ABLES® KOTA

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Q. Show that each of given three vectors is a unit vector which are mutually perpendicular to each other. let

$$\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}) = \vec{a}$$

$$\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}) = \vec{b}$$

$$\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k}) = \vec{c}$$

→ Show → $\vec{a}, \vec{b}, \vec{c}$ → unit vector

① ↓ i.e.

$$\left[\,|\vec{a}| = |\vec{b}| = |\vec{c}| = 1\,\right]$$

② $$\left[\,\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0\,\right]$$

→ $\vec{a} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$ → $|\vec{a}| = \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} = \sqrt{\frac{49}{49}} = 1$

$|\vec{b}| = |\vec{c}| = 1$

→ $\vec{a}\cdot\vec{b} = (2)(3) + (3)(-6) + (6)(2)$

$= 6 + (-18) + 12 = 18 - 18 = 0$

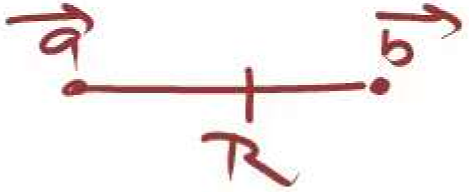
Vector

Sum of vector ✓

equility of vector ✓

unit vector ✓

sum → unit vector ✓



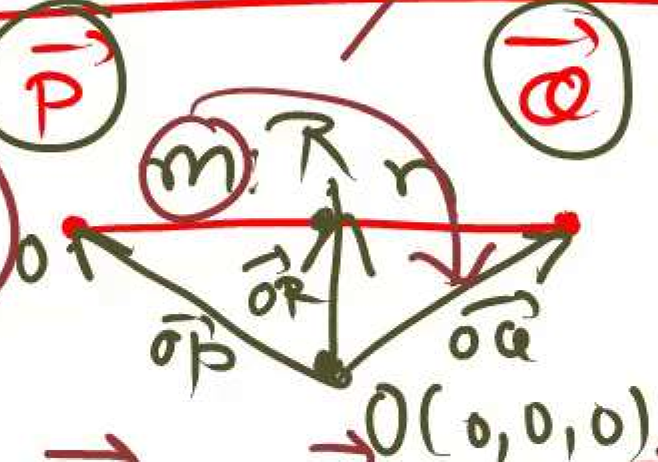
$\vec{OP} = \vec{a}$

$\vec{OQ} = \vec{b}$

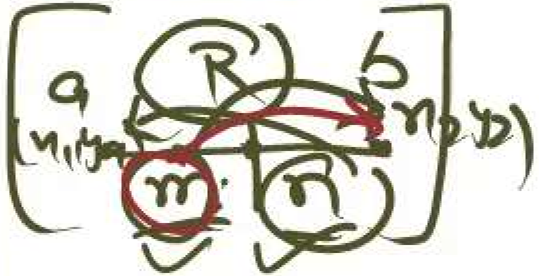
when R cuts externally.

$$\Rightarrow \vec{OR} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

Section formula



$$\vec{OR} = \frac{m\vec{OQ} + n\vec{OP}}{m+n} = \frac{m\vec{b} + n\vec{a}}{m+n}$$



$$\to \frac{my_2 + ny_1}{m+n}$$

→ when R cut internally.

Direction Ratio

$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Direction Cosine :-

$$l = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$m =$

$n =$

Co-linear → Parallel

→ $\vec{a}$ & $\vec{b}$ → ask → Colinear?

if :- $\vec{a} = \lambda\vec{b}$ OR $\vec{b} = \lambda\vec{a}$ → $\vec{a}$ & $\vec{b}$ are Collinear.

Ques :- Show the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ & $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are

Sol^n :- $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ | $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$\vec{b} = -2[2\hat{i} - 3\hat{j} + 4\hat{k}]$

$\vec{b} = -2\vec{a}$

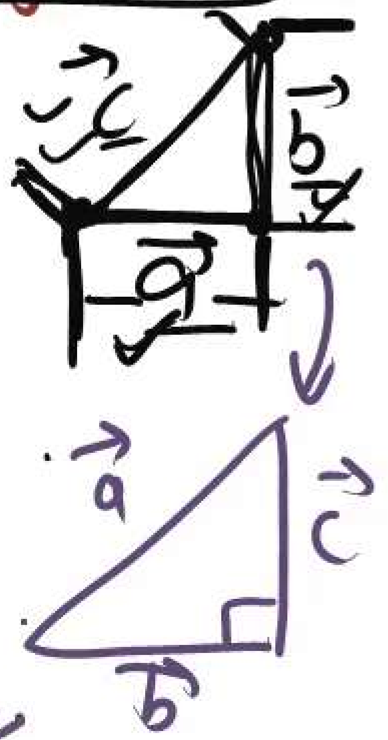
Ques:- Show that the points $\boxed{A, B, C}$ with position vectors

$$\vec a = 3\hat i - 4\hat j - 4\hat k$$
$$\vec b = 2\hat i - \hat j + \hat k$$
$$\vec c = \hat i - 3\hat j - 5\hat k$$

respectively form the vertices of a $\boxed{\text{Right angled } \Delta}$.

Sol^n:-

$$|\vec a| = \sqrt{9+16+16} = \sqrt{41}$$
$$|\vec b| = \sqrt{4+1+1} = \sqrt{6}$$
$$|\vec c| = \sqrt{1+9+25} = \sqrt{35}$$



Here: $|\vec a|^2 = |\vec b| + |\vec c|$

this is pythagoure theorm.

$\therefore\ 41 = 6 + 35 = \boxed{41 = 41} \rightarrow$ H.P.

Product of 2 vectors :-

① Scalar (Dot) — easy

② Vector (Cross) — Tough

* Scalar product :- $\vec{a}$ & $\vec{b}$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

angle b/w $\vec{a}$ & $\vec{b}$.

① $\vec{a}\cdot\vec{b} \rightarrow$ real no.

② $\vec{a}\cdot\vec{b} = 0$ where $\vec{a}$ & $\vec{b} \rightarrow$ non zero vector

then $\boxed{\theta = 90^\circ} \rightarrow \vec{a} \perp \vec{b}$

iii) $\boxed{\theta = 0^\circ}$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|$$

$$\vec{a}\cdot\vec{a} = |\vec{a}|\,|\vec{a}| = |\vec{a}|^2$$

So :- $x \rightarrow$ axis $\rightarrow$ unit vector $\rightarrow \hat{i}$

0°

$$\hat{i}\cdot\hat{i} = |\hat{i}|^2 = 1$$

$$\hat{j}\cdot\hat{j} = 1$$

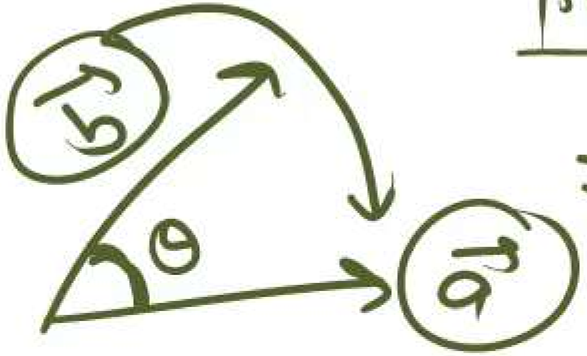
$$\hat{k}\cdot\hat{k} = 1$$

$$\boxed{\hat{i}\cdot\hat{j} = 0}$$

$$\hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

Projection of a vector on another vector :-

projection of vector $\vec{b}$ on $\vec{a}$



$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

* projection of vector $\vec{q}$ on $\vec{p}$.

$$\boxed{\Rightarrow \frac{\vec{p} \cdot \vec{q}}{|\vec{p}|}}$$