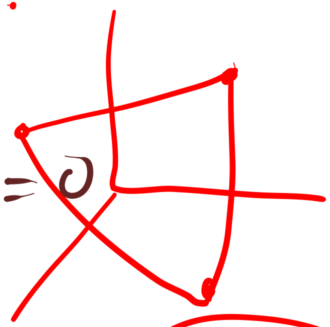
Equation of a plane passing through 3 non-collinear points.

vector form.

$$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$$



$\downarrow$

Cartesian

what is the eq. of plane

Collinear

$\downarrow$

pass $\rightarrow$ plane

$\downarrow$

∞

Q. Give vector form of a plane passing through the points

$\vec{a}$ R(2, 5, −3), $\vec{b}$ S(−2, −3, 5), $\vec{c}$ T(5, 3, −3)

Sol:- Here 1st we check collinearity of points

$$\frac{1}{2}\begin{vmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{vmatrix} = 0 \Rightarrow 2[9-15]-5(6-25)+(-3)(-6+15)$$

$$\Rightarrow -12+95-27 \neq 0$$

$= 0$

i.e. points are non-collinear.

So: eq. of plane :- $[\vec{r} - (2\hat{i}+5\hat{j}-3\hat{k})]\cdot[(-4\hat{i}-8\hat{j}+8\hat{k})\times(3\hat{i}-2\hat{j})] = 0$

Determinate ... →

[If three points are collinear then the area of the Δ made by them is zero.]

i.e.

Intercept form of equation of plane :-

→ e.g. the eq. of plane is :-

$$\boxed{Ax + By + Cz = D}$$

↓

Intercept form

$$\frac{Ax}{D} + \frac{By}{D} + \frac{Cz}{D} = \frac{D}{D} = 1$$

$$\frac{x}{D/A} + \frac{y}{D/B} + \frac{z}{D/C} = 1$$

$$\rightarrow \boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1} \rightarrow \text{I.F.}$$

$$6x + 4y + 3z = 12$$

↓

$$\boxed{\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1}$$

→ line :-

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

$$\frac{2x}{6} + \frac{3y}{6} = \frac{6}{6}$$

$$\boxed{\frac{x}{3} + \frac{y}{2} = 1}$$

Q. find the intercept cut by the plane

$[2x+y-z=5]$

$\downarrow$

Intercept form:- $\frac{2x}{5}+\frac{y}{5}-\frac{z}{5}=\frac{5}{5}$

$\rightarrow \left[\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1\right] \rightarrow$

Sol:- intercept cut by the plane are a, b, c at x, y, z axis respectively.

So: $a=5/2$, $b=5$, $c=-5$ Ans

Plane passing through the intersection of 2 given planes:-

Let:- $\vec{r}\cdot\vec{n_1}=d_1$ & $\vec{r}\cdot\vec{n_2}=d_2$

$\vec{r}\cdot\hat{n}=d$

$Ax+By+Cz=d$

?

vector form:- $\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2})=d_1+\lambda d_2$

$(x\hat{i}+y\hat{j}+z\hat{k})$

$\lambda \to$ Constant

this plane is also passing through a point (x, y, z)

further solve

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Ans: find vector eq. of plane passing through the intersection of the planes

$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})=9$

($\vec{n_1}$, d_1; $\vec{n_2}$, d_2)

& through the point $(2,1,3)$

Sol^n:- ∵ eq. of plane :- $\left(\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2}) = d_1+\lambda d_2\right)$

$\Rightarrow \vec{r}\cdot\left[2\hat{i}+2\hat{j}-3\hat{k}+\lambda(2\hat{i}+5\hat{j}+3\hat{k})\right]=7+\lambda 9$ —— (1)

∵ this plane is passing through $(2,1,3)$

∴ position vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}=2\hat{i}+\hat{j}+3\hat{k}$ —— (2)

from (1) & (2) satisfy the plane.

So: $(2\hat{i}+\hat{j}+3\hat{k})\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]=7+9\lambda$

$\rightarrow 4+4\lambda+2+5\lambda-9+9\lambda=7+9\lambda$

$\Rightarrow -3+18\lambda=7+9\lambda$

$\rightarrow 18\lambda-9\lambda=7+3$

$9\lambda=10 \rightarrow \lambda=10/9$

$\rightarrow$ So from eq.(1):-

$\vec{r}\cdot\left[\left(2+2\cdot\frac{10}{9}\right)\hat{i}+\left(2+5\cdot\frac{10}{9}\right)\hat{j}+\left(-3+3\cdot\frac{10}{9}\right)\hat{k}\right]=7+\frac{10\cdot 9}{9}$

$\vec{r}\cdot\left(\frac{38}{9}\hat{i}+\frac{68}{9}\hat{j}+\frac{3}{9}\hat{k}\right)=17$

$\vec{r}\cdot(38\hat{i}+68\hat{j}+3\hat{k})=153$

Ans

Ques:- Find the eq. of plane through the line of intersection of planes $x+y+z=1$ & $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$

$\rightarrow \lambda(\vec{n}_1+\lambda\vec{n}_2) = d_1+\lambda d_2$

Solⁿ:

$x+y+z=1$ —(1) $2x+3y+4z=5$ —(2)

$\boxed{x+y+z-1=0}$ (3) $[2x+3y+4z-5]=0$ —(4)

So: eq. of plane of intersection of 2 given planes,

$\Rightarrow [(x+y+z-1)+\lambda(2x+3y+4z-5)=0]$

$\rightarrow [(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-(1+5\lambda)]=0$ (1)

∵ the plane in eq (1) is perpendicular to the plane: $\rightarrow x-y+z=0$.

So:- $a_1a_2+b_1b_2+c_1c_2=0$

$(1+2\lambda)+(1+3\lambda)(-1)+(1+4\lambda)1=0$

$\cancel{1}+2\lambda-\cancel{1}-3\lambda+1+4\lambda=0$

$3\lambda=-1 \rightarrow \lambda=-1/3$

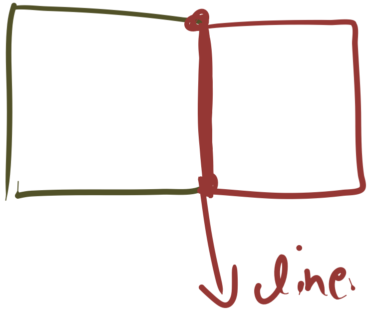
$\rightarrow$ So eq. of plane:- $\frac{1}{3}x+0+-\frac{1}{3}z+\frac{2}{3}=0 \rightarrow x-z+2=0$ Ans

→ when lines & are perpendicular

→ $\boxed{a_1 a_2 + b_1 b_2 + c_1 c_2 = 0}$

↓

plane.



Coplanarity of Two lines -

[· (dot product)

$a \cdot b = 0$

→ let 2 lines are

$\boxed{\vec{r} = \vec{a_1} + \lambda \vec{b_1}}$ & $\boxed{\vec{r} = \vec{a_2} + \lambda \vec{b_2}}$

So there: that 2 lines are coplaner if & only if $\vec{AB}$ is $\perp$ to $\vec{b_1} \times \vec{b_2}$.

Here: $\vec{AB} = \vec{a_2} - \vec{a_1}$

V: → $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$

C. → $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$