Session 12: Ray Optics – Refraction at Plane Surfaces

- Recap
- Laws of refraction
- Refractive Index
- Refraction through a glass slab
    - Lateral shift
    - Normal shift
    - Apparent depth
- Total Internal Reflection (TIR)
- **Prism & Dispersion - Examples**

Recap

↳ dispersive media

↳ Speed of wave depends on freq. of the wave

↳ $\mu(\lambda) = a + b/\lambda^2$

$v(\lambda) = c/\mu(\lambda)$

$\lambda_{med} = \dfrac{\lambda}{\mu(\lambda)}$.

↳ Prism

$\delta = i + e - A$

$= \delta_1 + \delta_2$

$\boxed{r_1 + r_2 = A}$

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Outline for today →

$\boxed{v = f\lambda} \rightarrow \boxed{f \propto 1/\lambda}$

$v = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$, $v_{string} = \sqrt{T/\mu}$, $v_{sound} = \sqrt{\dfrac{\gamma R T}{M}}$

Prism (for monochromatic light).

→ δ_{min} $(i = e)$

→ δ_{max} (TIR @ second surface) (Grazing emergence)

→ No emergence condⁿ (for any i)

↳ Dispersion → angular disp. / mean deviation

Prism (Monochromatic source) → Cond^n for min^m deviation (δ_{min})

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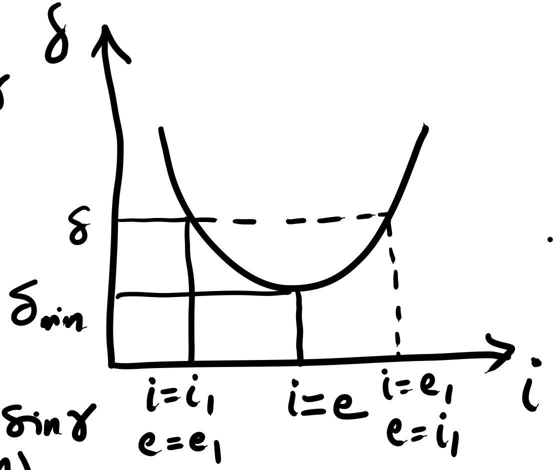
$\boxed{i = e}$

$\Rightarrow r_1 = r_2$

$r_1 + r_2 = A \Rightarrow r_1 = r_2 = A/2 = r$

$\delta_{min} = i + i - A = 2i - A$

$\Rightarrow i = \frac{\delta_{min} + A}{2} = e$



$\delta = \delta_1 + \delta_2$

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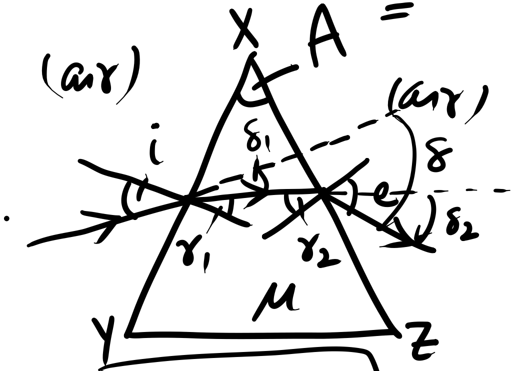


$\boxed{\delta = i + e - A}$

$r_1 + r_2 = A$

$\delta = f(i, A)$.

Snell's law @ XY → $1 \times \sin i = \mu \sin r$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin (A/2)}.$$

if A is small ($A < 10^\circ$). $\mu = \frac{\frac{\delta_{min} + A}{2}}{A/2}$

$\Rightarrow \boxed{\delta_{min} = (\mu - 1)A}$

@XY → $1 \times \sin i = \mu \sin r_1$

@XZ → $\mu \sin r_2 = 1 \times \sin e$

if $i = e \Rightarrow (r_1 = r_2)$

Prism (Monochromatic source) — Condⁿ for maxᵐ deviation (δ_{max})

For $i = i_g$, $e = 90^\circ \Rightarrow$ $\boxed{\delta_{max} = i_g + 90^\circ - A}$

→ TIR happens @ second face of prism (Grazing emergence)

Snell's law @ XY → $1 \times \sin i_g = \mu \sin r_1$

Snell's law @ XZ → $\mu \sin r_2 = 1 \times \sin 90^\circ = 1$

$\Rightarrow \sin r_2 = 1/\mu = \sin\theta_c$

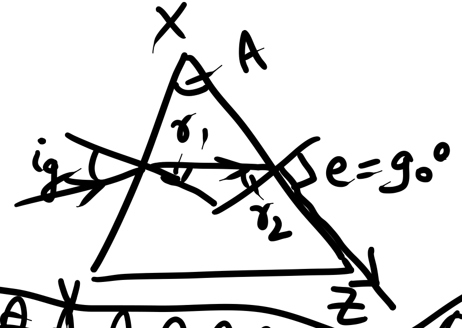
$r_1 + r_2 = A$

$r_1 = A - r_2 = A - \theta_c$

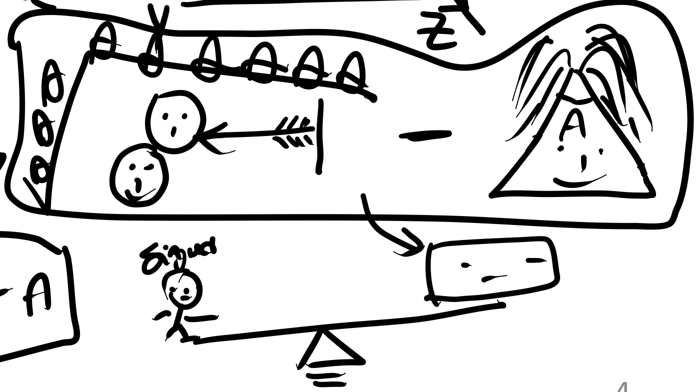
$\sin r_1 = \sin(A - \theta_c) = \sin A \cos\theta_c - \cos A \sin\theta_c$

$\mu \sin r_1 = \mu(\sin A \cos\theta_c - \cos A \sin\theta_c)$

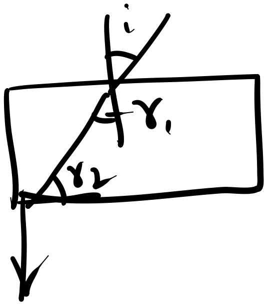
$\sin i_g = \mu\left(\sin A \frac{\sqrt{\mu^2 - 1}}{\mu} - \cos A \times \frac{1}{\mu}\right)$



$\boxed{\sin i_g = \sqrt{\mu^2 - 1}\,\sin A - \cos A}$ $\boxed{\delta_{max} = i_g + 90^\circ - A}$

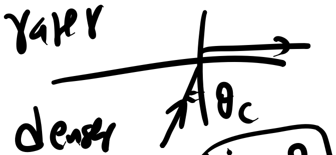


for $i < i_g \Rightarrow$ TIR happens @ XZ / $i > i_g \Rightarrow$ ray emerges.



for TIR to happen @ second face,

$i < i_g$



$i > \theta_c \Rightarrow$ TIR

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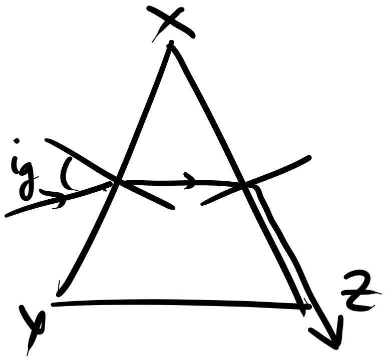
for max^n deviation →

$\sin i_g = \sqrt{\mu^2-1}\,\sin A - \cos A$ $(e=90°)$

$i < i_g \Rightarrow$ TIR happens @ 2^{nd} surface (XZ)

$i > i_g \Rightarrow$ ray emerges out $(e<90°)$



$\sin i_g = 1/\mu$

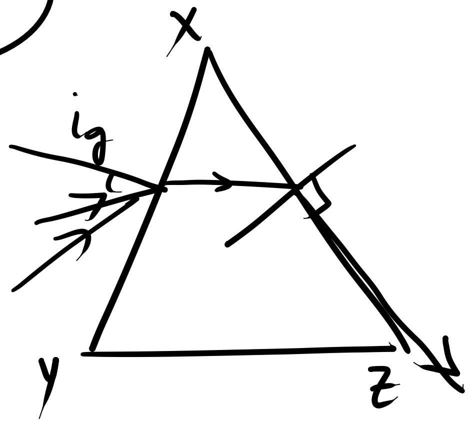
$i > i_g \Rightarrow$ TIR happens

$i < i_g \Rightarrow$ ray emerges out.

Prism (monochromatic source) → No emergence condition

→ Irrespective of the value of i, no ray should emerge from XZ surface

Set $\boxed{i_g = 90^\circ}$



$i < i_g \Rightarrow$ TIR happens @ XZ

$\Rightarrow$ ~~$\sin i_g \geq 1$~~

$i_g \geq 90^\circ$

$\sqrt{\mu^2-1}\,\sin A - \cos A \geq 1$

$\sqrt{\mu^2-1} \cdot 2\sin A/2 \cos A/2 \geq 2\cos^2 A/2$

$\Rightarrow \boxed{\mu > \frac{1}{\sin A/2}}$ $\boxed{A > 2\theta_c}$ $\boxed{A > 2\sin^{-1}\frac{1}{\mu}}$ $\boxed{r_2 > \theta_c}$ $\boxed{r_2 > \sin^{-1}\frac{1}{\mu}}$