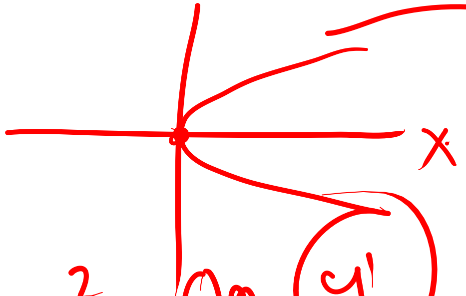
The differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis is

(a) $y^2y' - 2xy' = 0$ (b) $y^2 - 2xyy'' = 0$

(c) $y^2 - 2xyy' = 0$ (d) None of these



$\boxed{y^2 = 4ax}$

$\boxed{2y \cdot y' = 4a}$

$y^2 = 4ax$ y'

$y^2 = 2yy' \cdot x \rightarrow y^2 - 2xy \cdot y' = 0$

The differential equation which represent the family of curves $y = ae^{bx}$, where a and b are arbitrary constants.

(a) $y' = y^2$ (b) $y'' = y\,y'$

(c) $y\,y'' = y'$ (d) $y\,y'' = (y')^2$

$y = ae^{bx}$

$y' = a \cdot e^{bx} \cdot b \Rightarrow y' = y \cdot b \to b = \frac{y'}{y}$

$\to y'' = y' \cdot b \Rightarrow y'' = y' \times \frac{y'}{y}$

$\Rightarrow y \cdot y'' = (y')^2$

The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, respectively, are

y

$\frac{dy}{dx}$

power

fraction

allowed

(a) 2 and not defined (b) 2 and 2

(c) 2 and 3 (d) 3 and 3

Remove → power y

$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^{\frac{1}{4} \times 4} + \left(x^{\frac{1}{5}}\right)^4 = 0$

$\tan^{-1}x + \tan^{-1}y = c$ is the general solution of the differential equation

(a) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$

(b) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(c) $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$

(d) $(1 + x^2)\, dx + (1 + y^2)\, dy = 0$

$\dfrac{1}{1+x^2} + \dfrac{1}{1+y^2} \times \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{(1+y^2)}{(1+x^2)}$

$(1+x^2)\,dy + (1+y^2)\cdot dx = 0$

Which of the following equation has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(a) $\dfrac{d^2y}{dx^2} + y = 0$

(b) $\dfrac{d^2y}{dx^2} - y = 0$

(c) $\dfrac{d^2y}{dx^2} + 1 = 0$

(d) $\dfrac{d^2y}{dx^2} - 1 = 0$

$$\frac{dy}{dx} = c_1 e^x + c_2 e^{-x} \times (-1) = c_1 e^x - c_2 e^{-x}$$

$$\frac{d^2y}{dx^2} = c_1 e^x + c_2 e^{-x} \Rightarrow \frac{d^2y}{dx^2} = y$$

The differential equation representing the family of curves $y = A\cos(x + B)$, where A, B are parameters, is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} = \frac{dy}{dx} + y$ (d) $\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = -A\sin(x+B)(1+0)$

$\frac{d^2y}{dx^2} = -A\cos(x+B) = -y$

$\frac{d^2y}{dx^2} + y = 0$

The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^{x} + be^{-x}$ is

$\rightarrow x \cdot y' + y = ae^{x} + be^{-x}(-1)$

$\downarrow$

$x \cdot y'' + y' + y' = \underline{ae^{x} + be^{-x}}$

$x \cdot y'' + 2y' = xy$

$\underline{x \cdot y''} + \underline{2y'} - \underline{xy} = 0$

(a) $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy = 0$

(c) $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + xy = 0$

(b) $\dfrac{d^2y}{dx^2} + 2y\dfrac{dy}{dx} - xy = 0$

(d) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0$

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

Column-I Differential equations		Column-II Degree
A.	$\frac{dy}{dx} = e^x$	1. 1
B.	$\frac{d^2y}{dx^2} + y = 0$	2. 2
C.	$\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$	3. not defined
D.	$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$	4. 3
E.	$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - \sin^2 y = 0$	
F.	$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$	

$\frac{dy}{dx} - e^x = 0$

Trigono
log
expon

Codes

	A	B	C	D	E	F
(a)	2	2	4	3	1	1
(b)	1	1	1	1	2	3
(c)	3	4	1	1	2	3
(d)	1	1	1	3	4	2

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

Column-I (Solutions)	**Column-II Differential equations**
A. $y = e^x + 1$	1. $y' + \sin x = 0$
B. $y = x^2 + 2x + C$	2. $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0 \text{ and } x > y \text{ or } x < -y)$
C. $y = \cos x + C$	3. $y'' - y' = 0$
D. $y = \sqrt{1 + x^2}$	4. $xy' = y (x \neq 0)$
E. $y = Ax$	5. $y' - 2x - 2 = 0$
F. $y = x \sin x$	6. $y' = \dfrac{xy}{1 + x^2}$

Codes

	A	B	C	D	E	F
(a)	2	1	4	3	6	5
(b)	2	1	4	6	5	3
(c)	5	6	1	4	3	2
(d)	3	5	1	6	4	2

(E) $y = Ax$

$y' = A$

$y' = y/x$

$y'x - y = 0$

$y' = x\cos x + \sin x$

ABLES KOTA

2nd

For $y = \cos kx$ to be a solution of

differential equation $\dfrac{d^2y}{dx^2} + 4y = 0$,

the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

$y' = -\sin kx \cdot k$

$y'' = -\cos kx \cdot k^2$

$\Rightarrow -\cos kx \cdot k^2 + 4\cos kx = 0$

$\Rightarrow \cos kx[-k^2 + 4] = 0 \Rightarrow -k^2 + 4 = 0$

$k^2 = 4 \Rightarrow k = \pm 2$

1	2	3	4	5	6	7	8	9	10
C	D	A	C	B	A	A	B	D	A

$2xy + y^2 - 2x^2 \cdot \frac{dy}{dx} = 0$; $y = 2$, $x = 1$

$$\frac{dy}{dx} = \frac{\cancel{+}\left[2xy + y^2\right]}{\cancel{+}\,2x^2}$$

9.5 finished

9.6 → 2 lect

Q $\left[x \cdot \sin^2\left(\frac{y}{x}\right) - y\right] dx + x \cdot dy = 0 \quad \left(y = \frac{\pi}{4},\ x = 1\right)$

$$\rightarrow \frac{dy}{dx} = -\left[\frac{x \cdot \sin^2\left(\frac{y}{x}\right) - y}{x}\right] \rightarrow -\frac{\left[\lambda x \cdot \sin^2\left(\frac{\lambda y}{\lambda x}\right) - \lambda y\right]}{\lambda x}$$

$$= -\frac{\left[x \cdot \sin^2(y/x) - y\right]}{x} \rightarrow \text{Homo}$$

$$\underline{y = vx} \rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$\rightarrow v + x \cdot \frac{dv}{dx} = -\left[\frac{\cancel{x} \cdot \sin^2\left(\frac{v\cancel{x}}{\cancel{x}}\right) - v\cancel{x}}{\cancel{x}}\right] = -\left[\sin^2 v - v\right]$$

$$\rightarrow \cancel{v} + x \cdot \frac{dv}{dx} = \cancel{v} - \sin^2 v \rightarrow \int \frac{dv}{\sin^2 v} = \int -\frac{dx}{x}$$

$$\boxed{\cot(y/x) = \log x \cdot e}$$

$$\int \operatorname{cosec}^2 v \cdot dv = -\log x + \log c$$

$$\rightarrow -\cot v = -\log x - \log c$$

$$\rightarrow -\cot v = -[\log x c]$$

$$\rightarrow \cot v = \log x \cdot c$$

$$\rightarrow \boxed{\cot\left(\frac{y}{x}\right) = \log x \cdot c}$$

$$\cot\left[\frac{\pi}{4}\right] = \log 1 \cdot c$$

$$\boxed{\log c = 1} \rightarrow \boxed{c = e^1 = e}$$

ABLES KOTA

Q $\left[x^2 dy + (xy + y^2)\,dx\right]$ $[y = 1 \text{ when } x = 1]$

$\frac{dy}{dx} = -\frac{(xy+y^2)}{x^2}$ $\rightarrow$ $[y = vx]$

$\Rightarrow v + x\cdot\frac{dv}{dx} = -\frac{[x\cdot vx + v^2x^2]}{x^2} = -[v+v^2]$

$\rightarrow x\cdot\frac{dv}{dx} = -v - v^2 - v = -[2v+v^2]$

$= -v[v+2]$

$\Rightarrow \int\frac{dv}{v(v+2)} = \int -\frac{dx}{x}$

$\frac{1}{v(v+2)} = \int\frac{A}{v} + \int\frac{B}{v+2} =$

ABLES® KOTA

D.E.

Qus: $(x+y)\,dy + (x-y)\,dx = 0 \rightarrow \boxed{\begin{matrix} y = 1 \\ x = 1 \end{matrix}}$

$$\frac{dy}{dx} = \frac{-[x-y]}{[x+y]} \rightarrow \text{Homo.}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{-[x-vx]}{x+vx} = \frac{v-1-v}{v+1}$$

$$x\cdot\frac{dv}{dx} = \frac{\cancel{v}-1-v^2-\cancel{v}}{v+1} = \frac{-(1+v^2)}{1+v}$$

$$\int\left(\frac{1+v}{1+v^2}\right)dv = -\int\frac{dx}{x} \qquad \boxed{C = }$$