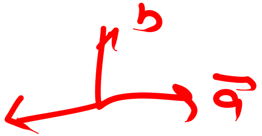
# Vector :-

* **Vector / Cross product :-**




$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \cdot \sin\theta \, \hat{n}$$

$\Rightarrow \sin\theta = \dfrac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$

# $[\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$

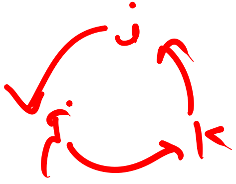
# $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

$[\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$

(1) $\vec{a} \parallel \vec{b} \rightarrow \vec{a} \times \vec{b} = 0$

$[\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0]$

(2) $\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{i} = \hat{j}$

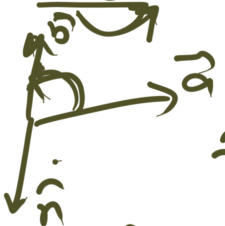


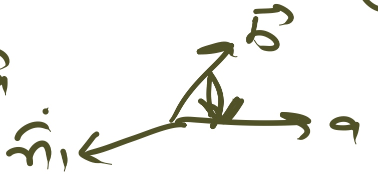

$\rightarrow$ But CW

(3) $\left[\begin{array}{l} \hat{j} \times \hat{k} = -\hat{i} \\ \hat{k} \times \hat{i} = -\hat{j} \\ \hat{i} \times \hat{j} = -\hat{k} \end{array}\right]$

Vector product

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$




$\vec{b} \times \vec{a} = |\vec{b}||\vec{a}| \sin\theta \, \hat{n}_1$

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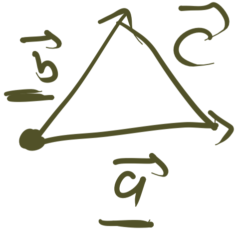
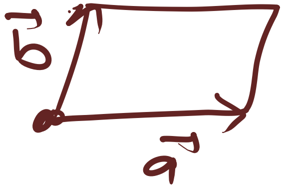
# if $\vec{a}$ & $\vec{b}$ are adjacent sides of a $\Delta$. then area of

$$\boxed{\Delta = \frac{1}{2}\left|\vec{a} \times \vec{b}\right|}$$




# if $\vec{a}$ & $\vec{b}$ are adjacent side of a parallelogram:

$\rightarrow$ area $= \left|\vec{a} \times \vec{b}\right|$




$\boxed{\vec{a} \times \vec{b}}$ $\qquad \vec{a} \cdot \vec{b} =$

# properties:

① $\vec{a} \times (\vec{b} + \vec{c})$

$\rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

② $\lambda(\vec{a} \times \vec{b})$

$= \lambda\vec{a} \times \vec{b}$

$= \vec{a} \times \lambda\vec{b}$

$\vec{a} =$ ____ $\downarrow \uparrow$

$\vec{b} =$

$S_\vartheta(\vec{a} \times \vec{b}) =$ ____

# $[\vec{a}\cdot\vec{b} = x_1x_2 + y_1y_2 + z_1z_2]$

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

# $\vec{a}\times\vec{b} = ?$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \rightarrow \text{Det}$$

$\vec{a}\times\vec{b}$

$$\left[\Rightarrow \hat{i}(a_2b_3 - b_2a_3) - \hat{j}(a_1b_3 - b_1a_3) + \hat{k}(a_1b_2 - a_2b_1)\right]$$

a. $|\vec{a} \times \vec{b}| \Rightarrow ?$

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ | $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i}(\cancel{-14} + \cancel{14}) - \hat{j}(2 - 21) + \hat{k}(-2 + 21)$$

$$= +19\hat{j} + 19\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = 19\sqrt{2}$$

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find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$, where $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

Solⁿ: Let unit vector $\rightarrow \hat{n} = ? = \dfrac{\vec{n}}{|\vec{n}|}$

$\vec{a}+\vec{b} = \vec{c} = 4\hat{i}+4\hat{j}$

$\vec{a}-\vec{b} = \vec{d} = 2\hat{j}+4\hat{k}$

perpendicular

$\vec{n} = \vec{c}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & - & - \\ - & - & - \end{vmatrix} = \underline{\quad} = \vec{n}$

$|\vec{n}| =$

Q. if $\vec{a}$ unit vector $\rightarrow |\vec{a}| = 1$

$\vec{a}$ makes an angle $\pi/3$ with $\hat{i}$

$\pi/4$ with $\hat{j}$

& an acute angle $\theta$ with $\hat{k}$

then find $\theta$ & Hence, the Component of $\vec{a}$

$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

**Sol^n:** let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\because \boxed{\cos\alpha = \dfrac{a_1}{|\vec{a}|}}$

$\rightarrow \cos\dfrac{\pi}{3} = \dfrac{a_1}{1} \Rightarrow \boxed{a_1 = \dfrac{1}{2}}$

$\rightarrow \cos\pi/4 = \dfrac{a_2}{1} \Rightarrow \boxed{a_2 = 1/\sqrt{2}}$

$\rightarrow \cos\theta = \dfrac{a_3}{1}$

$\boxed{a_3 = \cos\theta}$

$\boxed{a_3 = 1/2}$

$\vec{a}$ compo. $\rightarrow \left(\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}, 1/2\right)$

$\because \vec{a} \rightarrow$ unit vector

$|\vec{a}| = 1$

$\sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$\sqrt{1/4 + 1/2 + \cos^2\theta} = 1$

$\dfrac{3}{4} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{3}{4} = 1/4$

$\boxed{\cos\theta = 1/2}$

find $\lambda$ & $\mu$

if $(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+\lambda\hat{j}+\mu\hat{k}) = \vec{0}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$\lambda$, & $\mu$

Q. given $\boxed{\vec{a}\cdot\vec{b}=0}$ & $\boxed{\vec{a}\times\vec{b}=0}$

what you can conclude about $\vec{a}$ & $\vec{b}$?

Sol$^n$. $\vec{a}\cdot\vec{b}=0 \rightarrow \boxed{\vec{a}\perp\vec{b}=0} \rightarrow \boxed{\theta=90^\circ}$

$\vec{a}\times\vec{b}=0 \rightarrow \boxed{\vec{a}=0 \,/\, \vec{b}=0} \rightarrow \boxed{\theta=0^\circ}$

either $\vec{a}=0$ | $\vec{b}=0$ ✓

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if $\vec{a}, \vec{b}$ & $\vec{c}$ are :-

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{b}+\vec{c} = \boxed{(b_1+c_1)\hat{i} + \quad = \vec{d}}$

$\vec{a}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & - & - \\ - & - & - \end{vmatrix}$

✓

Show that

$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{c}\times\vec{a}$

LHS

$\vec{a}\times\vec{b} = \vec{p}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & - & - \\ - & - & - \end{vmatrix}$

$\vec{q}$

$\vec{p}+\vec{q}$

Q. either $\vec{a} = 0$ or $\vec{b} = 0 \rightarrow \vec{a} \times \vec{b} = 0$

its converse is true = ? → with exam

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ → $|\vec{a}|$

$\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ → $|\vec{b}|$

$\vec{a} \times \vec{b} =$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & - & - \\ - & - & - \end{vmatrix}$

Row ←

Same / same properties

find area of $\Delta$ with vertices

$A(1,1,2)$ $B(2,3,5)$ & $C(1,5,5)$

$\vec{a} = \vec{B} - \vec{A}$

$\vec{b} = \vec{C} - \vec{A}$


$\dfrac{|\vec{a} \times \vec{b}|}{2} \Rightarrow \Delta$ Area

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Q. find area of parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ & $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$\hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$$

$$20\hat{i} + 5\hat{j} - 5\hat{k} \rightarrow |\vec{a} \times \vec{b}| = \sqrt{400+25+25}$$

$$= \sqrt{450} \quad \sqrt{225 \times 2} = 15\sqrt{2} \text{ Ans}$$

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Q. Area of rectangle have vertices A, B, C, D

$-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$

$\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$

$\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

$-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$


parallelogram

$|\vec{AB} \times \vec{AD}|$

# Show that :-

$$(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = 2(\vec{a}\times\vec{b})$$

Sol$^n$: LHS → $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

→ $\vec{a}\times(\vec{a}+\vec{b}) - \vec{b}\times(\vec{a}+\vec{b})$

→ $\vec{a}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{a} - \vec{b}\times\vec{b}$

⇒ $0 + \vec{a}\times\vec{b} - [-\vec{a}\times\vec{b}] - 0$

⇒ $2(\vec{a}\times\vec{b})$ ⇒ RHS A.P.