Ques:- Show that the lines

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \quad \& \quad \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$ are $\perp$ to each other.

Soln:-

$a_1 \quad b_1 \quad c_1 \qquad a_2 \quad b_2 \quad c_2$

$$[a_1 \cdot a_2 + b_1 b_2 + c_1 c_2 = 0]$$

Shortest Distance b/w 2 lines

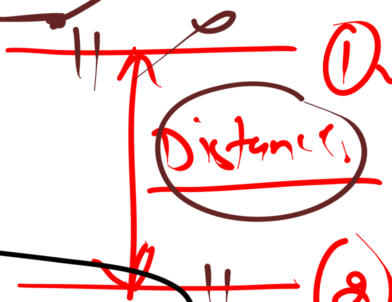
[Distance b/w 2 point]

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(1) vector:

let 1st line → $\vec{r}_1 = \vec{a}_1 + \lambda \vec{b}_1$

2nd line → $\vec{r}_2 = \vec{a}_2 + \mu \vec{b}_2$



$a_1 = x_1 y_1 z_1 \quad b_1 = a_1\ b_1\ c_1$

$a_2 = x_2 y_2 z_2 \quad b_2 = a_2\ b_2\ c_2$

$$\text{Dis} \rightarrow D|d = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Cartesian form: →

$$\text{Distance} = \left|\frac{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) & (z_2-z_1) \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

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Ques Find Shortest Dis. b/w lines l_1 & l_2

$l_1 \to \left[\ \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})\ \right]$

$\left[\ \vec{r} = 2\hat{i} + \hat{j} - k + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})\ \right]$

Sol.

$\because\ a_1 = \hat{i} + \hat{j}$ $\qquad b_1 = 2\hat{i} - \hat{j} + k$

$a_2 = 2\hat{i} + \hat{j} - \hat{k}$ $\qquad b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\Rightarrow$ SD Distance $= \left| \dfrac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$D = \left| \dfrac{10}{\sqrt{59}} \right| = \dfrac{10}{\sqrt{59}}$ Ans

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3) \to |\vec{b_1} \times \vec{b_2}| = \sqrt{(3)^2 + (1)^2 + (-7)^2}$

$= 3\hat{i} - \hat{j} - 7\hat{k}$ ✓ $\qquad = \sqrt{9+1+49} = \sqrt{59}$

$\star\ \vec{a_2} - \vec{a_1} = \hat{i} + 0\hat{j} - \hat{k} = \hat{i} - \hat{k}$

$\Rightarrow (\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k}) = 3 + 0 + 7 = 10$

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Ans. Find shortest distance b/w the lines

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \quad \& \quad \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

Solⁿ:- Here we know that :-

$a_1 = 7 \Rightarrow x_1 = -1$	$a_2 = 1 \rightarrow x_2 = 3$
$b_1 = -6 \rightarrow y_1 = -1$	$b_2 = -2 \rightarrow y_2 = 5$
$c_1 = 1 \rightarrow z_1 = -1$	$c_2 = 1 \rightarrow z_2 = 7$

So Distance $\Rightarrow$

$$\frac{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) & (z_2-z_1) \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(a_2c_1-a_1c_2)^2+(a_1b_2-a_2b_1)^2}}$$

$$D = \frac{\begin{vmatrix} (3+1) & (5+1) & (7+1) \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\sqrt{(-6+2)^2+(1-7)^2+(-14+6)^2}}$$

$$D = \left|\frac{4(-6+2)-6(7-1)+8(-14+6)}{\sqrt{16+36+64}}\right|$$

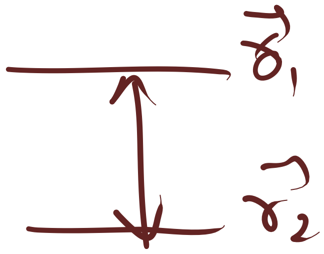
$$D = \left|\frac{-16-36-64}{\sqrt{116}}\right|$$

$$D = \left|\frac{-116}{\sqrt{116}}\right| = \frac{116}{\sqrt{116}}$$

$$D = \sqrt{116} = \sqrt{29\times4} = 2\sqrt{29}$$

Distance b/w 2 parallel lines

$$\text{Dis} = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$



Q. Find Distance b/w 2 lines given below

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Sol: $\because$ Here the both lines are parallel because $\vec{b_1} = \vec{b_2}$

$$\text{So Dis} = \left| \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})}{\sqrt{4+9+36}} \right| = ?$$

$$D = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{7}$$

$$D = \hat{i}(-3-6) - \hat{j}(-2-12) + \hat{k}(2-6)$$

$$D = \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{7}$$

$$D = \frac{\sqrt{81+196+16}}{7}$$

$$D = \frac{\sqrt{293}}{7} \quad \text{Ans.}$$

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Ques:- Find shortest Distance b/w the lines whose vector equations are $\vec{r} = \underline{\vec{a} + \lambda\vec{b}}$

$$D = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ &

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

Soln:-

$\vec{r} = \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$

$\vec{r} = \underbrace{\hat{i} - 2\hat{j} + 3\hat{k}}_{\vec{a_1}} + t\underbrace{(-\hat{i} + \hat{j} - 2\hat{k})}_{\vec{b_1}}$

$\vec{r} = s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$

$\vec{r} = \underbrace{(\hat{i} - \hat{j} - \hat{k})}_{\vec{a_2}} + s\underbrace{(\hat{i} + 2\hat{j} - 2\hat{k})}_{\vec{b_2}}$

eg. →

given Cartesian form

↓

Vector formula → easy

angle →

Dist →

Solve →

$\vec{a_1}$

$\vec{a_2}$

$\vec{b_1}$

$\vec{b_2}$

Equation of a plane in normal form.

points → line → plane

vector form :- →

Cartesian form →

angle → b/w 2 planes
dis → b/w 2 planes
→ b/w 2 parallel planes