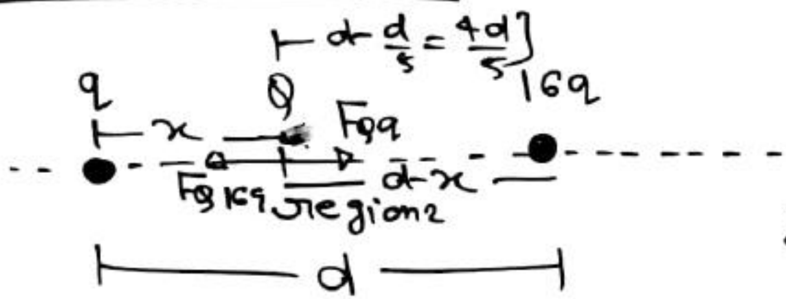
Equilibrium of charge :-

$\boxed{Q}$



$$F_{Qq} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} \quad \text{— (i)}$$

$$F_{Q(16q)} = \frac{1}{4\pi\varepsilon_0}\frac{Q(16q)}{(d-x)^2}$$

F_{net} on Q is zero then

$$F_{Qq} = F_{Q(16q)}$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}}\frac{\cancel{Q}\cancel{q}}{x^2} = \cancel{\frac{1}{4\pi\varepsilon_0}}\frac{\cancel{Q}(16\cancel{q})}{(d-x)^2}$$

$$\frac{1}{x^2} = \frac{16}{(d-x)^2}$$

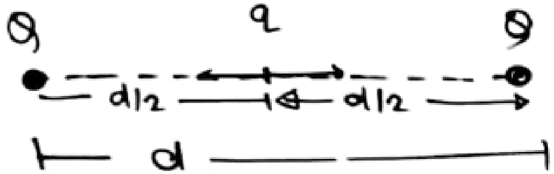
$$\frac{1}{x^2} = \left(\frac{4}{d-x}\right)^2$$

$$\frac{1}{x} = \frac{4}{d-x} \Rightarrow d - x = 4x$$

$$d = 5x$$

$x = \frac{d}{5}$ from q

Equilibrium of charge :-



$(a-b)(a+b)$
$= a^2 - b^2$

$$\boxed{(a-b)^2(a+b)^2 = (a^2-b^2)^2}$$

(q)

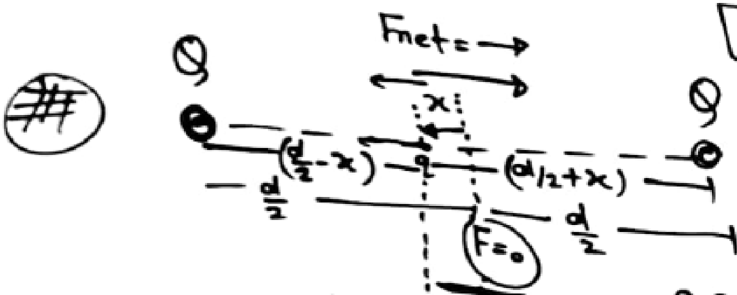
$$F_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{d}{2}-x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{d}{2}+x\right)^2}$$

$$F_{net} = \frac{1}{4\pi\varepsilon_0}qQ\left[\frac{1}{\left(\frac{d}{2}-x\right)^2} - \frac{1}{\left(\frac{d}{2}+x\right)^2}\right]$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{\left(\frac{d}{2}+x\right)^2 - \left(\frac{d}{2}-x\right)^2}{\left(\frac{d}{2}-x\right)^2\left(\frac{d}{2}+x\right)^2}\right]$$

[Condition $\left(x << \frac{d}{2}\right)$

$$F_{net} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{\cancel{\frac{d^2}{4}}+\cancel{x^2}+2\times\frac{d}{\cancel{2}}\times x - \cancel{\frac{d^2}{4}}-\cancel{x^2}+2\times x\times\frac{d}{2}}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}\right]$$



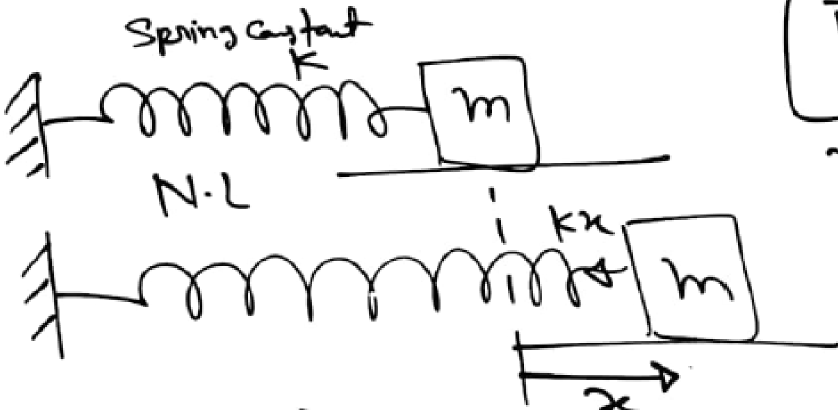
$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{d}{2}+x\right)^2} \leftarrow \quad \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{d}{2}-x\right)^2}$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{2xd}{\left[\left(\frac{d}{2}\right)^2\right]^2}\right] \qquad x^2 \to 0$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{2xd}{\frac{d^4}{16}}\right]$$

$$F_{net} = \frac{qQ}{\cancel{4}\pi\varepsilon_0}\times\frac{2x\cancel{d}}{d^4}\times 16\,4 \xrightarrow{\text{net}} \boxed{= \frac{8qQx}{\pi\varepsilon_0 d^3}}$$

⇒ S.H.M (Simple Harmonic motion)



$F_{spring} = kx$

$$\boxed{\vec{F} = -k\vec{x}}$$

S.H.M

$$\boxed{\vec{F} = -k\vec{x}} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$\Downarrow$

$ma = -kx$

$a = -\frac{k}{m}x$

$\Downarrow$

$-\omega^2 x = -\frac{k}{m}x$

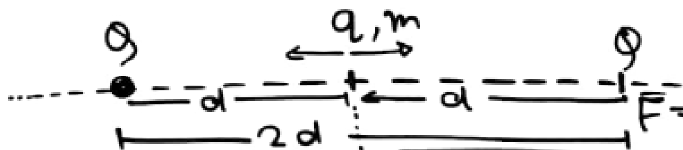
$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

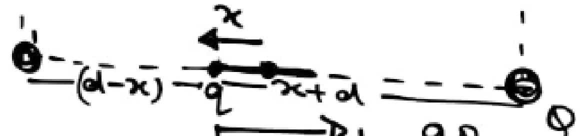
⟹ Time period of charge particle when its displace (x) from equilibrium <u>Position</u> :- [Equilibrium means $F_{net} = 0$]



$F = -Kx$

$\boxed{x << d}$

$$T = 2\pi\sqrt{\frac{m}{qQ/\pi\varepsilon_0 d^3}}$$



$\frac{1}{4\pi\varepsilon}\frac{qQ}{(d-x)^2}$ →

← $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{(d+x)^2}$

$$T = 2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{qQ}}$$

$F_{net} = \rightarrow$

$$F_{net} = \frac{1}{4\pi\varepsilon}\frac{qQ}{(d-x)^2} - \frac{1}{4\pi\varepsilon}\frac{qQ}{(d+x)^2}$$

q $\quad F_{net} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2}\right]$

$$F_{net} = \frac{qQ}{4\pi\varepsilon}\left[\frac{(d+x)^2 - (d-x)^2}{(d-x)^2(d+x)^2}\right]$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon}\left[\frac{\cancel{d^2} + \cancel{x^2} + 2xd - \cancel{d^2} - \cancel{x^2} + 2xd}{\underbrace{(d-x)(d-x)}\underbrace{(d+x)(d+x)}}\right]$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon}\left[\frac{4xd}{(d^2-x^2)^2}\right]$$

x - small
$x^2 \rightarrow 0$

$$F_{net} = \frac{qQ}{4\pi\varepsilon}\frac{4xd}{d^4}$$

$$F_{net} = \frac{qQ}{4\pi\varepsilon_0}\frac{4x}{d^3}$$

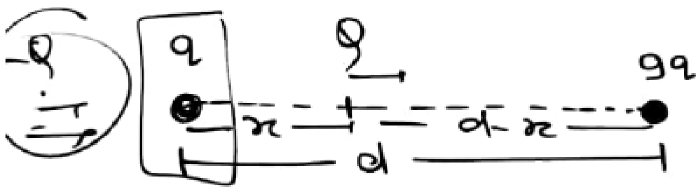
$$\vec{F}_{net} = -\frac{qQ \cdot x}{\pi\varepsilon_0 d^3}$$

$$F_{net} = \frac{qQx}{\pi\varepsilon_0 d^2}$$

$$K = \frac{qQ}{\pi\varepsilon_0 d^3}$$

ABLES KOTA

System Equilibrium :-



$$F_{Qq} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2}$$

$$F_{Q9q} = \frac{1}{4\pi\varepsilon_0}\frac{Q(9q)}{(d-x)^2}$$

$$F_{Qq} = F_{Q/9q}$$

$$\frac{1}{x^2} = \frac{9}{(d-x)^2}$$

$$d-x = 3x$$

$$x = \frac{d}{4}$$

Find Value of third charge & Location of third charge such that whole system in equilibrium.

↳ Step I :- Find Location of third Charge, where Net charge force On third charge is zero.

$\left[x = \frac{d}{3} \text{ from } q\right]$

↳ Step II :- Force on any one charge equal to zero.

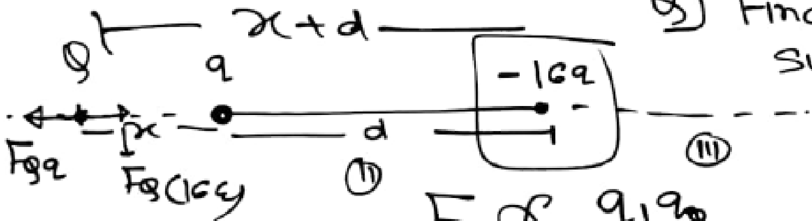
$$F_{qQ} + F_{q9q} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{(d/3)^2} + \frac{1}{4\pi\varepsilon_0}\frac{(q)(9q)}{d^2} = 0$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}}\frac{\cancel{q}Q \times 9}{\cancel{d^2}} = \cancel{\frac{-1}{4\pi\varepsilon_0}}\frac{9q^{\cancel{2}}}{\cancel{d^2}}$$

$$\boxed{Q = -q}$$

Q) Find magnitude & Location of third charge such that whole system in equilibrium.



Step I) Net Force on third charge is zero.

$F \propto q_1 q_2$

$F \propto \frac{1}{d^2}$

Step II) Net Force other than third charge equal to zero

$F_{Qq} = F_{Q/16q}$

$\cancel{\frac{1}{4\pi\varepsilon_0}} \frac{\cancel{Q}q}{x^2} = \cancel{\frac{1}{4\pi\varepsilon_0}} \frac{\cancel{Q}(16q)}{(d+x)^2}$

$\frac{1}{x^2} = \frac{16}{(d+x)^2} \Rightarrow (d+x)^2 = 16x^2$

$(d+x)^2 = (4x)^2$

$d+x = 4x$

$3x = d$

$x = d/3$ — q

$16q$ — $\frac{d}{3} + d = \frac{4d}{3}$

$\frac{1}{4\pi\varepsilon_0} \frac{Q(16q)}{\left(\frac{4d}{3}\right)^2} + \frac{1}{4\pi\varepsilon_0} \frac{q(16q)}{d^2} = 0$

$\cancel{\frac{1}{4\pi\varepsilon_0}} \frac{Q(\cancel{16q}) \times 9}{16\cancel{d^2}} = -\cancel{\frac{1}{4\pi\varepsilon_0}} \frac{q(\cancel{16q})}{\cancel{d^2}}$

$Q = \frac{-16q}{9}$