Session 12: Ray Optics – Refraction at Plane Surfaces

- Recap
- Laws of refraction
- Refractive Index
- Refraction through a glass slab
    - Lateral shift
    - Normal shift
    - Apparent depth
- Total Internal Reflection (TIR)
- **Prism & Dispersion - Examples**

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Recap

↳ dispersive media

↳ Speed of wave depends on freq. of the wave

↳ $\mu(\lambda) = a + b/\lambda^2$

$v(\lambda) = c/\mu(\lambda)$

$\lambda_{med} = \frac{\lambda}{\mu(\lambda)}$.

↳ Prism

$\delta = i + e - A$

$= \delta_1 + \delta_2$

$\boxed{r_1 + r_2 = A}$

Outline for today →

$\boxed{v = f\lambda} \rightarrow \boxed{f \propto 1/\lambda}$

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $v_{string} = \sqrt{T/\mu}$, $v_{sound} = \sqrt{\frac{\gamma RT}{M}}$

Prism (for monochromatic light) → δ_{min} $(i = e)$

↳ δ_{max} (TIR @ second surface) (Grazing emergence)

↳ No emergence cond^n (for any i)

↳ Dispersion → angular disp. / mean deviation

Prism (Monochromatic source) → Cond^n for min^m deviation (δ_{min})

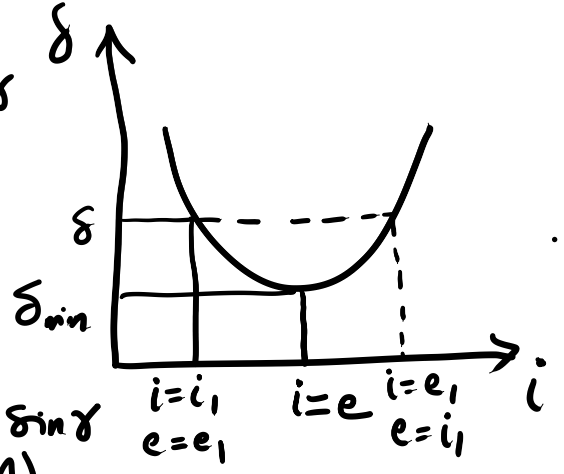
$$\boxed{i = e}$$

$\Rightarrow r_1 = r_2$

$r_1 + r_2 = A \Rightarrow r_1 = r_2 = A/2 = r$

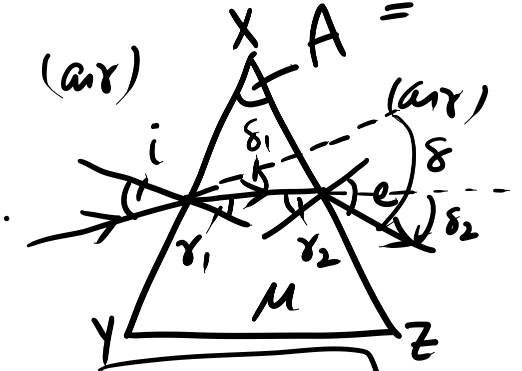
$\delta_{min} = i + i - A = 2i - A$

$$\Rightarrow i = \frac{\delta_{min} + A}{2} = e$$



$\delta = \delta_1 + \delta_2$

=



$$\boxed{\delta = i + e - A}$$

$r_1 + r_2 = A$

$\delta = f(i, A)$.

@XY → $1 \times \sin i = \mu \sin r_1$

@XZ → $\mu \sin r_2 = 1 \times \sin e$

if $i = e \Rightarrow (r_1 = r_2)$

Snell's law @ XY → $1 \times \sin i = \mu \sin r$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin (A/2)}.$$

if A is small ($A < 10^\circ$).

$$\mu = \frac{\frac{\delta_{min} + A}{2}}{A/2}$$

$$\Rightarrow \boxed{\delta_{min} = (\mu - 1) A}$$

Prism (Monochromatic source) — Cond^n for max^m deviation (δ_{max})

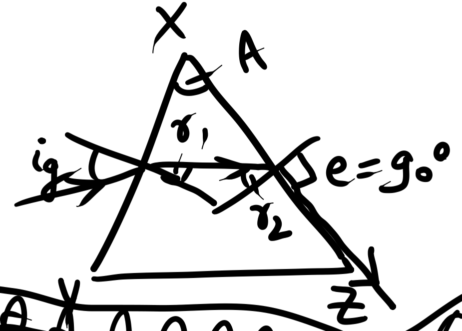
→ TIR happens @ second face of prism (Grazing emergence)

For $i = i_g$, $e = 90^\circ \Rightarrow$ $\boxed{\delta_{max} = i_g + 90^\circ - A}$

Snell's law @ XY → $1 \times \sin i_g = \mu \sin r_1$

Snell's law @ XZ → $\mu \sin r_2 = 1 \times \sin 90^\circ = 1$

$\Rightarrow \sin r_2 = 1/\mu = \sin \theta_c$



$r_1 + r_2 = A$

$r_1 = A - r_2 = A - \theta_c$

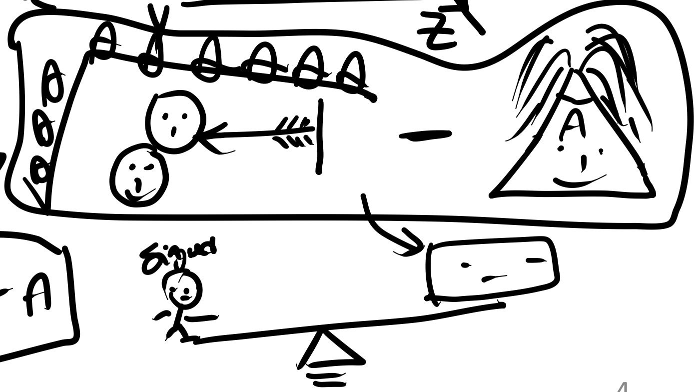
$\sin r_1 = \sin(A - \theta_c) = \sin A \cos\theta_c - \cos A \sin\theta_c$

$\mu \sin r_1 = \mu(\sin A \cos\theta_c - \cos A \sin\theta_c)$

$\sin i_g = \mu\left(\sin A \frac{\sqrt{\mu^2-1}}{\mu} - \cos A \times \frac{1}{\mu}\right)$

$\boxed{\sin i_g = \sqrt{\mu^2-1}\,\sin A - \cos A}$

$\boxed{\delta_{max} = i_g + 90^\circ - A}$



for $i < i_g \Rightarrow$ TIR happens @ XZ / $i > i_g \Rightarrow$ ray emerges.

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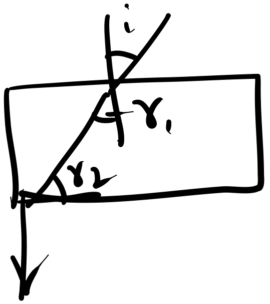
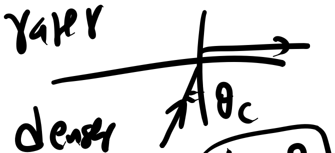


for TIR to happen @ second face,

$i < \cancel{\theta_c}\ i_g$



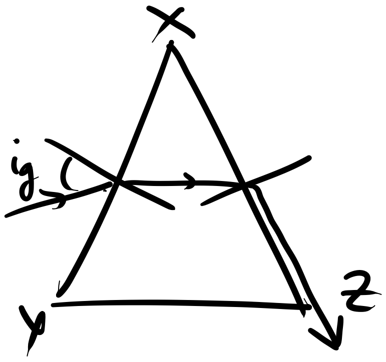
$i > \theta_c \Rightarrow$ TIR

for max^m deviation →

$\sin i_g = \sqrt{\mu^2 - 1}\,\sin A - \cos A$ $(e = 90°)$

$i < i_g \Rightarrow$ TIR happens @ 2nd surface (XZ)

$i > i_g \Rightarrow$ ray emerges out $(e < 90°)$



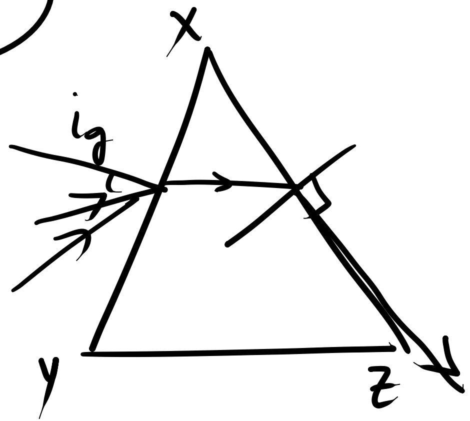
$\sin i_g = 1/\mu$

$i > i_g \Rightarrow$ TIR happens

$i < i_g \Rightarrow$ ray emerges out.

Prism (monochromatic source) → No emergence condition

→ Irrespective of the value of i, no ray should emerge from XZ surface



Set $i_g = 90^\circ$

$i < i_g \Rightarrow$ TIR happens @ XZ

$\Rightarrow$ ~~$\sin i_g >$~~ 1

$i_g \geq 90^\circ$

$\sqrt{\mu^2-1}\,\sin A - \cos A \geq 1$

$\sqrt{\mu^2-1} \times 2\sin A/2 \cos A/2 \geq 2\cos^2 A/2$

$\Rightarrow \mu > \frac{1}{\sin A/2}$

$A > 2\theta_c$

$A > 2\sin^{-1}\frac{1}{\mu}$

$r_2 > \theta_c$

$r_2 > \sin^{-1}\frac{1}{\mu}$

Session 13: Ray Optics – MicroNotes of 1st half

- Reflection at plane surfaces
  - MicroNotes (pMAVs - O1)
  - Concept review (Object & Image, Real & Virtual spaces, Ray diagram, Angle of deviation, Magnification, velocity of image)
- Refraction at plane surfaces
  - MicroNotes (pMAVs – O2)
  - Concept review (Ref index, App depth, normal shift, TIR, Dispersion, 3 conditions in Prism)

Recap – (Reflection @ plane surfaces) – MicroNote

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Plane Mirror

① → $i = r$

② → $\delta = \pi - 2i$
↳ $(2\pi - 2\theta)$

③ → $l = H/2$

④ → $m = \frac{360^\circ}{\theta}$

obj position → symm / odd } no of images → $m-1$ / m (if there are two odds) / closest even int to m (if $m \to$ fraction)

⑤ → magnification, $m = -\frac{v}{u} = \frac{h_I}{h_O}$
↳ $= 1$ (for plane mirror)

⑥ → $v_i = 2v_m - v_o$

⑦ → $\theta_i \to \theta_r = -\theta_i$
$\theta_m \to \theta_r = 2\theta_m$

Refraction @ plane surfaces – MicroNote-2

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① → $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 = \frac{{}_1\mu_2}{1}$

② → Lateral shift $= \frac{t}{\cos r} \sin(i-r)$

③ → Normal shift $= t_1\left(1 - \frac{1}{\mu_{1rel}}\right) + t_2\left(1 - \frac{1}{\mu_{2rel}}\right) + \cdots$

$\mu_{rel} d$ (single ref) d/μ_{rel} $(\mu_{rel} > 1)$

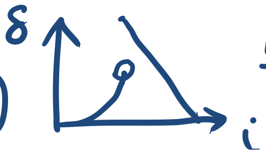
④ → App depth $= \frac{t_1}{\mu_{1rel}} + \frac{t_2}{\mu_{2rel}} + \cdots$

$\mu_{1rel} = \frac{\mu_1}{\mu_{obs}}$

⑤ → TIR, $\sin\theta_c = 1/\mu_{rel}$ $(\mu_{rel} = \mu_D/\mu_R)$

$r = d\tan\theta_c$

$\delta = |i - r|$ to $(\pi - 2i)$



⑥ → Prism → $\delta = i + e - A$

$r_1 + r_2 = A$

↳ 3 Conditions → ① δ_{min} $(i = e)$

$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin(A/2)}$

$A \to$ small

$\delta_{min} = (\mu - 1)A$

Mirage, $h\uparrow \mu\uparrow$ (inverted)

Looming, $h\uparrow \mu\downarrow$, (erect)

(6) → Prism

↳ (2) δ_{max} ($e = 90°$, TIR @ second surface, $i < i_g$)

↓

$$\sin i_g = \sqrt{\mu^2 - 1}\,\sin A - \cos A$$

(3) No Emergence condⁿ (Set $i_g = 90°$)

$\mu > \frac{1}{\sin(A/2)}$ | $A > 2\theta_c$ | $r_2 > \theta_c$

(7) → Dispersion

↳ (1) $\mu(\lambda) = a + b/\lambda^2 \rightarrow \lambda_{med} = \frac{\lambda}{\mu(\lambda)}$

$\left(\frac{d\mu(\lambda)}{d\lambda} < 0\right) \rightarrow$ Normal $\quad v_\lambda = \frac{c}{\mu(\lambda)}$

Concept Review (Reflection @ plane surfaces)

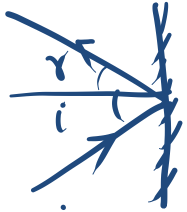
→ Fermat's Principle (least time)

- → $i = r$ (reflection)
- → $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$ (refraction)

→ object & Image

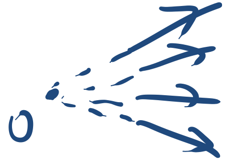
- object → (incident rays)
- Image → (reflected/refracted rays)

Ray diagram



→ Real & virtual obj

- incident rays diverging
- incident rays converging



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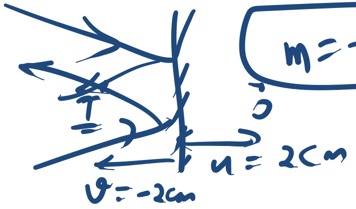
Magnification

$m = -\frac{v}{u} = \frac{h_I}{h_o}$ → (Mirrors)

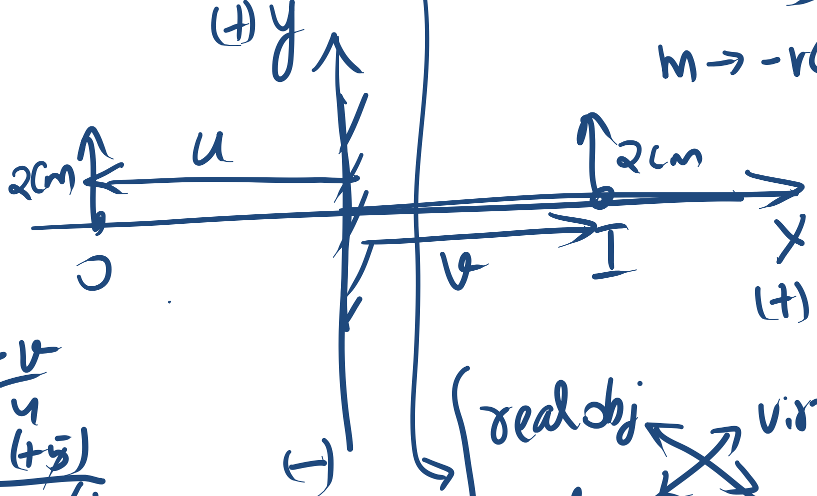
$= \frac{v}{u} = \frac{h_I}{h_o}$ (Lenses)

Magnification →
- Magnitude → small or large; → position
- Sign →



m = +1

$m \to -ve$: real obj ↔ real img; virtual obj ↔ virtual img



$u = -5$
$v = +5$

$m = \frac{-v}{u}$

$= -\frac{(+5)}{(-5)}$

$= +1 = \frac{h_I}{h_o}$

real obj ↔ virtual img; virtual obj ↔ real img → $m \to +ve$

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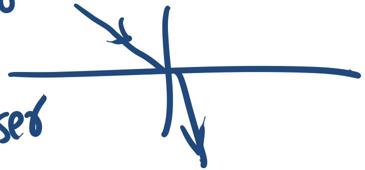
Concept Review → (Refraction).

($\mu > 1$) always.

rarer

denser



$$\mu = c/v$$

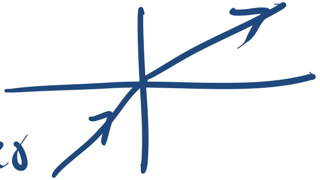
$$v \leq c$$

$$\mu \geq 1$$

$$v = c/\mu$$

rarer

denser



rarer to denser → light bends towards the normal

denser to rarer → " ——— away from " —