Session - 20^{th} – Refraction through Curved Surfaces

ABLES® KOTA

Outline –

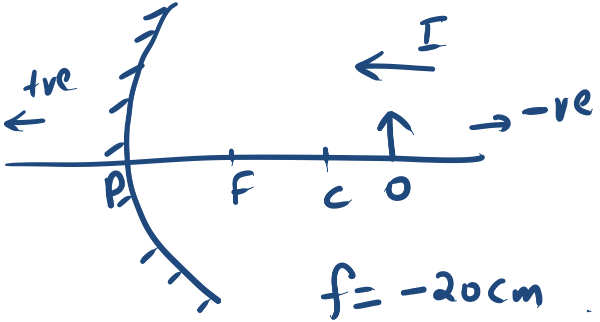
→ Convex Mirror - Recap

→ Newton's formula (Mirrors)

→ Refraction through curved surfaces

→ Examples

Recap (Convex Mirror)



Concave Mirror

$f = -20\text{cm}$

$\begin{cases} u_1 = -60\text{cm}, & x_1 = 40\text{cm} \\ v_1 = ? & x_2 = 10\text{cm} \end{cases}$

$v_1 = -30\text{cm}$

$\begin{cases} u_2 = -10\text{cm} & x_1 = 10\text{cm} \\ v_2 = ? & x_2 = 40\text{cm} \end{cases}$

$v_2 = +20\text{cm}$

$\begin{cases} u_3 = +20\text{cm} & x_1 = 40\text{cm} \\ v_3 = ? & x_2 = 10\text{cm} \end{cases}$

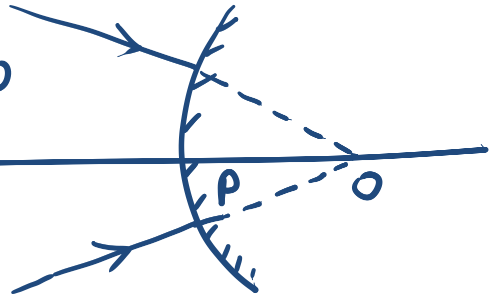
$v_3 = -10\text{cm}$

$$v = \frac{uf}{u-f} = \frac{20 \times (-20)}{20+20} = -10\text{cm}$$

Ex1 Find nature & position of image if pt O

a). $PO = 10\text{cm}$

b). $PO = 30\text{cm}$.



$f = 20\text{cm}$

$x_1 = 10\text{cm}$

$x_2 = 40\text{cm}$

a). $u_1 = +10\text{cm}$

$f = +20\text{cm}$

$v_1 = ?$

$$v_1 = \frac{u_1 f}{u_1 - f} = \frac{10 \times 20}{10 - 20} = -20\text{cm}$$

b). $u_2 = +30\text{cm}$

$f = +20\text{cm}$

$v_2 = ?$

$$v_2 = \frac{30 \times 20}{30 - 20} = +60\text{cm}.$$

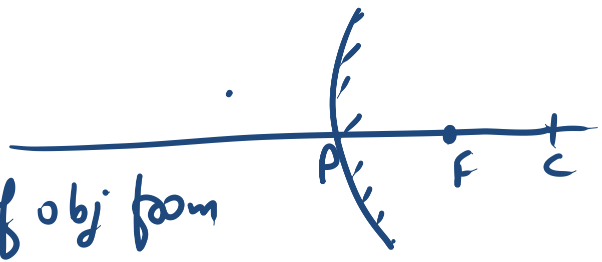
$x_1 = 10\text{cm}$

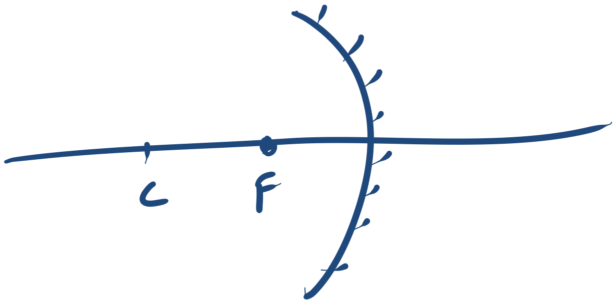
$x_2 = 40\text{cm}.$

Newton's Formula → Both obj & img always lie on the same side of a particular pt. on principal axis. (in both convex & concave mirror)

(Focus)

True / False ✓





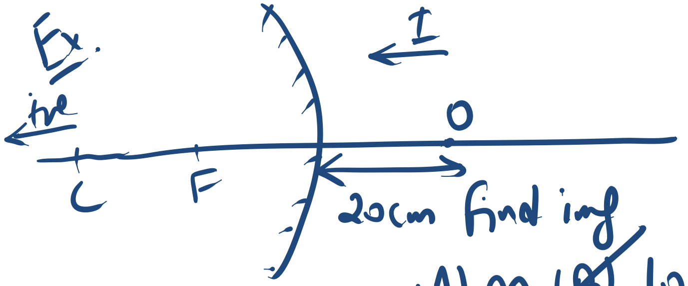
x_1 → dist. of obj from focus of mirror

x_2 → dist of img from focus of mirror

f → focal length of mirror.

$$\boxed{x_1 x_2 = f^2}$$

Ex.



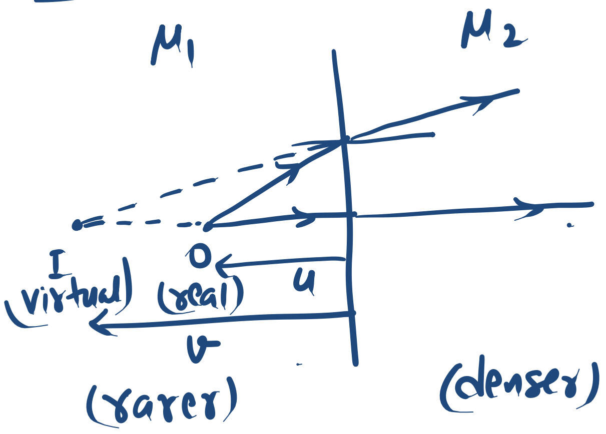
find img

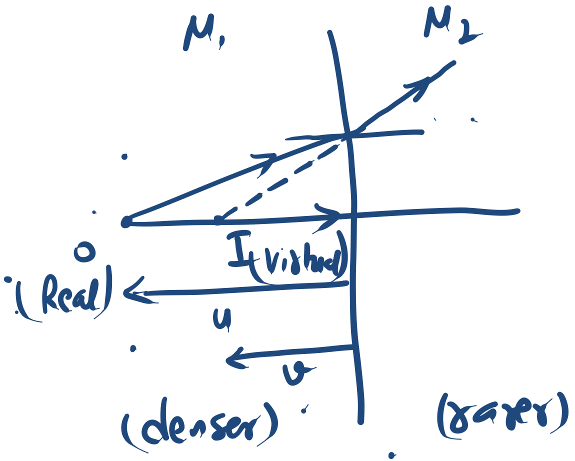
(A) ∞ (B). 10 cm (C) -10 cm

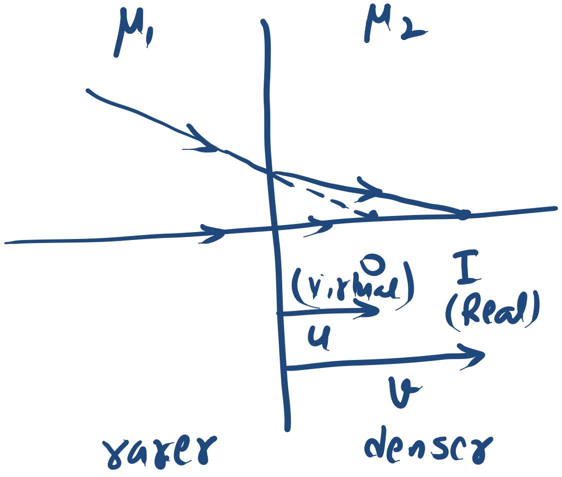
(D) -40cm.

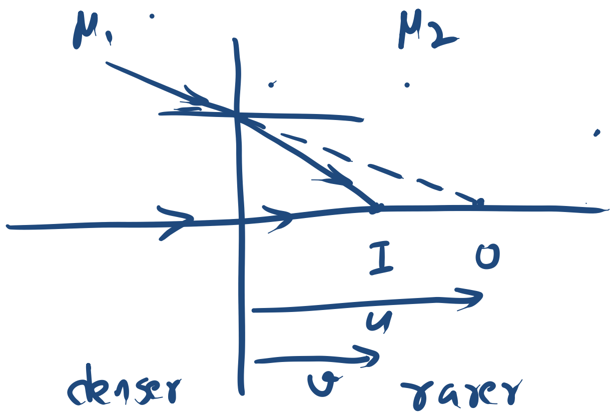
$f = 20cm$

Refraction through curved Surfaces (Recap).







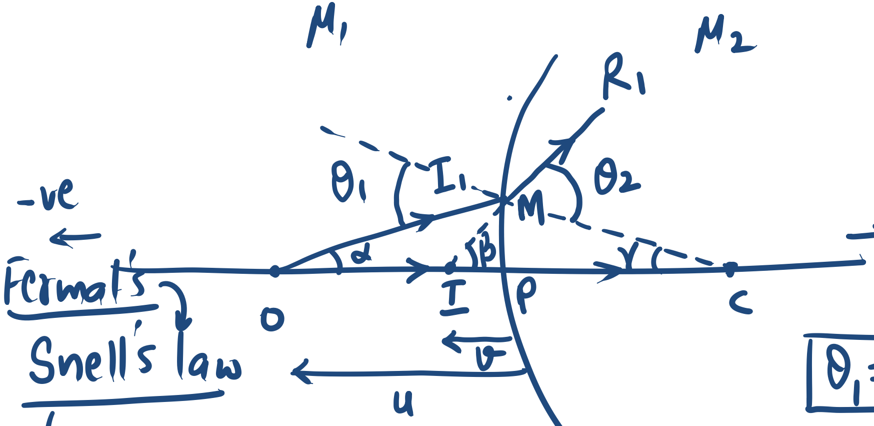


Q.1. Relation b/w u & v common for all the cases. → Yes it is common

Q.2. $\boxed{v = \left(\frac{\mu_2}{\mu_1}\right) u} \Rightarrow \boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \cancel{\frac{\mu_2 - \mu_1}{R}}} \rightarrow 0$

$R \rightarrow \infty$

Derivation of $\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right)$ for ref. thru curved surfaces



Fermat's → Snell's law

↳ $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

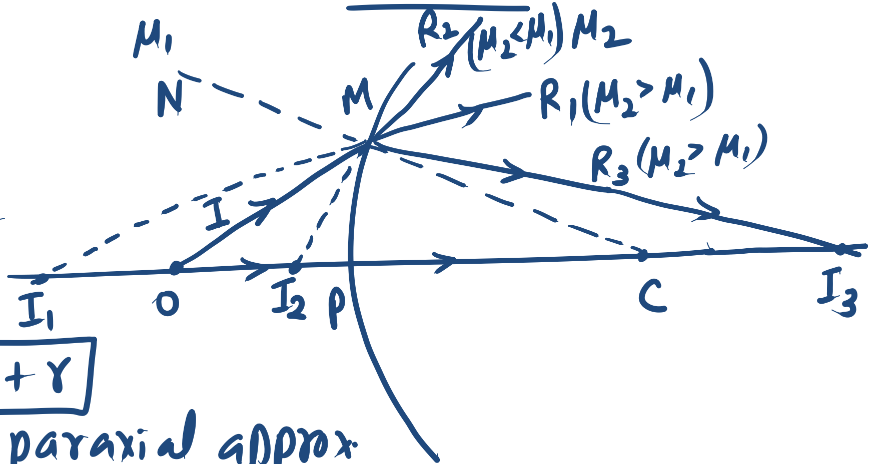
$\angle OMI = \theta_2 - \theta_1$

$\alpha + \angle OMI = \beta$ — ①

$\beta + \gamma + (\pi - \theta_2) = \pi$ — ②

$\boxed{\theta_2 = \beta + \gamma}$

$\alpha + (\theta_2 - \theta_1) = \beta \Rightarrow \alpha + \cancel{\beta} + \gamma - \theta_1 = \cancel{\beta}$



$\boxed{\theta_1 = \alpha + \gamma}$

Under paraxial approx

θ_1 & θ_2, α, β, γ → small.

$\Rightarrow \mu_1 \sin\theta_1 = \mu_2 \sin\theta$

$\sin\theta \simeq \theta \Rightarrow \mu_1\theta_1 = \mu_2\theta_2$

$\Rightarrow \mu_1(\alpha + \gamma) = \mu_2(\beta + \gamma)$

$\tan\alpha \sim \alpha = MP$, $\tan\beta \sim \beta = \frac{MP}{-v}$

$\tan\gamma \sim \gamma = \frac{MP}{R-u}$

$$\mu_1(\alpha+\gamma) = \mu_2(\beta+\gamma)$$

$$\Rightarrow \mu_2\beta - \mu_1\alpha = (\mu_1 - \mu_2)\gamma$$

$$\Rightarrow \alpha \approx \frac{MP}{-u}, \quad \beta = \frac{MP}{-v}, \quad \gamma \approx \frac{MP}{R}$$

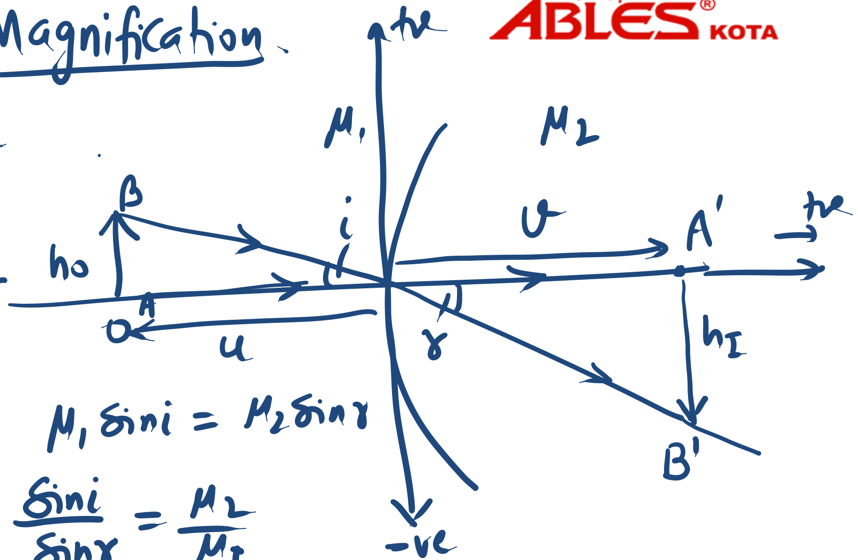
−ve ←

$$\Rightarrow \mu_2\left(\frac{MP}{-v}\right) - \mu_1\left(\frac{MP}{-u}\right) = (\mu_1 - \mu_2)\frac{MP}{R}$$

$$\Rightarrow \boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

$$\boxed{m = \frac{(v/\mu_2)}{(u/\mu_1)} = \frac{h_I}{h_o}}$$

Magnification.



$$\mu_1 \sin i = \mu_2 \sin \gamma$$

$$\frac{\sin i}{\sin \gamma} = \frac{\mu_2}{\mu_1}$$

paraxial approx :- $\sin i \approx \tan i$, $\sin \gamma \approx \tan \gamma$

$$\tan i = \frac{h_o}{-u}, \quad \tan \gamma = -h_I/v$$

$$\frac{\sin i}{\sin \gamma} \approx \frac{\tan i}{\tan \gamma} = \frac{h_o/-u}{-h_I/v} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow h_I/h_o = m = (v/\mu_2)/(u/\mu_1).$$

Ex. what will be image position & magnification?