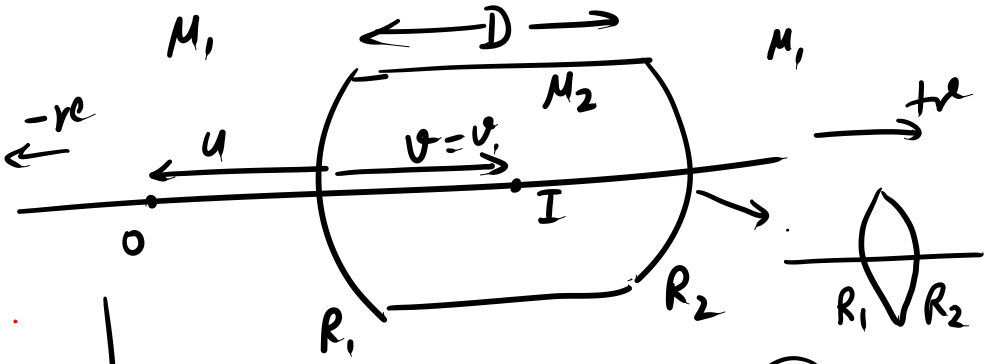
ABLES® KOTA

Session 22: Ray Optics – Refraction @ Lenses

- Recap
- Lens maker formula
- Focal length of lens
- Displacement method of finding focal length
- Ray diagram

Lens Maker Formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \left(\frac{\mu_2 - \mu_1}{R}\right)$$



@ I surface

$v = v_1 \quad R = +R_1$

$u = u$

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{— (1)}$$

@ II surface

$u = -(D - v_1)$

$v = v$

$R = -R_2$

$$\frac{\mu_1}{v} - \frac{\mu_2}{-(D - v_1)} = \frac{\mu_1 - \mu_2}{R_2} \quad \text{— (2)}$$

Let's take $D \to 0$ & eliminate v_1

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{— (1a)}$$

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{-R_2} \quad \text{— (2a)}$$

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

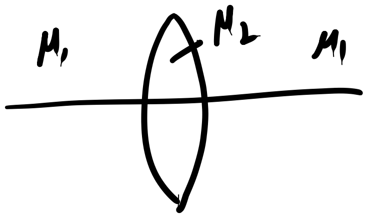
$$\boxed{\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

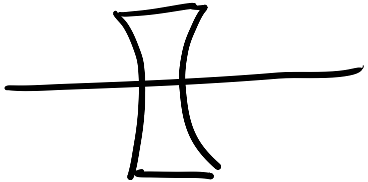
Lens Maker formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$f \rightarrow$ focal length of the lens

$\mu_{rel} = \mu_2 / \mu_1 = {}_1\mu_2$





Focal length of lenses

$$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$(\mu_{rel} - 1)$:

+ve if med → rarer, lens → denser

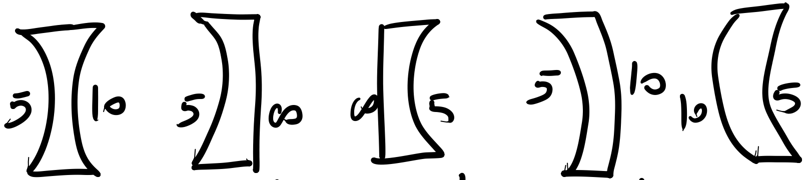
-ve if med → denser, lens → rarer.

$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$:

→ +ve for all convex lenses

$\left(\frac{1}{5} - \frac{1}{-10}\right) > 0$ $\quad \left(\frac{1}{5} - \frac{1}{-5}\right) > 0$ $\quad \left(\frac{1}{\infty} - \frac{1}{-10}\right) > 0$ $\quad \left(\frac{1}{10} - \frac{1}{\infty}\right) > 0$ $\quad \left(\frac{1}{-10} - \frac{1}{-5}\right) > 0$ $\quad \left(\frac{1}{5} - \frac{1}{10}\right) > 0$

→ -ve for all concave lenses.

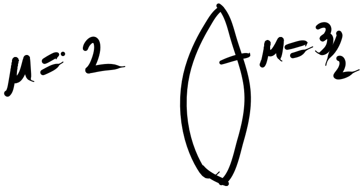


$\frac{1}{-5} - \frac{1}{10} < 0$ $\quad \frac{1}{-5} - \frac{1}{\infty} < 0$ $\quad \frac{1}{\infty} - \frac{1}{5} < 0$ $\quad \frac{1}{-5} - \frac{1}{-10} < 0$ $\quad \frac{1}{10} - \frac{1}{5} < 0$.

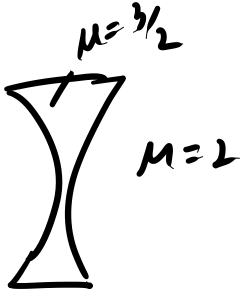
Lens → converging → if $f \to +ve \Rightarrow$ both factors are of same sign

→ diverging → if $f \to -ve \Rightarrow$ both factor are of opp sign

$$\frac{1}{f} = \underbrace{(\mu_{rel} - 1)}_{\text{1st factor}} \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{\text{2nd factor}}$$



diverging lens



Converging lens

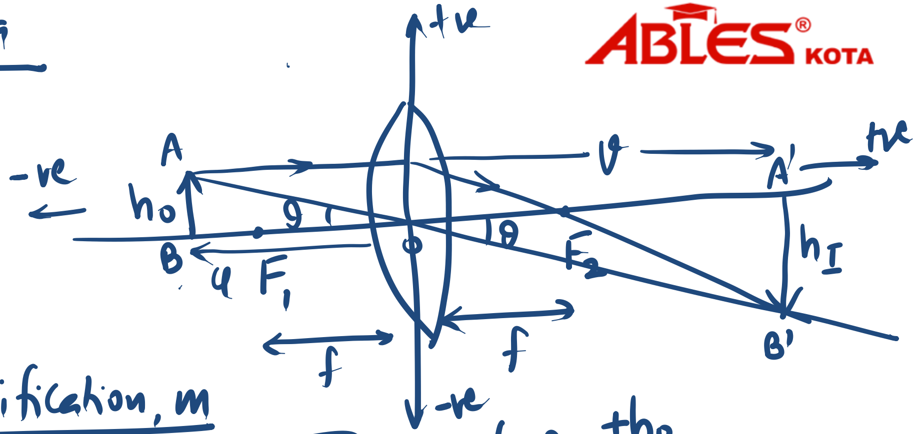
focal length also depends upon wavelength (λ) of light. (As $\lambda \uparrow$ $f \uparrow$)

Variations of Lens Maker formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$v = \frac{fu}{f+u}$$

$$u = \frac{fv}{f-v}$$



Magnification, m

$$m = \frac{h_I}{h_o} = \frac{v}{u}$$

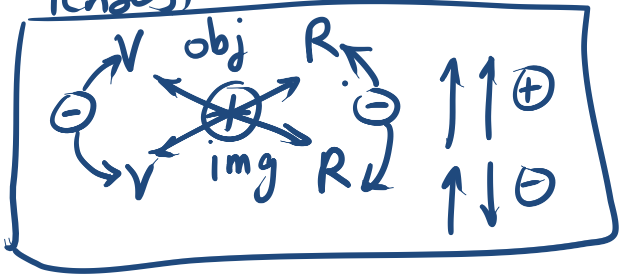
$$\tan\theta = \frac{+h_o}{-u}$$

$$\tan\theta = \frac{-h_I}{+v}$$

$$\frac{h_o}{-u} = -\frac{h_I}{v}$$

$$\frac{v}{u} = \frac{h_I}{h_o}$$

(Sign of m in lenses)



velocity of image

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

diff. w.r.t. time.

$$-\frac{1}{v^2}\frac{dv}{dt} - \left(\frac{1}{-u^2}\right)\frac{du}{dt} = 0.$$

$$\Rightarrow \underbrace{\frac{dv}{dt}}_{\text{vel. of img}} = \underbrace{\frac{v^2}{u^2}}_{(=m^2)} \underbrace{\frac{du}{dt}}_{\text{vel. of obj}} \Rightarrow \boxed{\left(V_{I/M}\right)_x = m^2\left(V_{O/M}\right)_x}$$

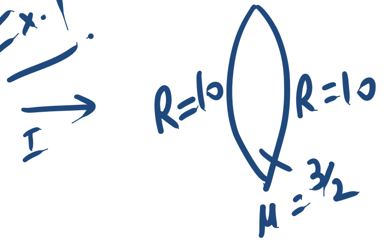
→ for lenses.

$$m = \frac{v}{u} = \frac{h_I}{h_O}$$

$$h_I = m\,h_O \Rightarrow \boxed{\left(V_{I/M}\right)_y = m\left(V_{O/M}\right)_x}$$

→ for mirror & lenses.

Ex. 1



Find f.

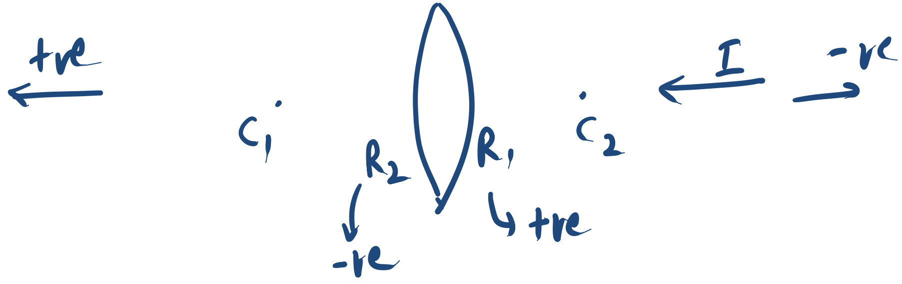
$\mu = 1$

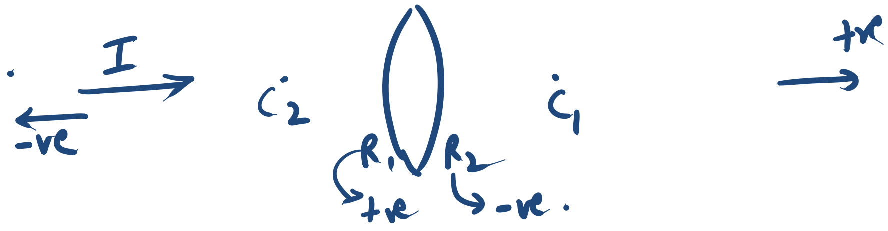
$$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\left(\frac{3/2}{1}\right) - 1\right)\left(\frac{1}{10} - \frac{1}{-10}\right)$$

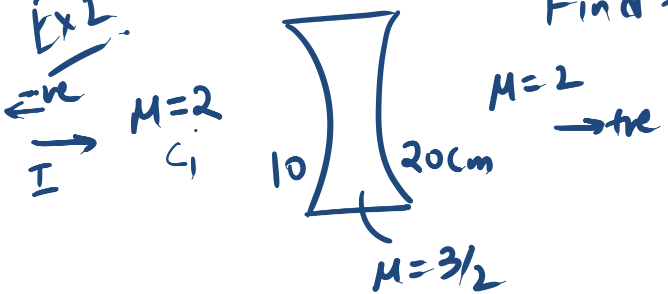
$$= \frac{1}{2} \times \frac{2}{10}$$

$$\Rightarrow \boxed{f = +10\text{ cm}}$$





Ex 2. Find focal length.

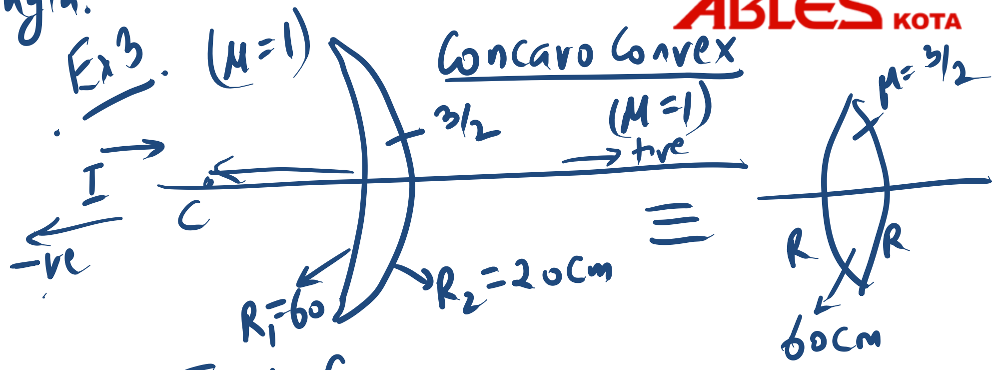


$$\frac{1}{f} = \left(\frac{3/2}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{+20}\right)$$

$$= -\frac{1}{4} \times \left(-\frac{3}{20}\right)$$

$$\boxed{f = +80/3}$$ Convex lens.

Ex 3. (μ=1) Concavo Convex



Find f.

Sign of R → 1). Look @ position of centre of the (C) surface

2). If C is towards Incident ray w.r.t. surface → R → +ve

else → R → -ve.

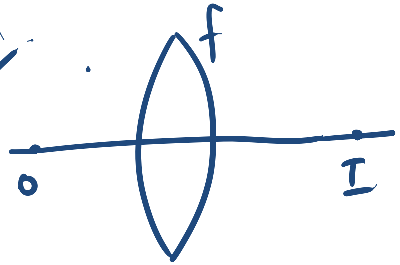
$$\frac{1}{f} = \left(\frac{3/2}{1} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$$

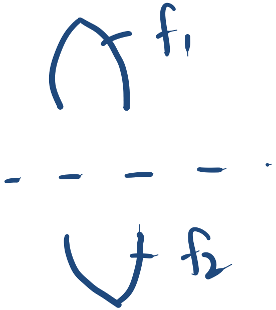
$$= \frac{1}{2} \times \frac{2}{60} \Rightarrow \boxed{f = +60\text{cm}}$$

$$\frac{1}{60} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

(R = 60cm).

Ex4





Find f_1 & f_2?

$f_1 = f$

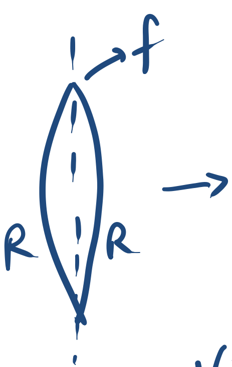
$f_2 = f$.

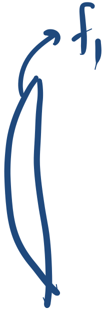
$$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Intensities will change.

$I \propto (\text{aperture})^2$

Ex5.





Find f_1

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$f = \frac{R}{2(\mu - 1)}$$

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$f_1 = \frac{R}{(\mu - 1)}$$

$\boxed{f_1 = 2f}$

→ Intensity will remain same.

→ Power will become half

$\boxed{P = 1/f}$