vector

Qus:- Find $|\vec{a}|$ & $|\vec{b}|$ if

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$ & $[|\vec{a}| = 8|\vec{b}|]$

Soln:-

$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$

$\vec{a}\cdot\vec{a} \boxed{\cancel{-\vec{a}\cdot\vec{b}} + \cancel{\vec{b}\cdot\vec{a}}} - \vec{b}\cdot\vec{b} = 8$

$|\vec{a}|^2 - |\vec{b}|^2 = 8$

$[8|\vec{b}|]^2 - |\vec{b}|^2 = 8$

$64 \cdot |\vec{b}|^2 - |\vec{b}|^2 = 8$

$63|\vec{b}|^2 = 8$

$|\vec{b}| = \sqrt{\frac{8}{63}}$ Ans $= \frac{2\sqrt{2}}{3\sqrt{7}}$ Ans

& $|\vec{a}| = 8 \times \sqrt{\frac{8}{63}}$

Ans

Find $|\vec{x}|$, if for a unit vector $\vec{a}$

$$(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 12$$

Sol:- $|\vec{a}| = 1$, $|\vec{x}| = ?$

$$|\vec{x}|^2 + \cancel{\vec{x}\cdot\vec{a}} - \cancel{\vec{a}\cdot\vec{x}} - |\vec{a}|^2 = 12$$

$$|\vec{x}|^2 - 1 = 12 \Rightarrow |\vec{x}| = \sqrt{13}$$

$\vec{a}\cdot\vec{a}$

$|\vec{a}|^2$

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$|\vec{a}| = \sqrt{3}$

$|\vec{a}|^2 = 3$

$\vec{a}\cdot\vec{a} = 1+1+1 = 3$

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Q. if $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k} \Rightarrow \lambda\vec{b} = -\lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

are such that $\boxed{\vec{a} + \lambda\vec{b}}$ is $\boxed{\text{perpendicular}}$ to $\vec{c}$. $\vec{d}$

then find value of λ ?

Sol^n:- $\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} + [-\lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}]$

$\vec{a} + \lambda\vec{b} = (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k} = \vec{d}$

now $\vec{d} \perp \vec{c} \rightarrow \vec{d}\cdot\vec{c} = 0$

$\Rightarrow \{(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}\}\cdot\{3\hat{i} + \hat{j}\} = 0$

$\Rightarrow (2-\lambda)3 + (2+2\lambda)1 + (3+\lambda).0 = 0$

$\rightarrow 6 - 3\lambda + 2 + 2\lambda + 0 = 0 \rightarrow \boxed{\lambda = 8}$ Ans.

Q. if $\vec{a}\cdot\vec{a} = 0$ & $\vec{a}\cdot\vec{b} = 0$, then what can be concluded about the vector $\vec{b} = ?$

$\hat{a}\cdot\hat{a} = 1$

Sol^n: $\vec{a}\cdot\vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow \boxed{|\vec{a}| = 0}$

$\downarrow$

mean $\vec{a}$ is zero vector.

& if $\vec{a}$ is $\boxed{\text{zero vector}}$

$\therefore \vec{a}\,\vec{b} = 0$

$\vec{a} = $ zero

$0\hat{i} + y\hat{j} + z\hat{k}$

thus $\vec{b}$ can be anything i.e. nonzero or zero vector

$\vec{a}$

0°

$\cos 0 = 1$

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Q. if $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that

$\vec{a}+\vec{b}+\vec{c}=0$, find value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=?$

Sol^n:- $\left[\,|\vec{a}|=|\vec{b}|=|\vec{c}|=1\,\right]$

$\Rightarrow \because \vec{a}+\vec{b}+\vec{c}=0 \rightarrow$ square of both side

$\rightarrow (\vec{a}+\vec{b}+\vec{c})^2=0 \Rightarrow$

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\cdot\vec{c}\cdot\vec{a}=0$

$1+1+1+2(\underbrace{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}}_{-3/2})=0$

$-3/2$ Ans

if either $\boxed{\vec{a} = 0}$ or $\boxed{\vec{b} = 0}$ then $\boxed{\vec{a} \cdot \vec{b} = 0}$

But the converse need not be true. Justify answer with example.

Soln: given $\vec{a} = 0$ or $\vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$

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$\rightarrow$ $\longrightarrow \vec{a} = 0$ or $\vec{b} = 0 \rightarrow \boxed{\vec{a} \cdot \vec{b} = 0}$ ✓

✓ $\longleftarrow$ ✓

$\downarrow$

Converse $\rightarrow$?

$\vec{a} = 1\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 1\hat{i} + 1\hat{j} - 1\hat{k}$

$\vec{a} \cdot \vec{b} = 0$

H.C

if $\vec{a} \cdot \vec{b} = 0$ then $\boxed{\vec{a} = 0}$ or $\boxed{\vec{b} = 0}$

Sol^n:- let $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k} \rightarrow |\vec{a}| = \sqrt{1+4+16} = \sqrt{21}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k} \rightarrow |\vec{b}| = \sqrt{4+9+4} = \sqrt{17}$

Now $|\vec{a}| = \sqrt{21} \neq$ and $|\vec{b}| = \sqrt{17} \neq 0$ $\rightarrow$ Converse is not true.

but $\vec{a} \cdot \vec{b} = (\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 2\hat{k})$

$\vec{a} \cdot \vec{b} = 2 + 6 - 8 = 0$ Here $\vec{a} \cdot \vec{b} = 0$ but $|\vec{a}| \neq 0$

$|\vec{b}| \neq 0$

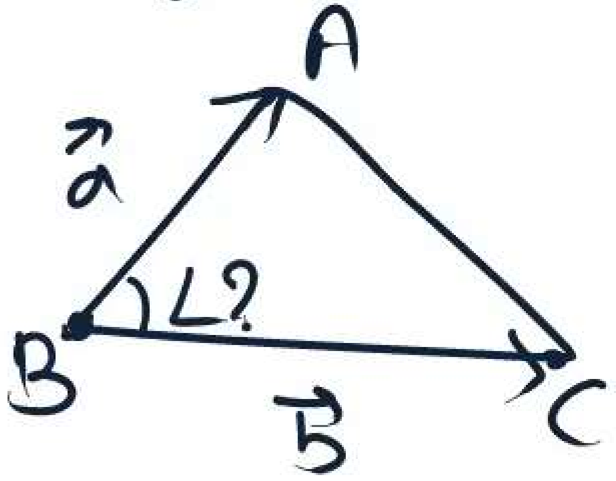
if vertices A, B, C of a ΔABC are $(1,2,3)$ $(-1,0,0)$ $(0,1,2)$ respectively, then find $\angle ABC = \angle B$

Soln:

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{B} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{C} = 0\hat{i} + \hat{j} + 2\hat{k}$



$\vec{a} = \overrightarrow{BA} = \vec{A} - \vec{B} =$ ______

$\vec{b} = \overrightarrow{BC} = \vec{C} - \vec{B} =$ ______

$\vec{a}$ & $\vec{b} \to$ angle $= \cos\theta$

$\vec{a} \cdot \vec{b} =$

$\to \cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} =$ ______

Q. Show that the points $A(1,2,7)$, $B(2,6,3)$, $C(3,10,-1)$ are collinear.

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Sol^n:- Collinear →

$\vec{a}$ $\vec{c}$ $\vec{b}$ → $\boxed{\vec{a} = \lambda \vec{b}}$

Free vector

$\Rightarrow$ let $\vec{A} = \hat{i} + 2\hat{j} + 7\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\vec{AB} = \hat{i} + 4\hat{j} - 4\hat{k} \rightarrow |\vec{AB}| = \sqrt{33}$

$\vec{BC} = \hat{i} + 4\hat{j} - 4\hat{k} \rightarrow |\vec{BC}| = \sqrt{33}$

$\vec{AC} = 2\hat{i} + 8\hat{j} - 8\hat{k} \rightarrow |\vec{AC}| = \sqrt{132} = \sqrt{4 \times 33} = 2\sqrt{33}$

$\vec{A}$ $\vec{B}$ $\vec{C}$

$\sqrt{33}$ $\sqrt{33}$

$\rightarrow \vec{AB} + \vec{BC} = \vec{CA}$

$\sqrt{33} + \sqrt{33} = 2\sqrt{33}$ H.P.

Q. Show $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to

$\vec{p}$

$\vec{q} \rightarrow |\vec{a}|\vec{b} - |\vec{b}|.\vec{a}$, for any two non-zero vectors $\vec{a}$ & $\vec{b}$

$\vec{p} \perp \vec{q}$

Soln:- if $\vec{a}$ & $\vec{b}$ are $\perp \rightarrow [\vec{a}.\vec{b} = 0]$

$\uparrow$

if $\vec{p} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ & $\vec{q} = |\vec{a}|\vec{b} - |\vec{b}|.\vec{a} \rightarrow \therefore \vec{p}.\vec{q} = 0$

$\rightarrow [|\vec{a}|.\vec{b} + |\vec{b}|.\vec{a}][|\vec{a}|\vec{b} - |\vec{b}|\vec{a}] = 0$

$\Rightarrow |\vec{a}|^2.|\vec{b}|^2 - \cancel{|\vec{a}||\vec{b}|\vec{a}.\vec{b}} + \cancel{|\vec{a}||\vec{b}|\vec{a}.\vec{b}} - |\vec{b}|^2|\vec{a}|^2 = 0$

$\rightarrow \cancel{|\vec{a}|^2.|\vec{b}|^2} - \cancel{|\vec{b}|^2|\vec{a}|^2} = 0$

$0 = 0$

Cross / vector product of 2 vectors :-

$\vec{a} \times \vec{b} \rightarrow \left[\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin\theta \; \hat{n}\right]$ $\left[\vec{a} \cdot \vec{b}\right]$ Cosine

$\hat{n}$ → unit vector → in Direction of $\vec{a} \times \vec{b}$

(1) $\vec{a} \times \vec{b} \rightarrow$ vector.

(2) $\vec{a} \times \vec{b} =$ zero → when $|\vec{a}| \neq 0$ & $|\vec{b}| \neq 0$

→ it can happen when $\vec{a}$ & $\vec{b}$ parallel.

$\vec{a} \times \vec{b} = |\vec{a}| . |\vec{b}| . \sin\theta \ \hat{n}$

(1) $\vec{a} \times \vec{b} = 0 \rightarrow$ angle $= 0$

(2) angle $= 90^\circ \rightarrow \vec{a} \times \vec{b} = |\vec{a}| . |\vec{b}|$

(3) $\vec{a} \times \vec{a} = 0 = \vec{b} \times \vec{b} = \vec{c} \times \vec{c}$

$[\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0]$

(4) $[\hat{i} \times \hat{j} = \hat{k} \ | \ \hat{j} \times \hat{k} = \hat{i} \ | \ \hat{k} \times \hat{i} = \hat{j}]$

$[\hat{k} \times \hat{j} = -\hat{i}$