3-D Geometry :-

Ans. → There is no such value of b for which f(x) is strictly ↓.

Value of b for which $\boxed{f(x) = x + \cos x + b}$ is $\boxed{\text{strictly decr.}}$ over R.

Sol^n:- a fun. is $\boxed{\text{strictly decreasing}}$

if $[f'(x) < 0]$ — (1)

Now $f(x) = x + \cos x + b$

$f'(x) = 1 + (-\sin x) + 0$

$[f'(x) = 1 - \sin x]$

∵ $f'(x) < 0$

$\boxed{1 - \sin x < 0}$

but: $\boxed{1 - \sin x < 0}$ ↓ not possible

∵ $-1 \leq \sin x \leq 1$

(−) mult. → $[1 \geq -\sin x \geq -1]$

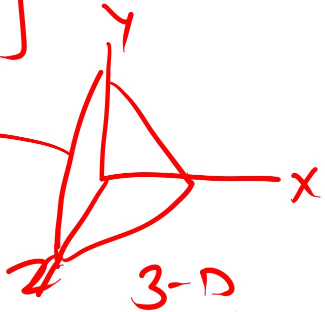
add (1) → $[1 + 1 \geq 1 - \sin x \geq -1 + 1]$

→ $2 \geq 1 - \sin x \geq 0$

$\boxed{1 - \sin x \in [0, 2]}$ → true conc.

Plane :-

$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$



3-D

① Equation of a plane in $\boxed{\text{normal form}}$

position vector $\vec{r}$

$\boxed{\vec{r} \cdot \hat{n} = d}$ → d → distance of plane with origin

$\hat{n}$ ⟹ unit normal vector

↓ $\vec{n}$ → $\boxed{\hat{n} = \dfrac{\vec{n}}{|\vec{n}|}}$

Cart. form :-

2nd

$\boxed{lx + my + nz = d}$

x, y, z → position vector DR.

$\boxed{l, m, n \to \text{Direction cosine of } \hat{n}}$

eq →

$[ax + by + cz = d]$ → DR

Cartesian form :-

$\boxed{ax + by + cz = d}$

a, b, c → DR

$\boxed{lx + my + nz = d}$

$l = \dfrac{a}{\boxed{\sqrt{a^2+b^2+c^2}}} = \sqrt{9} = 3$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{n}$ → DC → l, m, n

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Q. Find vector eq. of plane which is at distance of 7 unit from origin & normal to the vector $\boxed{3\hat{i}+5\hat{j}-6\hat{k}}$.

Sol: eq. of plane $\rightarrow \left[\vec{r}\cdot\hat{n}=d\right] \rightarrow$ given $\boxed{d=7}$

$\hat{n}=\dfrac{\vec{n}}{|\vec{n}|}$ $\quad \because \vec{n} \rightarrow$ normal vector

$\vec{n}=3\hat{i}+5\hat{j}-6\hat{k}$

$|\vec{n}|=\sqrt{9+25+36}=\sqrt{70}$

position

So: $\hat{n}=\dfrac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$

So: eq. is $=\vec{r}\cdot\left(\dfrac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$ Ans

$\vec{r}=\boxed{\vec{a}+\lambda\vec{b}}$

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Ans: find DC of the normal to the plane and find Distance from the origin

(i) $z = 2$ (ii) $2x + 3y - z = 5$

Sol^n: (i) $[z = 2] \rightarrow$ plane

$\because$ plane $\rightarrow [lx + my + nz = d]$

Divide

Here: $0 \cdot x + 0 \cdot y + 1z = 2 \rightarrow$ plane $\rightarrow$ Here Direction Ratio are, $[a = 0,\ b = 0,\ c = 1]$

So Direction Cosine are: $l = \dfrac{0}{\sqrt{0+0+1}} = 0 \;\Big|\; m = \dfrac{0}{\sqrt{0+0+1}} = 0 \;\Big|\; n = \dfrac{1}{\sqrt{0+0+1}} = 1$

$\rightarrow \boxed{0, 0, 1} \rightarrow$ A

$\rightarrow \sqrt{0+0+1} = 1$

& eq. of plane.

$0x + 0y + 1z = 2$

So:

DC are: $[0, 0, 1]$

& Distance of plane from origine $= 2$ A

Q. find Cart. form of given plane.

$\left[\vec{r}\cdot\left(\boxed{2\hat{i}+3\hat{j}-4\hat{k}}\right)=1\right]$ — (1)

Sol: $\because \left[\vec{r}\cdot\underline{\vec{n}}=d\right]$

let: $\left[\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}\right]$ — (2)

from (1) & (2):

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$

$\rightarrow \boxed{2x+3y-4z=1}$ Ans

Q. $\boxed{2x+3y-z=5}$

plane $\left[ax+by+cz=d\right]$

$a=2,\ b=3,\ c=-1$

So: $\sqrt{a^2+b^2+c^2}=\sqrt{4+9+1}=\sqrt{14}$

<u>Now</u> Dividing both side of eq. by $\sqrt{14}$

$\rightarrow \boxed{\frac{2}{\sqrt{14}}x+\frac{3}{\sqrt{14}}y-\frac{1}{\sqrt{14}}z=\frac{5}{\sqrt{14}}}$

$\left[lx+my+nz=d\right]$

So: DC $\rightarrow$ $2/\sqrt{14},\ 3/\sqrt{14},\ -1/\sqrt{14}$

$d=5/\sqrt{14}$ Ans

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Eq. of a plane perpendicular to a given vector & passing through a given point.

Eq. of plane: $\rightarrow \left[(\vec{r} - \vec{a}) \cdot \vec{N} = 0 \right]$

$\vec{r} \rightarrow$ position vector.

$\vec{a}$ = point from which plane is passing. $= \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{N}$ = vector which is $\perp$ to plane. $= \vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$

Carti. form: $\rightarrow \boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$

Q. Find vector & Cartesian form of the plane which passes through the points (5, 2, -4) & ⊥ to the line with Direction Ratio 2, 3, -1.

Sol^n: given: $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ & $\vec{N} = 2\hat{i} + 3\hat{j} - \hat{k}$

So vector form: $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$\rightarrow \left[\vec{r} - (5\hat{i} + 2\hat{j} - 4\hat{k})\right] \cdot \left[2\hat{i} + 3\hat{j} - \hat{k}\right] = 0$ Ans

Cartesian form: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

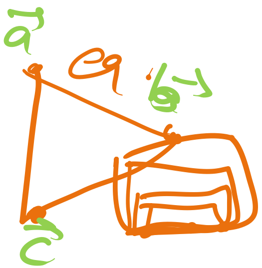
$\rightarrow \left[(x\hat{i} + y\hat{j} + z\hat{k}) - (5\hat{i} + 2\hat{j} - 4\hat{k})\right] \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$

$\rightarrow (x-5)2 + (y-2)3 + (z+4)(-1) = 0$ Ans

$\rightarrow 2x + 3y - z - 10 - 6 - 4 = 0 \rightarrow \boxed{2x + 3y - z = 20}$

③ Eq. of a plane passing through 3-non Collinear points.

$\vec{a}, \vec{b}, \vec{c}$



$\left[(\vec{r}-\vec{a})\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]\right] = 0$

[illegible]

Solve नहीं करना

Conclusion:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$