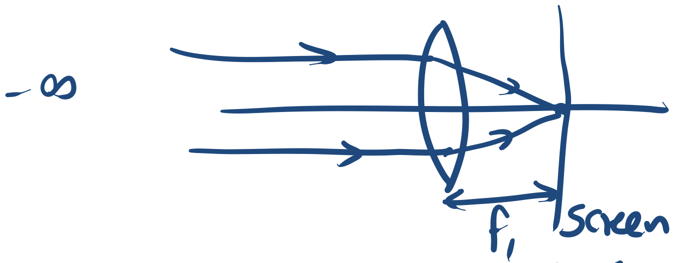


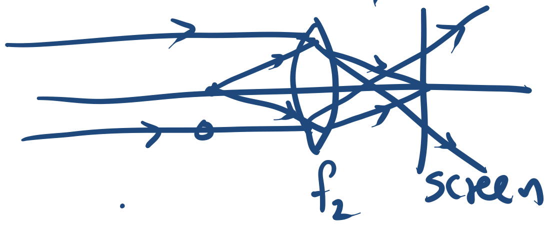
$$\mu_{rel} = \frac{1.396}{1.336}$$

$$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

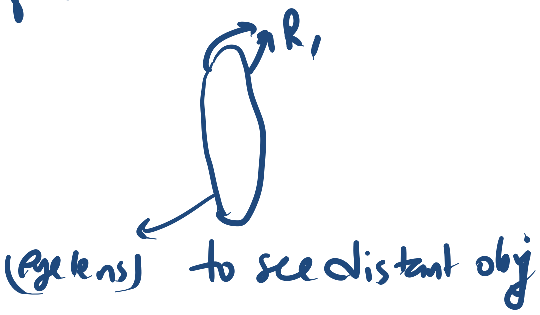
1). Eyes are naturally most relaxed to see distant obj.
Ciliary muscles are least strained

2). when we see a nearby obj., focal length of eyelens decreases. ⇒ Ciliary muscles are most strained.





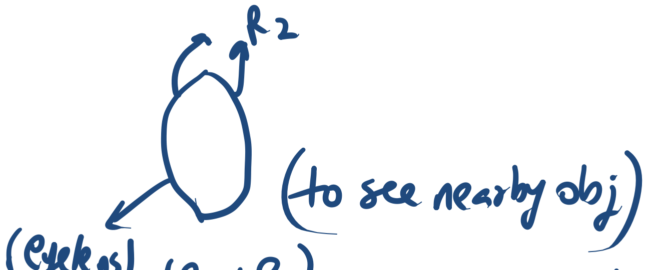
$(f_2 < f_1)$.



$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\frac{1}{f} = (\mu - 1) \times \frac{2}{R}$$

$$f\downarrow \Rightarrow R\downarrow$$



$(R_2 < R_1)$.