Session 23: Ray Optics – Refraction @ Lenses

- Recap
- Displacement method
- Ray Diagram for lenses
- Equivalent power of lenses and mirrors
- Chromatic aberration

Recap

1). Lens Maker formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

2). $\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\mu_{rel.} = \frac{\mu_{Lens}}{\mu_{surr.}}$$

Displacement Method

↳ Used to find focal length of converging lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

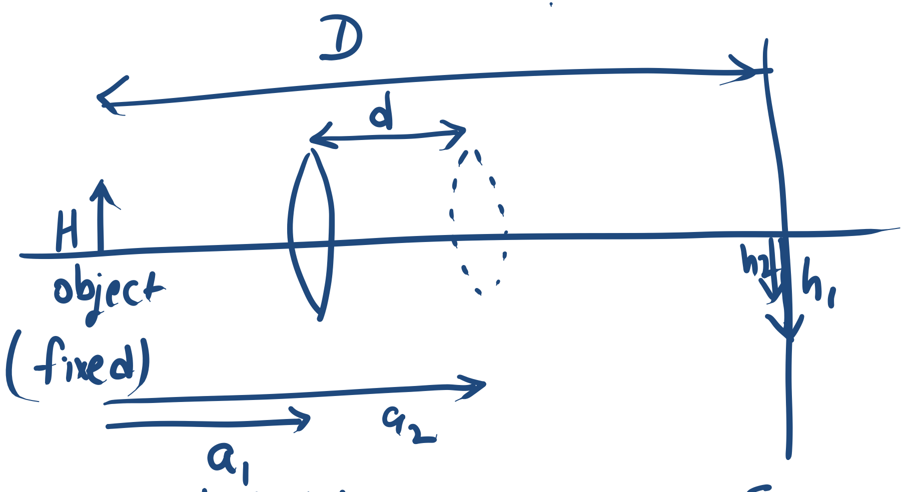
$u = -a, \; v = (D - a)$

$$\frac{1}{D-a} - \frac{1}{-a} = \frac{1}{f}$$

$$\frac{\cancel{a} + (D - \cancel{a})}{(D-a)a} = \frac{1}{f}$$

$$aD - a^2 = fD$$

$$a^2 - aD + fD = 0 \begin{array}{l} \nearrow a_1 \\ \searrow a_2 \end{array}$$



(dist of lens from obj)

Screen (fixed)

$$a_1 + a_2 = D \qquad a_1 a_2 = fD$$

$$|a_1 - a_2| = d = \sqrt{(a_1 + a_2)^2 - 4a_1a_2} = \sqrt{D^2 - 4Df} = d$$

$$\boxed{\begin{array}{l} ax^2 + bx + c = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array} \\ \alpha + \beta = -b/a \\ \alpha\beta = c/a \end{array}}$$

Displacement Method

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$$|a_1 - a_2| = d = \sqrt{D^2 - 4fD}$$

1st case: $\left(f > D/4\right)$ or $\left(D < 4f\right)$ $\Big\}$ $\Rightarrow d \rightarrow$ img.

(No lens position will give real image on screen)

2nd case: $D = 4f$, $f = D/4$ $\Big\} \longrightarrow d = 0$

$\Rightarrow a_1 = a_2 = D/2$

$(a_1 + a_2 = D)$

3rd case: $D > 4f$, $f < D/4$ $\Big\} \longrightarrow \boxed{f = \dfrac{D^2 - d^2}{4D}}$ ✓ $\boxed{D \geq 4f}$

Magnification, m :-

$$m_1 = \frac{v_1}{u_1} = \frac{(D - a_1)}{-a_1} = \frac{a_2}{-a_1} = \frac{-h_1}{H}$$

$$m_2 = \frac{v_2}{u_2} = \frac{(D - a_2)}{-a_2} = \frac{a_1}{-a_2} = \frac{-h_2}{H}$$

$$m_1 \times m_2 = +1$$

$$\boxed{h_1 h_2 = H^2}$$

Ex: (Disp. Method)

Convex lens

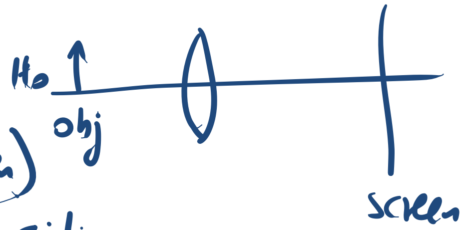
$h_I = 4$ cm (on screen)

lens → shifted to new position

↳ New image obtained.

$h'_I = 16$ cm (on screen)

Find H_o?



Sol^n

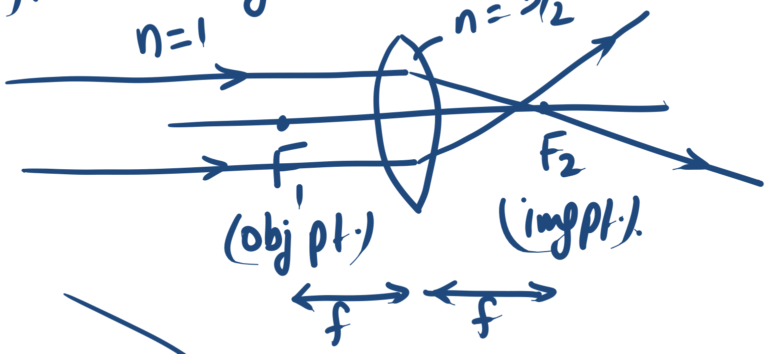
$H_o^2 = h_1 h_2$

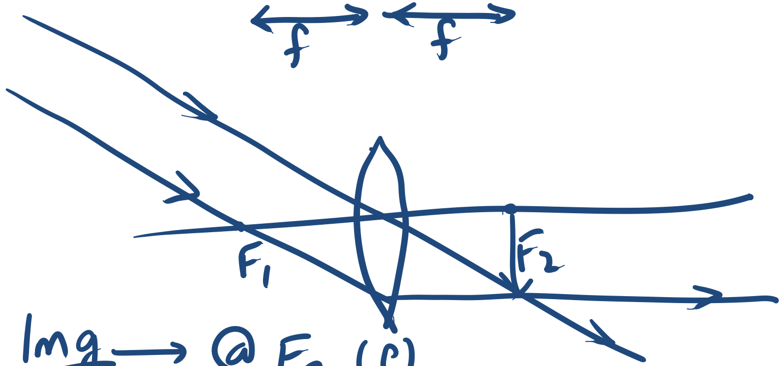
$H_o = \sqrt{h_1 h_2} = \sqrt{4 \times 16} = \underline{8\text{ cm}}$

Ray diagrams through Convex lens

(Similar to Concave Mirror ray diagrams)

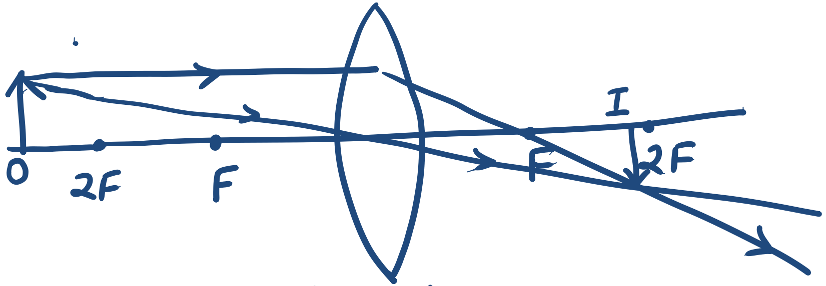
(i). Real obj @ $-\infty$





Img → @ F_2 (f)
- → real
- → inv
- → diminished.

(ii). Real obj → $(-\infty, -2F)$



Img → b/w $(+F, +2F)$
- → Real
- → Inv
- → Smaller

Ray Diagrams (Convex lens)

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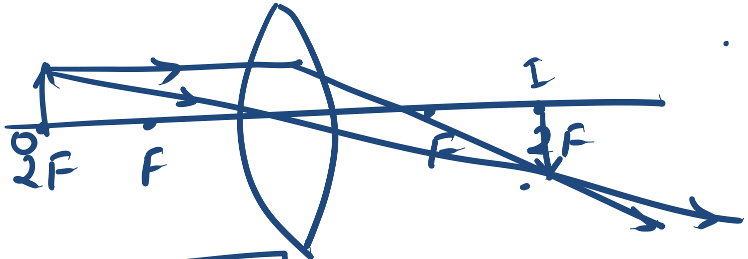
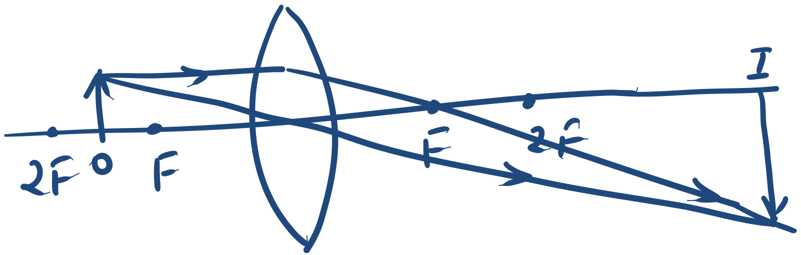
(iii). Real obj @ -2F



$\boxed{\begin{array}{l} \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \\ u = -2F \\ v = 2F \end{array}}$

img → @2F
↳ Real
↳ inv
↳ same size.

(iv). Real obj b/w (-2F, -F)



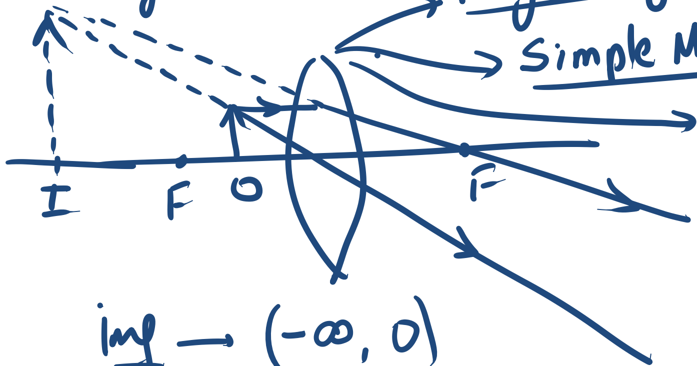
Img → b/w (2F, ∞)
↳ Real
↳ Inv
↳ larger.

(V) Real obj @ F

img → ∞.

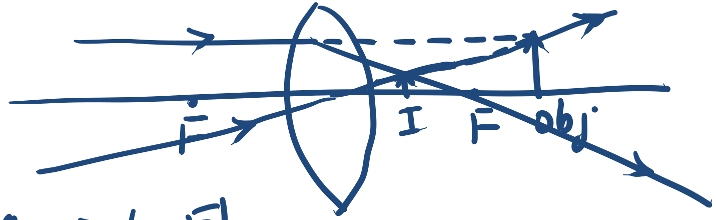
(vi). Real obj b/w (-F, 0)

→ ~~Magnifying glass~~
→ Simple Microscope
→ Eyepiece (in Microscopes & telescopes)



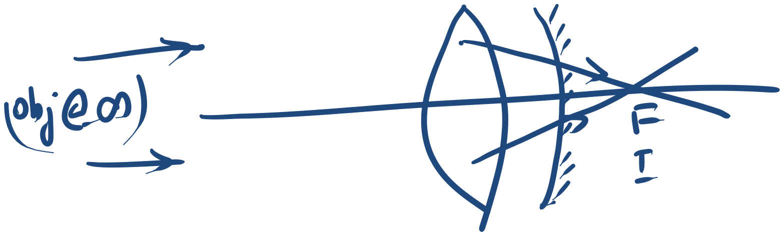
img → (-∞, 0)
↳ Virtual
↳ erect
↳ larger

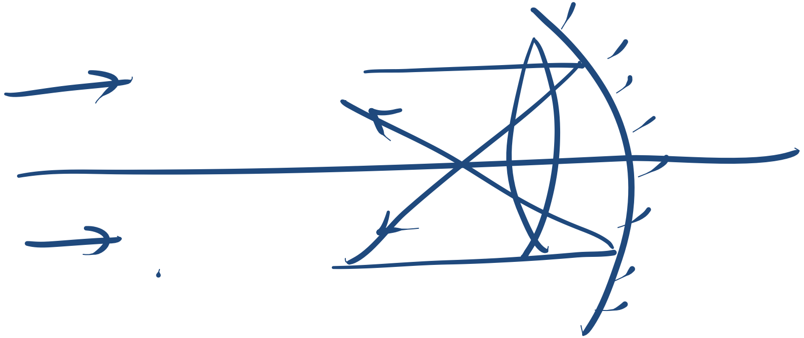
(vii) Virtual obj (0, ∞)



Img → (0, F)
↳ Real
↳ small
↳ erect.

How can we make virtual obj in real life?

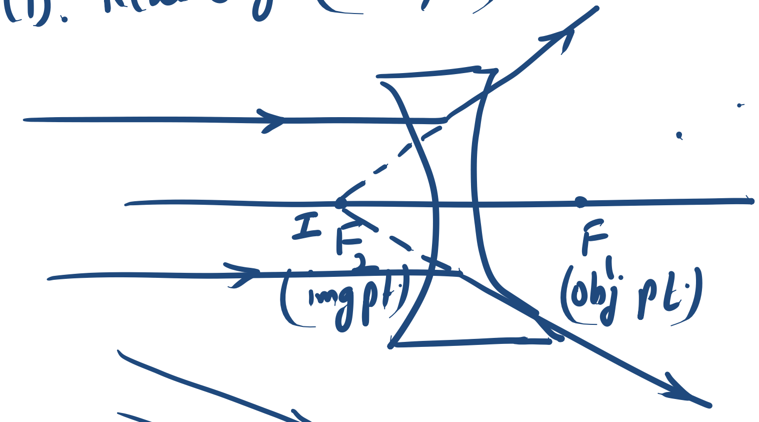


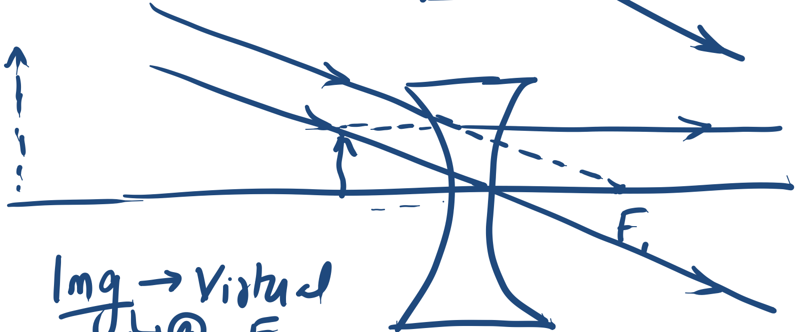


Ray Diagram for Concave Lens :-

(very similar to convex Mirror ray diagrams).

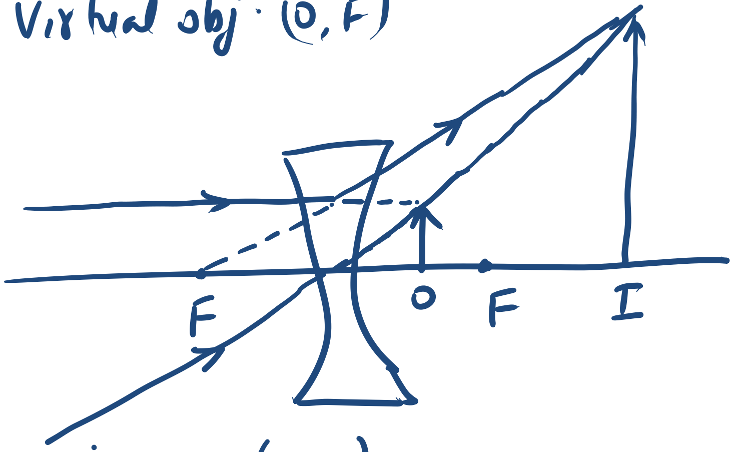
(i). Real obj $(-\infty, 0)$





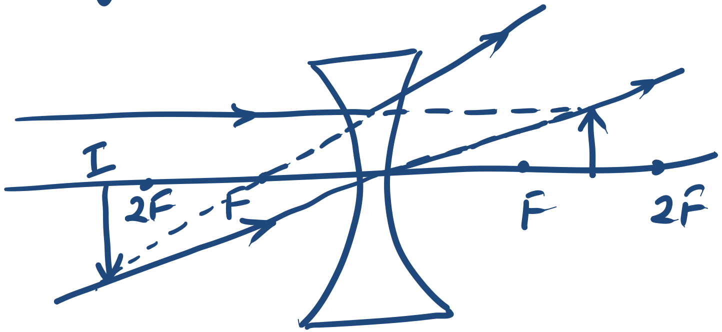
Img → Virtual
↳ @ −F
↳ diminished
↳ erect.

(ii). Virtual obj · (0, F)



img → $(0, \infty)$
↳ Real
↳ erect
↳ large.

(iii) Virtual obj b/w F & 2F.



img → Virtual
↳ inv
↳ $(-\infty, -2F)$
↳ larger

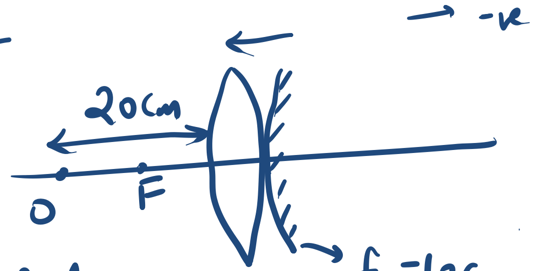
Comparison b/w plane surface & lenses

plane surface	Convex lens	Concave lens
Real obj → Virtual img	Real obj → virtual img	Real obj → Virtual img
Virtual obj → Real image	Virtual obj → Real img	Virtual obj → Real img
	Real obj → Real img	Virtual obj → Virtual img.
$\boxed{m=1}$	$m \to \gtrless 1$ ↳ +ve, −ve	$m \to \gtrless 1$ ↳ +ve, −ve
$m = \left(\frac{v/\mu_2}{u/\mu_1}\right)$		

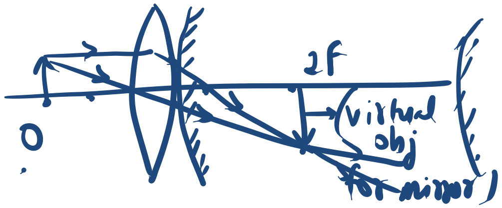
Vel. of img is along vel. of obj (always).

Examples

Ex.1



Find v of final image.



Steps :-

1). Refraction @ lens v_1

2). $v_1 \rightarrow$ obj for mirror get v_2

3). $v_2 \rightarrow$ obj for lens (Coordinate syst change)

($v_3 \rightarrow$ final image)

Sol^n → S 1). $v_1 = \frac{fu}{f+u} = \frac{(+10)(-20)}{10+(-20)} = \frac{-200}{-10} = +20cm$. (Real img)

2). $u_2 = +20cm$

$v_2 = \frac{uf}{u-f} = \frac{20 \times (+10)}{20-10} = +20cm$

3). $u_3 = -20cm$

$v_3 = \frac{fu}{f+u} = \frac{(+10)(-20)}{10+(-20)} = \frac{+200}{-10} = +20cm$.

~~Image~~ (Image will form over O)

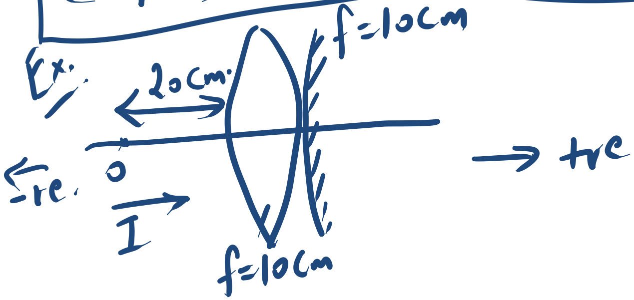
Equivalent Power of a combination of lenses & mirror

(lenses are thin & all of the lenses & mirrors are close to each other)

$$P_{eq} = P_1 + P_2 + \dots\dots$$

$$\underset{\text{(Diopters)}}{P_M} = \frac{-1}{f_M(\text{in m})} \qquad \underset{\text{(Diopters)}}{P_L} = \frac{1}{f_L(\text{in m})}$$

Ex.



Sol^n → Equivalent system → Mirror

$$P_{eq} = \frac{-1}{f_{eq}} = P_L + P_M + P_L$$

$$= \frac{1}{f_L} + \left(\frac{-1}{f_M}\right) + \frac{1}{f_L}$$

$$\frac{-1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_M}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_M} - \frac{2}{f_L}$$

$$= \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$$

$$\Rightarrow f_{eq} = -10\text{ cm}$$

$$u = -20\text{ cm}$$

$$v = \frac{uf}{u-f} = \frac{(-20)(-10)}{-20-(-10)} = \frac{200}{-10} = \boxed{-20}$$