

D.E.

Ques: $(x-y) dy - (x+y) dx = 0$

$$(x-y) dy = (x+y) dx$$

$$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{x+y}{x-y} = F(x, y) \quad \text{--- (1)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \lambda^0 \left[\frac{x+y}{x-y} \right]$$

$[F(\lambda x, \lambda y) = \lambda^0 \cdot F(x, y)] \rightarrow$ Homogeneous.

Let:- $y = vx \rightarrow v = \frac{y}{x}$ --- (2)

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx} \quad \text{--- (3)}$$

\Rightarrow Put (1), (2) & (3):-

$$v + \left[x \cdot \frac{dv}{dx} \right] = \frac{x + vx}{x - vx} = \frac{1+v}{1-v}$$

$$x \frac{dv}{dx} = \frac{1+v-v}{1-v} = \frac{1+\cancel{v}-\cancel{v}+v^2}{1-v} = \frac{1+v^2}{1-v}$$

$$\left[\frac{(1-v) dv}{1+v^2} \right] = \left[\frac{dx}{x} \right]$$

$$\frac{1}{1+v^2} \cdot dv - \frac{v}{1+v^2} \cdot dv \rightarrow t$$

D.E.

Ans:- $x dy - y dx = \sqrt{x^2 + y^2} \cdot dx$

$\Rightarrow x dy = [y + \sqrt{x^2 + y^2}] \cdot dx$

$\Rightarrow \left(\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \right) = F(x, y)$

$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda y + \sqrt{\lambda^2 x^2 + \lambda^2 y^2}}{\lambda x}$

$F(\lambda x, \lambda y) = \frac{\lambda \left[y + \sqrt{x^2 + y^2} \right]}{\lambda x} \rightarrow \text{Homo.}$

Solve:

$\Rightarrow y = vx \rightarrow v = \frac{y}{x} \rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$\Rightarrow \frac{y}{x} + x \cdot \frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2 x^2}}{x} = \frac{x \left[v + \sqrt{1 + v^2} \right]}{x}$

$\Rightarrow x \frac{dv}{dx} = \sqrt{1 + v^2}$

$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x} \Rightarrow \log |v + \sqrt{1+v^2}| = \log x + \log C$
 $\Rightarrow \left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} \right) = x C$

$\Rightarrow \frac{y + \sqrt{x^2 + y^2}}{x} = x C$

$\Rightarrow y + \sqrt{x^2 + y^2} = x^2 C \quad \checkmark$

D.E.

$$\text{Ex: } \left\{ x \cos\left(\frac{y}{x}\right) + y \cdot \sin\left(\frac{y}{x}\right) \right\} y \cdot dx = \left\{ y \cdot \sin\left(\frac{y}{x}\right) - x \cdot \cos\left(\frac{y}{x}\right) \right\} x \cdot dy$$

$$\Rightarrow \left[\frac{dy}{dx} = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \cdot \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cdot \cos\left(\frac{y}{x}\right) \right\} x} \right] = F(x, y)$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\left\{ \lambda x \cdot \cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y \cdot \sin\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda y}{\left\{ \lambda y \cdot \sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x \cdot \cos\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda x}$$

it is homogeneous

So: $y = vx \rightarrow v = y/x \rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$\rightarrow \frac{v + x \cdot \frac{dv}{dx}}{v + x \cdot \frac{dv}{dx}} = \frac{\left\{ x \cdot \cos(v) + vx \cdot \sin v \right\} vx}{\left\{ vx \cdot \sin v - x \cdot \cos v \right\} x}$$

$$\Rightarrow \left(\frac{v + x \cdot \frac{dv}{dx}}{v + x \cdot \frac{dv}{dx}} \right) = \frac{x \left[\cos v + v \sin v \right] \cdot vx}{x \left[v \sin v - \cos v \right] x} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v}$$

$$x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v - v^2 \sin v + v \cos v}{v \sin v - \cos v}$$

$$x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$$

$$\Rightarrow \int \frac{v \sin v - \cos v dv}{v \cos v} = \int \frac{2}{x} dx$$

$$\rightarrow \int \tan v \cdot dv - \int \frac{1}{v} \cdot dv = 2 \int \frac{1}{x} \cdot dx$$

~~dx~~

Sol. $y \cdot dx + x \cdot \log\left(\frac{y}{x}\right) dy - 2xy dy = 0$ $v = \frac{y}{x}$ # D.E. #

$\rightarrow y dx = [2x - x \log\left(\frac{y}{x}\right)] dy$

$\rightarrow \frac{dy}{dx} = \frac{y}{x[2 - \log\left(\frac{y}{x}\right)]} = F(x, y)$

So $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x [2 - \log\left(\frac{\lambda y}{\lambda x}\right)]} = \lambda^0 \cdot \frac{y}{x[2 - \log\left(\frac{y}{x}\right)]}$

$F(\lambda x, \lambda y) = \lambda^0 \cdot F(x, y) \rightarrow$ Homogeneous.

$\Rightarrow y = vx \rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\Rightarrow \frac{v + x \frac{dv}{dx}}{1} = \frac{v}{x[2 - \log\left(\frac{vx}{x}\right)]}$

$\therefore x \frac{dv}{dx} = \frac{v}{2 - \log v} - v = \frac{v - 2v + v \log v}{2 - \log v}$

$x \frac{dv}{dx} = \frac{v \log v - v}{2 - \log v} \Rightarrow \left[\frac{2 - \log v}{v(\log v - 1)} dv = \frac{dx}{x} \right]$

$\Rightarrow \int \frac{2}{v(\log v - 1)} dv - \int \frac{\log v}{v(\log v - 1)} dv = \log x + \log c$

$I_1 = 2 \int \frac{1}{v(\log v - 1)} \cdot dv$

Let $\log v - 1 = t \Rightarrow \frac{1}{v} \cdot dv = dt$

$I_1 = 2 \int \frac{dt}{t} = 2 \cdot \log t = 2 \log(\log v - 1)$

$I_2 = \int \frac{\log v}{v(\log v - 1)} dv = \int \frac{(\log v - 1) + 1}{v(\log v - 1)} \cdot dv$

$I_2 = \int \frac{(\log v - 1)}{v(\log v - 1)} \cdot dv + \int \frac{1}{v(\log v - 1)} \cdot dv$

$\left[I_2 = \log v + \log(\log v - 1) \right]$

D.E.

$$\frac{2 \log(\log v - 1) - \log v - \log(\log v - 1)}{\log(\log v - 1) - \log v} = \log x \cdot c$$

$$\log(\log v - 1) - \log v = \log x \cdot c$$

$$\log\left(\frac{\log v - 1}{v}\right) = \log x \cdot c$$

$$\Rightarrow (\log v - 1) = v \cdot x \cdot c$$

$$\Rightarrow \left[\log\left(\frac{y}{x}\right) - 1\right] = \left(\frac{y}{x}\right) \cdot x \cdot c$$

$$\Rightarrow \left[\log\left(\frac{y}{x}\right) = 1 + (y)\right] \quad \checkmark$$

$$x \cdot \frac{dv}{dx} = \frac{v \log v - v}{2 - \log v} \Rightarrow \left(\frac{2 - \log v}{v(\log v - 1)} dv = \frac{dx}{x} \right)$$

$$\Rightarrow \int \frac{2}{v(\log v - 1)} dv - \int \frac{\log v}{v(\log v - 1)} \cdot dv = \log x + \log c$$

$$I_1 = 2 \int \frac{1}{v(\log v - 1)} \cdot dv$$

$$\left. \begin{array}{l} \int \\ \int \end{array} \right\} \text{let } \log v - 1 = t \Rightarrow \frac{1}{v} \cdot dv = dt$$

$$\underline{I_1} = 2 \int \frac{dt}{t} \Rightarrow 2 \cdot \log t \Rightarrow 2 \log(\log v - 1)$$

$$I_2 = \int \frac{\log v}{v(\log v - 1)} dv = \int \frac{(\log v - 1) + 1}{v(\log v - 1)} \cdot dv$$

$$I_2 = \int \frac{(\log v - 1)}{v(\log v - 1)} \cdot dv + \int \frac{1}{v(\log v - 1)} \cdot dv$$

$$\left[\underline{I_2} = \log v + \log(\log v - 1) \right]$$

$x = vy \rightarrow y/x \rightarrow y = vx$
 Q. $(1 + e^{x/y}) dx + e^{x/y} (1 - x/y) dy = 0$ $\neq D.E.$ $\neq y = g$, when $x=1$

$\Rightarrow \left(\frac{dx}{dy} \right) = \frac{-e^{x/y} [1 - x/y]}{1 + e^{x/y}} = F(x, y)$

\Rightarrow let:- $f(\lambda x, \lambda y) = -\frac{e^{\lambda x/\lambda y} [1 - \lambda x/\lambda y]}{1 + e^{\lambda x/\lambda y}}$

$f(\lambda x, \lambda y) = \frac{-e^{x/y} [1 - x/y]}{1 + e^{x/y}} \rightarrow$ Homogenous.

Let:- $x = vy \rightarrow v = x/y \Rightarrow \left(\frac{dx}{dy} = v + y \cdot \frac{dv}{dy} \right)$

So: $v + y \cdot \frac{dv}{dy} = \frac{-e^v [1 - v]}{1 + e^v}$

$y \cdot \frac{dv}{dy} = \frac{-e^v + v e^v - v - y \cdot e^v}{1 + e^v} = \frac{-[e^v + v]}{1 + e^v}$

$\Rightarrow y \cdot \frac{dv}{dy} = -\frac{[e^v + v]}{1 + e^v}$

$\rightarrow \int \left(\frac{1 + e^v}{e^v + v} \right) dv = \int \frac{-dy}{y}$

$\Rightarrow e^v + v = t$

$(e^v + 1) \cdot dv = dt$

$\Rightarrow \log [e^v + 1] = -\log y + \log c$

$\Rightarrow e^v + v = \frac{c}{y}$

$= e^{x/y} + x/y = \frac{c}{y}$

$= [y \cdot e^{x/y} + x = c]$