1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

5 → R   5 → B → Total 10

i) 1st ball Drawn → Red → put 2 extra Red

New → [R → 7] [B → 5] → Total 12

→ Now the prob. when 2nd time ball drawn is red.

⇒ $\boxed{7/12}$

② 1st ball Drawn → black → put 2 Extra black

New [R → 5] [B → 7] → Total → 12

Now again prob. when 2nd time ball Drawn is Red: 5/12

Now the final probability

$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12}$

$= \frac{1}{2}(1) = 1/2$

Ans:- Prob. of solving specific problem independently by A & B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively. if both try to solve the problem independently, find the prob.

i) the problem is solved.

2) exactly one of them solve the problem.

if A & B are indendent

$P(A \cap B) = P(A) \times P(B)$

$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Sol^n:- $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{2}{3}$

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

② $[P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)]$

$= \frac{1}{2} \times (1 - \frac{1}{3}) + (1 - \frac{1}{2}) \times \frac{1}{3} = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$

2. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Let $E_1 \to$ 1st bag select $\Rightarrow P(E_1) = \frac{1}{2}$

$E_2 \to$ 2nd bag select $\Rightarrow P(E_2) = \frac{1}{2}$

let $A \to$ a ball is Drawn from bag which is red

$\therefore P(A/E_1) \to \frac{4}{8} = \frac{1}{2}$

$P(A/E_2) = \frac{2}{8} = \frac{1}{4}$

find :- $\boxed{P(E_1/A)} = \dfrac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_2) + P(E_2)\cdot P(A/E_2)} = \dfrac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}}$

$= \dfrac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}} = \dfrac{\frac{1}{4}}{\frac{3}{8}} = \frac{2}{3}$ Ans

4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

$E_1 \to$ Ans known $\to P(E_1) = 3/4$

$E_2 \to$ Ans guess $\to P(E_2) = 1/4$

[$A \to$ Ans is Correct]

$\boxed{P(A/E_1)} = 1$

$P(A/E_2) = 1/4$

Find :- $P(E_1/A) = ? = P(E_1) \cdot P(A/E_1)$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005 the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

60% Maths $\frac{60}{100}$ $\frac{3}{5}$

$1 - 0.001$

find → $\boxed{P(E_1/A)} = \dfrac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$

E_1 → person has Disease → $P(E_1) = 0.1\% = 0.001$

E_2 → person is Healthy | no Disease ; → $P(E_2) = 0.999$

A → Test result positive. →

$P(A/E_1) = 99\% = 0.99$

$P(A/E_2) = 0.005$

6. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?

$\Rightarrow E_1 \to$ 1st coin selection

$E_2 \to$ 2nd coin — " —

$E_3 \to$ 3rd coin selection

$A \to$ Head shows

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$P(E_1) = P(E_2) = P(E_3) = 1/3$

$P(A/E_1) = 1$ | $P(A/E_2) = .75 = 0.75$ | $P(A/E_3) = 1/2$