D.E. $\rightarrow$ $\left[\boxed{\frac{dy}{dx}} + \boxed{P}y = \boxed{Q}\right]$

Where $\rightarrow$ P & Q are fun. of x.

$\Rightarrow$ find I.F $\Rightarrow$ Integration factor $\Rightarrow$ $\boxed{e^{\int P.dx}}$

Sol^n: $\Rightarrow$ $\left[y \times I.F. = \int Q \times I.F\, dx + C\right]$

D.E. $\rightarrow$ $\frac{dx}{dy} + Px = Q$

Where: P & Q $\rightarrow$ fun. of y.

So: I.F $\Rightarrow$ $e^{\int P.dy}$

& sol^n:- $\left[x \times I.F = \int Q \times I.F \times dy + C\right]$

Q. 1 $\frac{dy}{dx} + 2y = \sin x$ $\longrightarrow$ $P = 2$, $Q = \sin x$

$\therefore$ $\left[\frac{dy}{dx} + Py = Q\right]$

$\Rightarrow I.F. = e^{\int P\,dx} = e^{\int 2\,dx} = e^{2x}$

$\Rightarrow \left[y \times e^{2x} = \int \underbrace{\sin x \times e^{2x}\,dx}_{I}\right]$ — 1

$I = \int \sin x \cdot e^{2x}\,dx = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2}\,dx$

$= \frac{1}{2}\sin x \cdot e^{2x} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} + \int \sin x \frac{e^{2x}}{2}\,dx\right]$

$= \frac{1}{2}\sin x \cdot e^{2x} - \frac{1}{4}\cos x \cdot e^{2x} - \frac{1}{4}\int \sin x \cdot e^{2x}\,dx$ — I

$I + \frac{1}{4}I = \frac{1}{4}e^{2x}[2\sin x - \cos x]$

$I = \frac{1}{5}e^{2x}[2\sin x - \cos x]$

From 1

$y \cdot \cancel{e^{2x}} = \frac{1}{5}\cancel{e^{2x}}[2\sin x - \cos x]$

$y = \frac{1}{5}[2\sin x - \cos x]$ Ans

Q. $\cos^2 x \cdot \frac{dy}{dx} + y = \tan x$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$

Q

$$\Rightarrow \frac{dy}{dx} + \boxed{\sec^2 x} \cdot y = \boxed{\tan x \cdot \sec^2 x}$$

P

So: $I.F. = e^{\int P \cdot dx} = e^{\int \sec^2 x \cdot dx} = e^{\tan x} = I.F.$

$$\Rightarrow y \times e^{\tan x} = \int \tan x \cdot \sec^2 x \cdot e^{\tan x} \cdot dx \longrightarrow \tan x = t \Rightarrow \sec^2 x \cdot dx = dt$$

$$\Rightarrow = \int e^t \, t \, dt = t \cdot e^t - \int 1 \cdot e^t \cdot dt = t \cdot e^t - e^t = e^t (t-1) + c$$

$$y \times \cancel{e^{\tan x}} = \cancel{e^{\tan x}} \left[\tan x - 1\right] + c \cdot e^{-\tan x} \ \text{Ans}$$

$Q.\ \left[x\frac{dy}{dx}+y-x+xy\cot x=0\right]$

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}-1+y\cot x=0$

$\rightarrow \frac{dy}{dx}+y\left[\frac{1}{x}+\cot x\right]=1$

So $P=\frac{1}{x}+\cot x$, $Q=1$

Then $I.F.=e^{\int\left(\frac{1}{x}+\cot x\right).dx}=e^{\left[\log x+\log(\sin x)\right]}=e^{\log x.\sin x}$

$\boxed{I.F=x.\sin x}$

$\therefore \Rightarrow y\times x.\sin x=\int \overset{①}{x}.\overset{②}{\sin x}.dx$

Q. $(x+3y^3)\frac{dy}{dx} = y$ $\qquad x = \frac{3}{2}y^3 + Cy$

$$\frac{dy}{dx} = \frac{y}{x+3y^3} \rightarrow \text{Flip.}$$

$$\frac{dx}{dy} = \frac{x+3y^3}{y} = \frac{x}{y} + \frac{3y^{\cancel{3}}}{\cancel{y}}$$

$$\left[\frac{dx}{dy} - \frac{x}{y} = 3y^2\right] \rightarrow \left[\frac{dx}{dy} + Px = Q\right]$$

So! $P = \left(-\frac{1}{y}\right)$ & $Q = 3y^2$ $\left[\therefore \text{I.F.} = e^{\int P\cdot dy} = e^{\int -\frac{1}{y}\cdot dy} = e^{\log y^{-1}} = \frac{1}{y}\right.$

So! $x \times \frac{1}{y} = \int 3y^{\cancel{2}} \times \frac{1}{\cancel{y}}\cdot dy = \frac{x}{y} = 3\int y\cdot dy = 3\cdot\frac{y^2}{2} + C$

Q $\frac{dy}{dx} + 2y \cdot \tan x = \sin x$; $y = 0$ when $x = \pi/3$

$\therefore \frac{dy}{dx} + Py = Q$

$\therefore P = 2\tan x$, $Q = \sin x$

$I.F. = e^{\int 2\tan x \cdot dx} = e^{2\log\sec x} = e^{\log \sec^2 x} = \sec^2 x$

$\therefore y \times \sec^2 x = \int \sin x \cdot \sec^2 x \cdot dx$

$= \int \sin x \cdot \frac{1}{\cos^2 x} \cdot dx = \int \tan x \cdot \sec x \cdot dx = \sec x + c$

$[\, y \times \sec^2 x = \sec x - 2 \,]$

$\Rightarrow 0 = \sec \frac{\pi}{3} + c \Rightarrow c = -2$

Q. $\frac{dy}{dx} - 3y\cot x = \sin 2x$ $\left[y = 2 \text{ when } x = \pi/2 \right]$

$$\frac{dy}{dx} - 3y\cot x = \sin 2x \quad \left[y = 2,\ x = \pi/2 \right]$$

Q. Find the eq. of a curve passing through the origin $(0,0)$ given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of coordinates of the point.

Sol^n:- slope of Tangent :- $\left.\frac{dy}{dx}\right|_{(x,y)}$

ATQ:- $\left.\frac{dy}{dx}\right|_{(x,y)} = x + y$

$$\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$$

$P = -1$ $\quad Q = x$

C

$x = 0$
$y = 0$

Find

Ans:- Find the eq. of curve passing through $(0,2)$ given that The sum of coordinate of any point in the curve exceeds The magnitude of the slope of the tangent to the curve at any point by 5

ATQ:- $x + y = \frac{dy}{dx} + 5$

Yes

$\frac{dy}{dx} - y = x - 5$

$P = -1$ $\quad$ $Q = x - 5$