

Independent Event

A & B → Independent event

if occurrence of one event doesn't affect the other.

$$\# \left[P(E \cap F) = P(E) \times P(F) \right]$$

$$\left[P(E \cap F) = P(E) + P(F) - P(E \cup F) \right]$$

Cards \rightarrow 52

✓ 2 Cards \rightarrow

$$\left(\frac{1}{52} \times \frac{1}{51} \right)$$

$$\left(\frac{1}{52} \times \frac{1}{52} \right)$$

\rightarrow 1 Card
 $\downarrow \Rightarrow$ $\left(\frac{1}{52} \right)$

$\cap \rightarrow$ and

$\cup \rightarrow$ OR

$P(A) \rightarrow$ prob. of event A ✓

$P(A') = P(\bar{A}) =$ prob. of event not A .

$$\overline{[P(A') = 1 - P(A)]}$$

$$\# P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$\# P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$$

$$\# \frac{P(A \cap B')}{P(A' \cap B)} = P(A) - P(A \cap B)$$

$$\# \frac{P(A \cap B')}{P(A' \cap B)} = P(B) - P(A \cap B)$$

Q. 2 Two cards are drawn at random and without replace.

from 52 playing cards. Find prob. that both are black?

Solⁿ - Total = 52

Drawn = 2

∴ Black cards = 26

Let E = when 1st card is drawn ⇒ $P(E) = \frac{26}{52} = \frac{1}{2}$

F = when 2nd card is drawn ⇒ $P(F) = \frac{25}{51}$

So the final prob = $\frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

Q. A fair Coin & a Die are tossed. Let A be event 'Head appears on coin' & B is event '3 on the Die'. Check whether A & B are independent event or not?

Solⁿ: - Sample space = $\{ H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6 \}$

A \Rightarrow 'Head appears' $\rightarrow \{ H_1, H_2, H_3, H_4, H_5, H_6 \}$
 $P(A) = \frac{6}{12} = \frac{1}{2}$

B \rightarrow '3 on Die' $\rightarrow \{ H_3, T_3 \} \rightarrow P(B) = \frac{2}{12} = \frac{1}{6}$

$\therefore A \cap B = \{ H_3 \} \rightarrow P(A \cap B) = \frac{1}{12}$

Now $P(A) \times P(B) = P(A \cap B) \rightarrow$ events independent
 $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \rightarrow$ Yes \checkmark

Q. Given that the events A & B are such that
 $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ & $P(B) = P \rightarrow$ find P if

- i) mutually exclusive ii) Independent.

when events are mutually

exclusive = $E \cap F = \emptyset$

$\therefore P(E \cap F) = 0$

so $P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$

$$\frac{3}{5} = \frac{1}{2} + P - 0$$

$$\Rightarrow P = \frac{1}{10} \quad \checkmark$$

- iii) when events are independent.

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = \frac{1}{2} \times P = \frac{P}{2}$$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{5} = \frac{1}{2} + P - \frac{P}{2}$$

$$\frac{3}{5} - \frac{1}{2} = \frac{P}{2} \Rightarrow \frac{P}{2} = \frac{1}{10} \Rightarrow$$

$$P = \frac{2}{10} = \frac{1}{5} \quad \checkmark$$

Q. Given two independent events A & B such that
 $P(A) = 0.3$, $P(B) = 0.6$

① $P(A \text{ and } B) = P(A \cap B) = 0.3 \times 0.6 = 0.18 \checkmark$

② $P(A \text{ and not } B) = P(A \cap B') = P(A) - P(A \cap B)$

③ $P(A \text{ or } B) = P(A \cup B) = 0.3 + 0.6 - 0.18$
 $= 0.72 \checkmark$

④ $P(\text{neither } A \text{ nor } B) = P(A' \cap B')$
 $= P(A \cup B)' = 1 - P(A \cup B)$
 $= 1 - 0.72 = 0.28 \checkmark$

Ans: A Die tossed thrice. Find the prob. of getting an odd number at least once.

Solⁿ: When Die is thrown 1 time = 6 (sample space)
odd no. $\Rightarrow \{1, 3, 5\}$

\rightarrow so getting an odd no. when Die is thrown 1 time = $\frac{3}{6} = \frac{1}{2}$

then the prob. of not getting an odd no. (even no.) = $\left(\frac{1}{2}\right)$

Now probability of getting even no. all three times = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

so prob. of getting odd no. at least one = $1 - \left\{ \text{pro. of getting even no. all three time} \right\}$
 $= 1 - \frac{1}{8} = \frac{7}{8}$ ✓

$$\Rightarrow S = \left\{ \begin{array}{l} \text{HHH, HH}\uparrow, \text{H}\uparrow\text{H,}\uparrow\text{HH,} \\ \text{H}\uparrow\uparrow, \uparrow\text{H}\uparrow, \uparrow\uparrow\text{H,} \uparrow\uparrow\uparrow \end{array} \right\}$$

$$\begin{array}{c} \uparrow\uparrow\uparrow\uparrow \\ \hline 1/16 \end{array}$$

$$\text{at least one head} = \left\{ \begin{array}{l} \text{HHH, HH}\uparrow, \text{H}\uparrow\text{H,}\uparrow\text{HH,} \\ \text{H}\uparrow\uparrow, \uparrow\text{H}\uparrow, \uparrow\uparrow\text{H} \end{array} \right\}$$

$$\text{Coin} \rightarrow 4 \text{ types} = 7/8 \checkmark$$

$$\rightarrow \text{at least one head} = 1 - \frac{1}{16} = \frac{15}{16} \checkmark$$