Session 9: Ray Optics – Refraction at Plane Surfaces

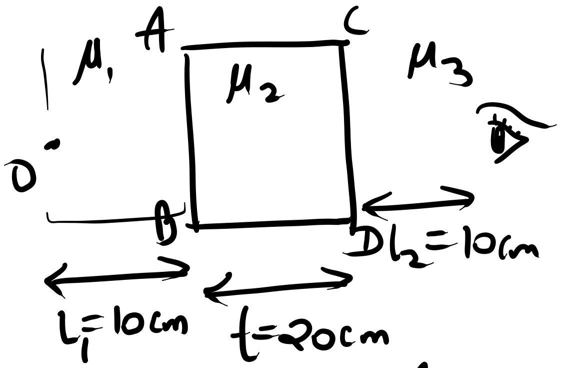
- ~~Sunday Test Discussion~~
- Recap
- Laws of refraction
- Refractive Index
- Refraction through a glass slab
    - Lateral shift
    - Normal shift
    - Apparent depth
- **Total Internal Reflection (TIR)**
- Prism

(Normal Shift)

Ex.

for multiple refraction with/without reflection →

1). Start with the root/first obj.
2). Follow the incident rays
3). Solve one by one
4). $\boxed{\text{Image of previous refraction} \rightarrow \text{obj. for next}}$



Find position of image.

① Refraction @ AB → I_1 ② Refraction @ CD

I_1 becomes obj

$I_2 \rightarrow$ final image.

$$I_1 = \frac{L_1}{(\mu_1/\mu_2)}$$

dist of obj from CD $= I_1 + t$

$$L_0 = \frac{L_1}{(\mu_1/\mu_2)} + t$$

$$I_2 = \frac{L_0}{(\mu_2/\mu_3)}$$

$$= I_2 + L_2$$

$$= \frac{L_0}{\mu_2/\mu_3} + L_2$$

$$= \frac{\mu_3}{\mu_2}\left(L_1\frac{\mu_2}{\mu_1} + t\right) + L_2$$

$$= L_1\frac{\mu_3}{\mu_1} + t\frac{\mu_3}{\mu_2} + L_2$$

Normal shift thru multiple slabs →

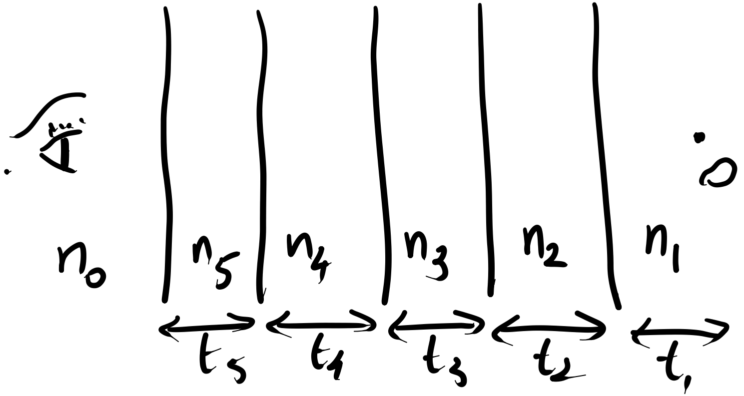
$$d_{ap} = \frac{t_1}{\mu_{1rel}} + \frac{t_2}{\mu_{2rel}} + \frac{t_3}{\mu_{3rel}} + \dots$$

$$x = t_1\left(1 - \frac{1}{\mu_{1rel}}\right) + t_2\left(1 - \frac{1}{\mu_{2rel}}\right) + \dots$$

$$\mu_{rel} = \frac{\mu_1}{\mu_{obser}}$$

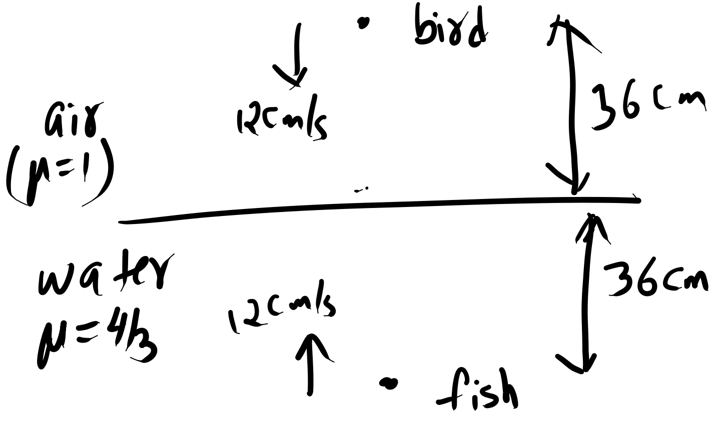
$$d_{app} = \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \cdots$$

$$x = t_1\left(1 - \frac{1}{n_{1rel}}\right) + t_2\left(1 - \frac{1}{n_{2rel}}\right) + \cdots$$



$$n_{1rel} = \frac{n_1}{n_0}$$

$$n_{2rel} = \frac{n_2}{n_0}$$



Q.1. Apparent height of bird

$d_{ap} = 36 \times 4/3 = 48\text{ cm} = 36 \times \mu$

Q.2. App. depth of fish

$v = \frac{u}{\mu}$

$\frac{dv}{dt} = \frac{1}{\mu}\left(\frac{du}{dt}\right)$

$d_{ap} = \frac{36}{\mu} = \frac{36 \times 3}{4} = 27\text{cm}$

Q.3. At what dist will bird find fish

$\text{dist} = 36 + 27 = 63\text{cm}$

Q.4. At what dist will fish find bird?

$\text{dist} = 36 + 48 = 84\text{cm}$

$\uparrow$ -ve

$\downarrow$ +ve

$d_{ap} = \frac{d_{ac}}{\mu}$

$v_{BI} = \frac{1}{\mu} v_{BO}$

$v_{B/water} = \frac{12}{(4/3)} = +9\text{ cm/s}$

Q.5. $v_{B/F}$? $\rightarrow$ $v_{B/F} = v_{B/water} - v_{F/water}$

$= +9 - (-12)$

$= +21\text{ cm/sec}$

$v_{F/B}$?

$v_{F/B} = v_{F/air} - v_{B/air}$

$= -12 \times 4/3 - 12 = \boxed{-28\text{ cm/s}}$

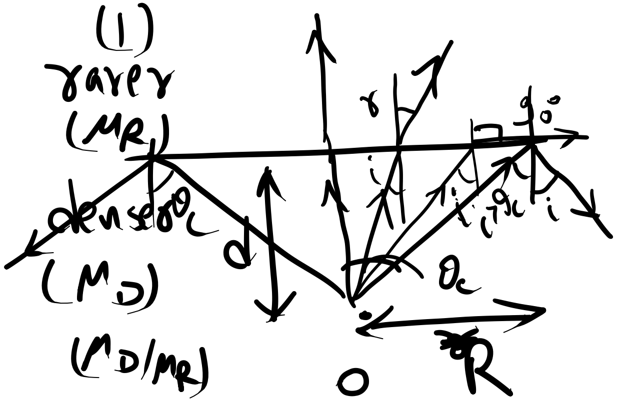
TIR (Total Internal Reflection) → (denser to rarer medium)

When angle of incidence reaches a critical value, θ_c

angle of refraction, $r \to 90^\circ$

for $\theta > \theta_c$, incident rays are totally reflected into the denser medium.

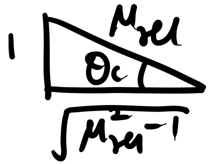
This is called TIR



Apply Snell's law

$$\mu_D \sin\theta_c = \mu_R \sin 90^\circ$$

$$\boxed{\sin\theta_c = \frac{1}{(\mu_D/\mu_R)} = \frac{1}{\mu_{rel}}}$$



$$\tan\theta_c = \frac{1}{\sqrt{\mu_{rel}^2-1}}$$

$$\frac{R}{d} = \tan\theta_c \Rightarrow R = d\tan\theta_c = \left(\frac{d}{\sqrt{\mu_{rel}^2-1}}\right)$$

Angle of Deviation in TIR →

$\delta = r - i$

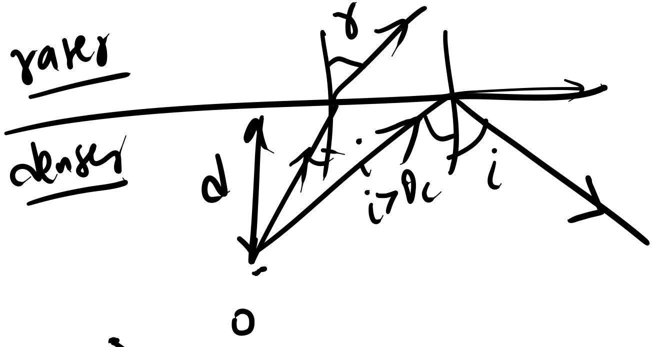
$\mu_{rel} \sin i = \sin r$

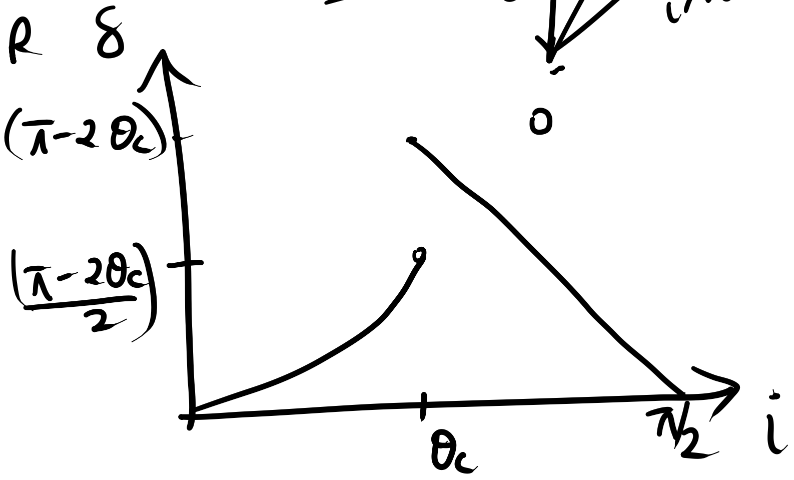
$r = \sin^{-1}(\mu_{rel} \sin i)$ → $\left(\frac{\pi}{2} - \theta_c\right) = \left(\frac{\pi - 2\theta_c}{2}\right)$

$\boxed{\delta = \sin^{-1}(\mu_{rel} \sin i) - i}$ → before TIR happens $(i < \theta_c)$

$\boxed{\delta = \pi - 2i}$ → after TIR $(i > \theta_c)$

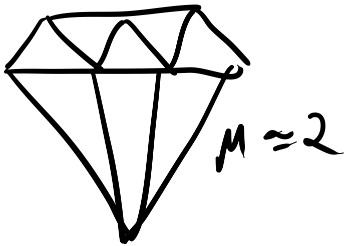
$= \pi - 2\theta_c$





Examples of TIR →

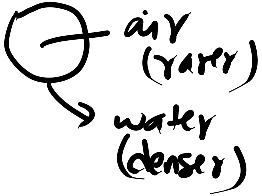
1).



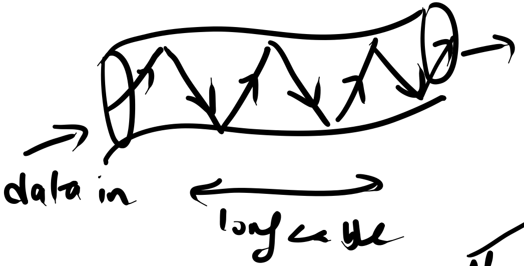
$\theta_c = 24^\circ$

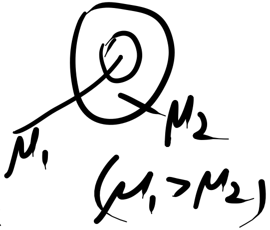
Sparkling of diamond

2). Shining of bubbles



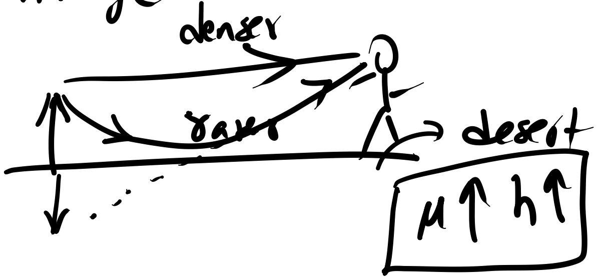
3). Optical fiber cable
(Light to send data)



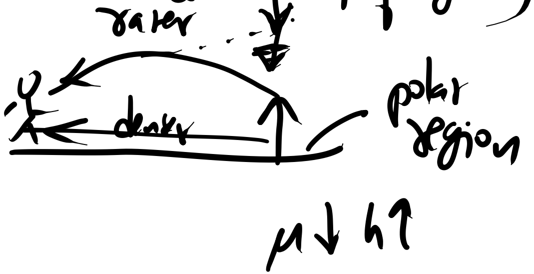


(Min^m loss of data or intensity of light)

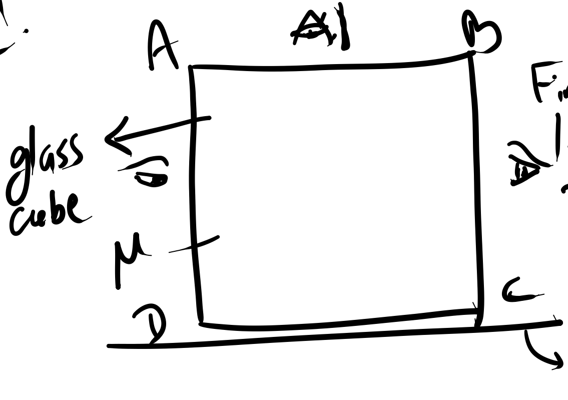
4). Mirage



5) Looming.

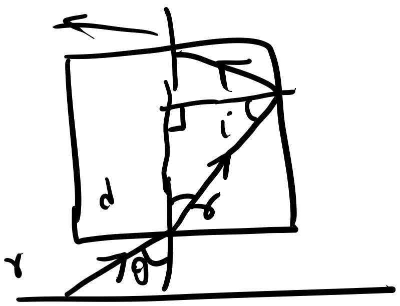


Ex!



Find μ such that letters are not visible from any vertical faces of the glass cube.

$i > \sin^{-1}(1/\mu)$



$i > \theta_c$ (for all values of θ.)

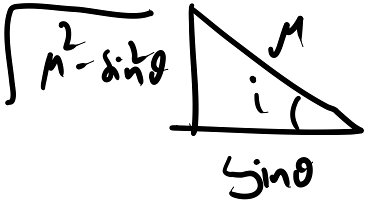
$i + r = \pi - \pi/2 = \pi/2$

$r = \pi/2 - i$

$1 \times \sin\theta = \mu \sin r$

$= \mu \sin(\pi/2 - i) = \mu \cos i$

$\cos i = \sin\theta/\mu \Rightarrow i = \cos^{-1}\left(\frac{\sin\theta}{\mu}\right) > \sin^{-1}(1/\mu)$



$\sin i = \frac{\sqrt{\mu^2 - \sin^2\theta}}{\cancel{\mu}} > \frac{1}{\cancel{\mu}}$

$\mu^2 - \sin^2\theta > 1$

$\mu^2 > 1 + \sin^2\theta$

$\mu^2 > 1 + 1$

$\boxed{\mu > \sqrt{2}}$