Probability

* Binomial Distribution :-

$\rightarrow {}^{6}C_{4}(p)^{4}(q)^{6-4}$

$\star\ {}^{n}C_{x}(p)^{x}(q)^{n-x}$

1 Coin → {H, T}

6 time tossed

X → 'Head' → Success → $P = 1/2$, $q = 1 - 1/2$

Unsuccess →

X	0	1
P(X)	1/2	1/2

n = 6 ✓ (4) success

${}^{6}C_{4}(p)^{4}(q)^{2}$

${}^{n}C_{x}(p)^{x}(q)^{n-x}$

Q 1 Die is thrown (6 time), if getting an odd no. is success, what is the probability of

a) 5 successes b) at least 5 c) at most 5

$P(X \le 5) = P(5) + P(4) + P(3) + P(2) + P(1) + P(0)$

$= 1 - P(X > 5)$

$= 1 - P(X = 6)$

Soln: ∵ a Die is thrown $= \{1, 2, 3, 4, 5, 6\}$

& $n = 6$ ✓

∴ Success ⇒ 'getting odd no' → $p = \frac{3}{6} = \frac{1}{2}$

∴ Unsuccess ⇒ 'not getting odd no' → $q = 1 - \frac{1}{2} = \frac{1}{2}$

a) 5 Success :-

$P(X = 5) \Rightarrow {}^{n}C_{x} (p)^{x} (q)^{n-x}$

$= {}^{6}C_{5} \left(\frac{1}{2}\right)^{5} \left(\frac{1}{2}\right)^{6-5} = 6 \times \left(\frac{1}{2}\right)^{6}$

b) at least 5.

$P(X \ge 5) = P(X = 5) + P(X = 6)$

$\Rightarrow {}^{6}C_{6} \left(\frac{1}{2}\right)^{6} \left(\frac{1}{2}\right)^{0}$

(5) There are 5% Defective items. What is the proba. that a Sample of 10 items will include not more than (at most) one Defective

Soln:- let X = Defective item = Success

→ $p = 5\% = \frac{5}{100} = \frac{1}{20}$ ∴ $q = 1 - \frac{1}{20} = \frac{19}{20}$

Now → Sample of 10 items is selected

∴ $n = 10$

So prob. of not including Defective item more than 1

$= P(X \leq 1) = P(X=1) + P(X=0)$

$= {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^9 + {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10}$

Q Five cards drawn successively with replacent from a well-suff. deck of 52 cards. prob.

i) all the five cards are spades

iii) none is a spade.

Soln:- Total ⇒ 52 → Drawn 5

let X = spade card ⇒ success → $P = \frac{13}{52} = \frac{1}{4}$ ⇒ $q = \frac{3}{4}$

i) ∵ $n = 5 \rightarrow P(X=5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^5$ ✓

(iii) $n = 5 \rightarrow P(X=0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^5$ ✓

Q. 20 Ques. of T-f type are asked. a student tosses a coin to give answer. if its Head He ans true, if is tail He ans. False. find prob. that He answers at least 12 Ques. Correctl.

Soln:- Let X = Correct answer = Success.

Sol:- $\left[p = \frac{1}{2}\right], \left[q = \frac{1}{2}\right]$

Sol:- at least 12 $\rightarrow$ $P(X \geq 12) = P(12) + P(13) + \_\_\_ + P(20)$

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{12}\left(\frac{1}{2}\right)^{8} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{13}\left(\frac{1}{2}\right)^{7} + \_\_\_ \cdot {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{0}$$

$$= \left(\frac{1}{2}\right)^{20}\left[{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + \_\_\_ {}^{20}C_{20}\right]$$ Ans

Q Suppose X has a binno. Distr. $B\left(6, \frac{1}{2}\right)$, show $X=3$ is the most likely outcome.

Sol:

$\therefore$ X has B.D. as $B\left(6, \frac{1}{2}\right)$ → mean i.e. → $n=6$, $p=\frac{1}{2}$ → $q=\frac{1}{2}$

Show → $X=3$ → most likely outcome.

So! $P(X=0) = {}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 = 1\times\left(\frac{1}{2}\right)^6$ ✓

$P(X=1) = {}^6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 = 6\times\left(\frac{1}{2}\right)^6$ ✓

$P(X=2) = {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 = 15\times\left(\frac{1}{2}\right)^6$ ✓

$P(X=3) = {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 = 20\times\left(\frac{1}{2}\right)^6$ ✓

$P(X=4) = 15\times$ ——

$P(X=5) = 6\times$ ——

$P(X=6) = 1\times$ ——

Q a person buys a lottery ticket in 50 lotteries. in each of which his chance of winning a prize is $\frac{1}{100}$ what is the proba. that He will win a prize.

a) atleast once b) exactly one c) at least twice.

Soln:- Let X = win → success → $\left[p = \frac{1}{100}\right] \left| \left(q = \frac{99}{100}\right)\right.$

a) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{50}C_0 \left(\frac{1}{100}\right)^0 \left(\frac{99}{100}\right)^{50}$

c) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X = 1) + P(X = 0)]$

Q Find the prob of getting 5 exactly twice in 7 throws of a Die.

Soln:- $S = \{1, 2, 3, 4, 5, 6\}$

X = getting 5 $\Rightarrow$ Success $\rightarrow$ $p = 1/6$ $q = 5/6$

$\Rightarrow {}^{7}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{5}$ Ans