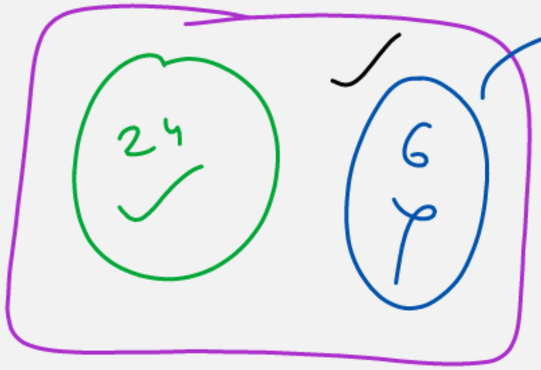
Ques:- From a lot of 30 bulbs which include 6 Defective a sample of 4 bulbs is drawn with Replacement. Find Prob. Dis. of no. of Defective bulbs.

Soln:-



Total 30

Sample → 4 bulbs.

let X = no. of defective bulbs:

X	0	1	2	3	4
P(X)	$\frac{^{24}C_4}{^{30}C_4}$	$\frac{^{6}C_1 \times ^{24}C_3}{^{30}C_4}$	$\frac{^{6}C_2 \times ^{24}C_2}{^{30}C_4}$	$\frac{^{6}C_3 \times ^{24}C_1}{^{30}C_4}$	$\frac{^{6}C_4}{^{30}C_4}$

$$^nC_r = \frac{\lfloor n}{\lfloor n-r \lfloor r}$$

$$\frac{24 \times 23 \times 22 \times 21}{4} = \frac{}{30 \times 29 \times 28 \times 27}$$

Q. A coin is biased so that the Head is 3 times as likely to occur as tail if the coin is tossed twice, find the prob. Dis. of no. of Tails.

Soln:- Let Tail occurs = x time $= \frac{1}{4} \to P(T)$

Head occurs = $3x$ time $= \frac{3}{4} \to P(H)$

$\therefore$ Total pro = Head + Tail = 1

$= 3x + x = 1$

$\Rightarrow x = \frac{1}{4}$

$\therefore$ Coin tossed 2 time

$\{HH, TT, HT, TH\}$

$X \to$ no. of Tails

$X = 0 \to P(X=0) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$X = 1 \to P(X=1) = \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} = \frac{6}{16}$

$X = 2 \to P(X=2) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

X	0	1	2
P(X)	9/16	6/16	1/16

Q The random variable X has proba. P(x) of Bell Form, where K is some no.

$$P(x) = \begin{cases} k & \text{if } x=0 \\ 2k & \text{if } x=1 \\ 3k & \text{if } x=2 \\ 0 & \text{otherwise} \end{cases}$$

Determine:-

1) Value of k
2) find $P(x<2)$, $P(x\leq 2)$, $P(x>2)$

→ Soln:-

X	0	1	2	otherwise
P(x)	k	2k	3k	0

i) $\sum P(x) = k+2k+3k+0 = 1$

$k = 1/6$

ii) $P(x<2) = P(x=1) + P(x=0)$

$= 2k + k = 3k = 3 \times 1/6 = 1/2$

$P(x\leq 2) = P(x=2) + P(x<2)$

$= 3k + \frac{1}{2} = 3\times\frac{1}{6} + \frac{1}{2} = 1$

$P(x>2) = P(x=2) + P(\text{otherwise})$

$= 3\times\frac{1}{6} + 0 = \frac{1}{2}$

Pro. Distri. →

mean = $\bar{x}$

X	0	1	2
P(x)	—	—	—

① Mean $= \sum x_i \cdot P(x_i) = E(x_i)$ (mean)

② # Variance $= E(x_i^2) - [E(x_i)]^2$

$E(x_i^2) = \sum x_i^2 \cdot P(x_i)$

③ # Standard Deviation $= \sqrt{\text{Variance}}$

Q Let X Denote Sum of no. obtained when two fair Dice are rolled. Find the variance & standard Deviation. of X.

Sol:- Total sample space = $6 \times 6 = 36$

∵ X = Sum of numbers when 2 Dice are rolled.

∴ X Can be → (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)

Sol:- $X = 2 \rightarrow P(2) = \frac{1}{36} = \{(1,1)\}$

$X = 3 \rightarrow P(3) = \frac{2}{36} \{(1,2)(2,1)\}$

$X = 4 \rightarrow P(4) = \frac{3}{36} \{(1,3)(3,1)(2,2)\}$

$X = 5 \rightarrow P(5) = \frac{4}{36}$

$X = 6 = P(6) = 5/36$

$X = 7 = P(7) = 6/36$

$X = 8 = P(8) = 5/36$

$X = 9 = P(9) = 4/36$

$X = 10 = P(10) = 3/36$

$X = 11 = P(11) = 2/36$

$X = 12 = P(12) = 1/36$

Pro. distri. [Stand. Devi = $\sqrt{5.8} = 2.4$ ✓] 218 1974 2074 (3) (3)

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

∴ Mean $\Rightarrow E(x_i) = \sum x_i \cdot P(x_i)$

$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$

$= \frac{252}{36} = \frac{42}{6} = 7 = E(x_i)$

∴ Variance = $Var(X_i) = E(x_i^2) - [E(x_i)]^2$ { $\because E(x_i^2) = \sum x^2 \cdot P(x_i)$ }

$E(x_i^2) = \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}$

$= \frac{1974}{36} = \frac{329}{6} = 54.8$ (Var $= 54.8 - (7)^2 = 54.8 - 49 = 5.8$ ✓)

Q Suppose that two cards are drawn at random from a Dack of cards. Let X be no. of aces obtained. Then value of $E(X) = ?$

Soln, $52 \rightarrow 2$

$X \rightarrow$ no. of aces.

∴ there are 4 aces in 52 Cards | other cards = 48

$X=0 \rightarrow \frac{{}^{48}C_2}{{}^{52}C_2}$ | $X=1 = \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}$ | $X=2 = \frac{{}^{4}C_2}{{}^{52}C_2}$

Ex:- 25 → find Prob. Dis. of number of doublets in three throws of a pair of Dice.

X = no. of Doublets

↓ Total Condition = 36

$X = \{(2,2)(3,3)(1,1)(4,4)(5,5)(6,6)\} = 6$

so prob. of occuring doublets = $\frac{6}{36} = \frac{1}{6}$

—||— not occuring doublets = $1 - \frac{1}{6} = \frac{5}{6}$

[∵ pair is throw 3 time]

Now $X = 0 = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$P(X=1)$

$3\left[\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right]$

$X = 2$

$3\left[\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right]$

$X = 3$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$ ✓