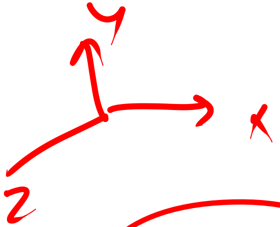
Three Dimensional Geometry :-

→ 2-D

geo-

Ref: x, y
z

→ 3-D →



vector

Orteg.

→ vector → 3-D Geo. (3-D) → (x, y, z)

↓ AOG

A(x, y, z) 3D

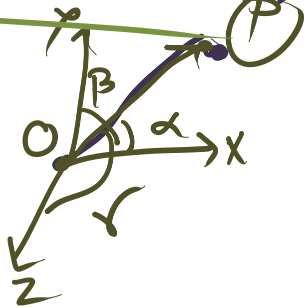
$$\boxed{\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}}$$

Diff → App x, y
z

Direction Cosine :

$\alpha, \beta, \gamma \to$ Direction angle

$\underset{l}{\cos\alpha}, \underset{m}{\cos\beta} \;\&\; \underset{n}{\cos\gamma} \to \boxed{DC}$



$\Rightarrow \left[k = \frac{l}{a} = \frac{m}{b} = \frac{n}{c} \right]$ ②

Now $\to \boxed{l^2 + m^2 + n^2 = 1}$

$k^2a^2 + k^2b^2 + k^2c^2 = 1$ from ①

$(a^2 + b^2 + c^2)k^2 = 1$

$$\boxed{k = \pm \frac{1}{\sqrt{a^2+b^2+c^2}}}$$

Direction Ratio :- $\left[\vec{P} = a\hat{i} + b\hat{j} + c\hat{k}\right]$

$\to \boxed{a, b, c} \to$ D.R.

Direction Cosine are proportional to Direction Ratios :

$l \propto a \;|\; m \propto b \;|\; n \propto c$

$\boxed{l = ka \;|\; m = kb \;|\; n = kc}$ ①

& from eq ② :

$$\pm \frac{1}{\sqrt{a^2+b^2+c^2}} = \frac{l}{a} \Rightarrow \boxed{l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}}$$

$$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}} \;\Big|\; n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Relation b/w Direction Cosine & Ratio $\rightarrow$ $l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$, m, n

$\Rightarrow \checkmark \left[l^2 + m^2 + n^2 = 1 \right]$

D(ratios) of line PQ = $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$

$\Rightarrow$ Direction Cosine of a line passing through two points

let P (x_1, y_1, z_1) —— Q (x_2, y_2, z_2)

$P(a\hat{i} + b\hat{j} + c\hat{k})$, Q

DC = $l = \frac{a}{|\vec{OP}|} \rightarrow x_2 - x_1$

$m = b/\vec{OP} \rightarrow y_2 - y_1$

$n = c/\vec{OP} \rightarrow z_2 - z_1$

$\Rightarrow$ DC of line segment joining P & Q are $\rightarrow$ $\boxed{\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}}$

where PQ =

$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Qus:- if a line make angles 90°, 135°, 45° with x, y & z respect. find its Direction Cosine.

α, β, γ

$\cos\alpha, \cos\beta, \cos\gamma$

$\cos 90$	$\cos 135$	$\cos 45$
$l = 0$	$m = -\frac{1}{\sqrt{2}}$	$n = \frac{1}{\sqrt{2}}$

Direction Ratios of line joining P & Q are

$$(x_2 - x_1)\ (y_2 - y_1)\ (z_2 - z_1)$$

ABLES KOTA

Q. find Direction cosine l, m & n of a line which makes equal angles with the coordinate axes

Solⁿ :- ∵ given line makes equal angle with axes.

let :- x axis → α → $l = \cos\alpha$
y axis → α → $m = \cos\alpha$
z axis → α → $n = \cos\alpha$

∵ $l^2 + m^2 + n^2 = 1$

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$\cos\alpha = \pm\frac{1}{\sqrt{3}} = l = m = n$

Que: if a line has DR $-18, 12, -4$ then what are its Direction cosine.

Direction Ration

a b c

Sol^n: $\because \ l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$

$m = $

$\because \sqrt{a^2+b^2+c^2} = \sqrt{(-18)^2+(12)^2+(-4)^2}$

Q. Show that the points $(2, 3, 4)$ $(-1, -2, 1)$ $(5, 8, 7)$ are collinear

$\underline{A}$ $\quad$ $\underline{B}$ $\quad$ C

$$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$$

IInd Method: DR → AB, DR → BC are proportional

↓

ABC → collinear

$DR \to AB = (x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

$= -3, -5, -3$

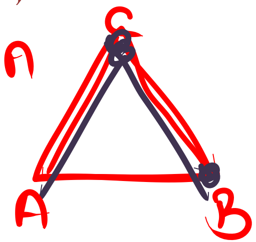
$BC = 6, 10, 6$

Here: $\frac{-3}{6} = \frac{-5}{10} = \frac{-3}{6} = \frac{1}{2}$ → So A, B, C are collinear

Q. find $\boxed{\text{DC of the sides of}}$ a Δ whose vertices are A $(3, 5, -4)$ B $(-1, 1, 2)$ C $(-5, -5, 2)$

Sol^n:- there are 3 sides AB, BC & CA

So we have find DC of all 3 sides



for AB → DC are

$$\boxed{\frac{x_2-x_1}{AB}}, \frac{y_2-y_1}{AB}, \frac{z_2-z_1}{AB}$$

A $\boxed{\text{DC}}$ B

$\boxed{(3, 5, -4)}$ $\boxed{(-1, 1, 2)}$

$$AB = \sqrt{(-1-3)^2 + (1-5)^2 + (2+4)^2} = \sqrt{16+16+36}$$

$$AB = \sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}$$

Sol:- $l = \frac{-1-3}{2\sqrt{17}} = \frac{-4}{2\sqrt{17}}$

$m = \frac{1-5}{2\sqrt{17}} = \frac{-4}{2\sqrt{17}}$

$n = \frac{2+4}{2\sqrt{17}} = \frac{6}{2\sqrt{17}}$ Ans

Now BC = H.W.

CA = H.W

Equation of a line in space :-

$y = mx + c$

① When line passes through a point & has given Direction is called

② When line passes through 2 points.

$A \longrightarrow B$

① Eq. of a line through a given point & parallel to a given vector → Direction

Vector form :- $\vec{r} = \vec{a} + \lambda \vec{b}$

→ where $\vec{a}$ → from which line passes

$\vec{b}$ → vector whose parallel your line is.

Cartesian form :- let $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

& $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

then $\vec{r}$ $\left[\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}\right]$

$\cdot P\ (a\hat{i} + b\hat{j} + c\hat{k})$

→ ∥ vector → $\vec{b}$

A → vector when your line

Q. find the vector & Cartesian eq. of the line through the points (5, 2, −4) & which is parallel to vector $(3\hat{i}+2\hat{j}-8\hat{k})$

Sol:- given ∵ line passes through $(5, 2, -4) \rightarrow \boxed{\vec{a} = 5\hat{i}+2\hat{j}-4\hat{k}}$

& $\vec{b} = 3\hat{i}+2\hat{j}-8\hat{k}$

vector eq. of line = $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = 5\hat{i}+2\hat{j}-4\hat{k} + \lambda(3\hat{i}+2\hat{j}-8\hat{k})$

$\vec{a} = \vec{b}$

$x_2 - x_1 \; (\vec{b}-\vec{a})$

Ans $\boxed{\vec{r} = (5+3\lambda)\hat{i} + (2+2\lambda)\hat{j} - (4+8\lambda)\hat{k}}$

Cartesian eq of line ⇒ $\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$

where $a, b, c \rightarrow 3, 2, -8 \rightarrow \frac{x-5}{3}, \frac{y-2}{2}, \frac{z+4}{-8}$ Ans

$x_1, y_1, z_1 \rightarrow 5, 2, -4$

2 points :- eq. of line :

$\vec{a}\,(x_1, y_1, z_1)$ $\quad$ $\vec{b}\,(x_2, y_2, z_2)$

$\vec{r} = \vec{a} + \lambda \vec{b}$

vector → $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Cartesian → $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - y_1}$

over