3-D Geometry :-

* Coplanarity of 2 lines :-

vector → $\boxed{\overrightarrow{AB}\cdot(\vec{b_1}\times\vec{b_2})=0}$ $\overrightarrow{AB}=\vec{a_2}-\vec{a_1}$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \end{vmatrix}$$

Ans :- Show that the lines

$$\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5} \;\&\; \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$$

are coplanner.

Solⁿ :- ∵ lines are coplanner if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \Rightarrow \begin{vmatrix} -1+3 & 2-1 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}=0$$

$2[5-10]-1[-15+5]=0$

$\Rightarrow -\cancel{10}+\cancel{10}=0$

$\boxed{0=0}$ H.P.

Angle b/w 2 planes :-

$\rightarrow \vec{r}\cdot\vec{n_1} = d_1$ & $\vec{r}\cdot\vec{n_2} = d_2$

so angle $\cos\theta = \left|\dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\cdot|\vec{n_2}|}\right|$

vectors

Carti $\rightarrow \cos\theta = \left|\dfrac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

Que: find angle b/w 2 planes $[Ax+By+Cz=d]$

$3x-6y+2z=7$ & $2x+2y-2z=5$

Sol^n: ∵ from the given plane:

$A_1 = 3 \qquad A_2 = 2$

$B_1 = -6 \qquad B_2 = 2$

$C_1 = 2 \qquad C_2 = -2$

$$\cos\theta = \left|\frac{3\times 2+(-6)(2)+(2)(-2)}{\sqrt{9+36+4}\cdot\sqrt{4+4+4}}\right|$$

$$\cos\theta = \left|\frac{6-12-4}{7\cdot 2\sqrt{3}}\right| = \frac{10}{14\cdot\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{7\sqrt{3}}\right)$$

Distance of a point from a plane :-

→ plane → $\vec{r} \cdot \vec{n} = d$ $\quad A\hat{i}+B\hat{j}+C\hat{k}$ $\quad \vec{A} \to (x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$

Distance in vector form :- $\dfrac{|\vec{a}\cdot\vec{N} - d|}{|\vec{N}|}$ → ive → +ive

Cartisian → → $\dfrac{|Ax_1 + By_1 + Cz_1 - d|}{\sqrt{A^2+B^2+C^2}}$

plane
$Ax+By+Cz=d$

Q. find distance of a point $(2,3,-5)$ → $x_1=2$, $y_1=3$, $z_1=-5$

from plane $2x-3y+6z-2=0$ → $A=2$, $B=-3$, $C=6$, d

$2x-3y+6z=2$ d

39 6   33   4 4   37

Sol^n :- Let point → $2,3,-5$

$\hookrightarrow \vec{a} = 2\hat{i}+3\hat{j}-5\hat{k}$

$\rightarrow D = \left|\dfrac{4-9-30+2}{7}\right|$

Cart form = $\left|\dfrac{Ax_1+By_1+Cz_1-d}{\sqrt{A^2+B^2+C^2}}\right|$

$\boxed{D = 37/7}$ Ans

$\Rightarrow \left|\dfrac{2(2)+(-3)(3)+6(-5)-2}{\sqrt{4+9+36}}\right|$

Qus. Find Distance of a point $(2, 5, -3)$ from the plane.

$\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$

Solⁿ:-

vector from $D = \left| \dfrac{\vec{a} \cdot \vec{N} - d}{|\vec{N}|} \right|$

$$D = \left| \frac{(2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) - 4}{\sqrt{36+9+4}} \right|$$

$$D = \left| \frac{12 - 15 - 6 - 4}{7} \right| \Rightarrow D = 13/7 \text{ Ans}$$

☆ Angle b/w a line & a plane :- ☆

2 lines
2 plane — $\cos\theta$

→ angle b/w a line & normal to the plane is given by $\cos\theta$.

$$\cos\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\,|\vec{n}|}$$

$\sin\phi$

$\cos\theta =$ → $\theta = ?$ Complementary angle

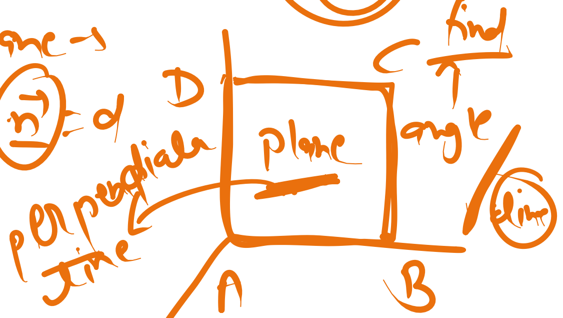
Angle b/w line & plane:-

So this angle ϕ b/w the line & the plane is given by $(90-\theta)$.

$$\sin\phi = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\,|\vec{n}|}$$

line → $\vec{r} = \vec{a} + \lambda\vec{b}$

plane → $\vec{r}\cdot\vec{n} = d$



$$\sin\phi = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\,|\vec{n}|}$$

Ans: find angle b/w the line & plane

$$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6} \quad \& \quad \text{plane} \rightarrow 10x+2y-11z=3$$

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \rightarrow \vec{b} = a\hat{i}+b\hat{j}+c\hat{k}$

225

Sol^n: $\sin\phi = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$

$\rightarrow (\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k})$ & $\vec{n} = 10\hat{i}+2\hat{j}-11\hat{k}$

Sol: $\sin\phi = \left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\cdot(10\hat{i}+2\hat{j}-11\hat{k})}{\sqrt{4+9+36}\cdot\sqrt{100+4+121}}\right|$

$\sin\phi = \left|\frac{20+6-66}{7\cdot 15}\right| = \left|\frac{-40}{105}\right| = \frac{8}{21} \rightarrow \phi = \sin^{-1}\left(\frac{8}{21}\right)$

ABLES KOTA

Ques: Determine whether the given planes are parallel or perpendicular & in case they are neither, find angle b/w them.

→ $[2x + y + 3z - 2 = 0 \ \& \ x - 2y + 5 = 0]$

Solⁿ:- $\because$ $a_1x + b_1y + c_1z = d$ & $a_2x + b_2y + c_2z = d$

⇒ $\because$ plane are parallel → $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ⇒ $\boxed{\frac{2}{1} \neq \frac{1}{-2} \neq \frac{3}{0}}$ → not ∥

→ $\because$ plane are ⊥ → $\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

→ $(2)(1) + (1)(-2) + (3)(0) = 0$ plane 1 & 2 are

→ $2 - 2 + 0 = 0$ ⇒ $\boxed{0 = 0}$ → ⊥. So; angle $= 90°$

ABLES KOTA

13 × 8 → per chapter → $\boxed{\text{8 hours}}$

2

104

↓

Integ. → ITF

7 → 9 Hours → Maths

↓

Extra $\boxed{\text{8 Hours}}$

12 → graph

↓

4 → Types

↓

1 Hour

$\boxed{\text{3 months}}$ → 13 + 14

$$\begin{array}{r} 14 \\ \times\ 9 \\ \hline \boxed{126} \end{array}$$

3 Hours

22 Hours