3-D Geometry :-

* DR → DR

p. → $\vec{a}$; q. → $\vec{b}$

line → 1 point
↳ 1 line → parallel
→ direction

→ vector →] eq. of line → $\vec{r} = a + \lambda b$

Cart →] eq. of line → $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

DR → a, b, c

(a) A ($\vec{a}$) —— B ($\vec{b}$) → $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

→ $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$

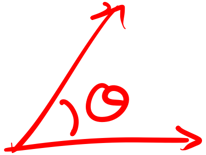
DR → x_2-x_1, y_2-y_1, z_2-z_1

$\vec{a} = x_1, y_1, z_1$ $\quad \vec{b} = x_2, y_2, z_2$

Angle between 2 lines :-

$\vec{b_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b_2} = a_2\hat{i} + b_2\hat{j} + 3\hat{k}$



→ vector form of lines :-

$\vec{r_1} = \vec{a_1} + \lambda\vec{b_1}$ & $\vec{r_2} = \vec{a_2} + \lambda\vec{b_2}$

then angle b/w $\vec{r_1}$ & $\vec{r_2}$ = $\boxed{\cos\theta = \dfrac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\cdot|\vec{b_2}|}}$

Cart. form of lines

$$\boxed{\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}}$$

Ques: find angle b/w pairs of lines given as.

Line passes | Direction/parallel

(1) $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ — (1)

(2) $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ — (2)

Sol^n: from eq (1) :-

$\Rightarrow \vec{r_1} = \vec{a_1} + \lambda\vec{b_1} \Rightarrow \vec{a_1} = 5\hat{i} - 2\hat{j} + 0\cdot\hat{k}$

$x\hat{i} + y\hat{j} + z\hat{k} =$

$\boxed{\vec{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}}$

from eq (2), $\vec{a_2} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = C$

$\vec{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

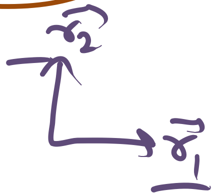
So angle $\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\cdot|\vec{b_2}|} = \frac{(3\hat{i} + 2\hat{j} + 6\hat{k})(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{9+4+36}\cdot\sqrt{1+4+4}}$

$\Rightarrow \cos\theta = \frac{3\times1 + 2\times2 + 12}{7\cdot3}$

$\cos\theta = \frac{19}{21} \rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$

A

if direction cosine of 2 lines are perpendicular than :-



→ 1 line → l_1, m_1, n_1

2^{nd} line → l_2, m_2, n_2

(1) if perpendicular :- $[l_1 l_2 + m_1 m_2 + n_1 n_2 = 0]$

(2) if parallel :- $\left[\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}\right] = \lambda$

But in case of Direction Ratio → if $\perp$ → $[a_1 a_2 + b_1 b_2 + c_1 c_2 = 0]$

if $\parallel$ → $\left[\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k\right]$

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Qus :- Show that the three lines with DC

A $\left(\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}\right)$, B $\left(\frac{4}{13}, \frac{12}{13}, \frac{3}{13}\right)$ C $\left(\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}\right)$

are mutually perpendicular.

Solⁿ :- if 2 lines are perpend. the their DC

$\Rightarrow$ $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

Sol :- from point A & B

$l_1 = \frac{12}{13}$, $m_1 = \frac{-3}{13}$, $n_1 = -\frac{4}{13}$

$l_2 = \frac{4}{13}$, $m_2 = \frac{12}{13}$, $n_2 = \frac{3}{13}$

$\Rightarrow \therefore$ if DC are perpend.

then: $\left(\frac{12}{13}\right)\left(\frac{4}{13}\right) + \left(\frac{-3}{13}\right)\left(\frac{12}{13}\right) + \left(\frac{-4}{13}\right)\left(\frac{3}{13}\right)$

$\frac{48}{169} - \frac{36}{169} - \frac{12}{169} = \frac{48-48}{169} = 0$

it means A & B are perpendicular.

similarly :-

B = l_1 m_1 n_1

C = l_2 m_2 n_2

Q. Show that the line through the point (4, 7, 8) (2, 3, 4) is ~~parallel~~ perpendicular to the line through the points (−1, −2, 1) (1, 2, 5).

Solⁿ :-

A ——— DR ——— B

(4, 7, 8) ——— (2, 3, 4)

AB ∥ CD

C ——— DR ——— D

(−1, −2, 1) ——— (1, 2, 5)

1st we find DR of AB & CD

AB = $a_1 = (2-4) = -2$

$b_1 = (3-7) = -4$

$c_1 = 4-8 = -4$

CD = $a_2 = 1+1 = 2$

$b_2 = 2+2 = 4$

$c_2 = 5-1 = 4$ → $\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

H.P.

∴ both line AB & CD are parallel if

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\Rightarrow \dfrac{-2}{2} = \dfrac{-4}{4} = \dfrac{-4}{4}$

$\boxed{-1 = -1 = -1}$ H.P.

Que: Find eq. of line in vector & Carte. form that passes through the point with vector $2\hat{i}-\hat{j}+4\hat{k}$ & in the Direction of $\hat{i}+2\hat{j}-\hat{k}$

Sol:
$\vec{a} = 2\hat{i}-\hat{j}+4\hat{k} \rightarrow x_1=2, y_1=-1, z_1=4$

$\vec{b} = \hat{i}+2\hat{j}-\hat{k} \rightarrow a=1, b=2, c=-1$

$\checkmark \ \vec{a}+\lambda\vec{b}$

$\frac{x-x_1}{a}$

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Q. find Cart. form of line which pass through point $(-2, 4, -5)$ & parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6} \rightarrow \text{Cartesian}$$

$(-3, 4, -8) \rightarrow \vec{a}$

$(3, 5, 6) \rightarrow \vec{b}$

Soln Cart $\rightarrow$ $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

Here $\vec{a} = -2\hat{i} + 4\hat{j} - 5\hat{k}$

& $\vec{b} = 3\hat{i} + 5\hat{j} + 6\hat{k}$

A

B

C

Sol:- Cart. $\rightarrow$ $\dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+5}{6}$

A

$\Rightarrow \underset{\text{vector}}{\vec{r}} = \underline{\vec{a}} + \lambda \underline{\vec{b}} \longrightarrow$ Cart'

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

let $\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

Now $\vec{r} =$

$\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$x\hat{i} + y\hat{j} + z\hat{k} = (x_1 + \lambda a)\hat{i} + (y_1 + \lambda b)\hat{j} + (z_1 + \lambda c)\hat{k}$

on Comparing $x = x_1 + \lambda a$ | $y = y_1 + \lambda b$ | $z = z_1 + \lambda c$

$\frac{x-x_1}{a} = \lambda$ — (1) | $\frac{y-y_1}{b} = \lambda$ — (2) | $\frac{z-z_1}{c} = \lambda$ (3)

from (1) (2) (3) :-

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

Af

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Q. The Cart. form of a line is

$$\frac{x+5}{3} = \frac{y+4}{7} = \frac{z+6}{2}$$, write its vector form.

Ans: Find vector & Cart. form of eq. of line passes through the points

$(3,-2,-5)$ $(3,-2,6)$

$\vec{a}$ $\vec{b}$

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$

angle b/w 2 lines :- vector form

Que: find angle b/w following pair of lines:

$$\boxed{\frac{x}{2}=\frac{y}{2}=\frac{z}{1}} \ \& \ \boxed{\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}}$$

Sol^n. :- ∵ it Cart form:

$$\Rightarrow \quad a_1 = 2 \qquad a_2 = 4$$
$$b_1 = 2 \qquad b_2 = 1$$
$$c_1 = 1 \qquad c_2 = 8$$

64
1
16
81

1

$$\because \text{angle} = \cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{18}{3\cdot 9} = \frac{2}{3} \qquad \cos^{-1}\left(\frac{2}{3}\right)$$

Q. find value of p so that the line

$$\frac{1-x}{-3} = \frac{7y-14}{2p} = \frac{z-3}{2}$$ &

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$ are at right angles.

$$\frac{x-x_1}{a} = \frac{y-y_1}{b}$$

$$\frac{x-1}{}\quad \frac{y-2}{2p/7}$$

Soln. ∵ Cart. form → lines ⊥

$$\boxed{a_1 \cdot a_2 + b_1 b_2 + c_1 c_2 = 0}$$

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$$p = 7.$$