D.E.

Ans:- in a bank, principle increase Continuously at rate of $r\%$ per year. find value of r if ₹ 100 double itself in 10 years.

$\log 2 = \boxed{0.629}$

Sol:- rate = $r\%$ = find

Principle = 100 ₹

time = 10 years

$P + I = 200 = \text{Amount}$

☆ $A = P\left(1 + \frac{R}{100}\right)^{T}$

given

$$\frac{dP}{dt} = (r\%) \times P$$

$$\Rightarrow \frac{dP}{dt} = \left(\frac{r}{100}\right) \times P$$

$$\Rightarrow \boxed{\int \frac{dP}{P} = \int \frac{r}{100} \cdot dt}$$

$$\Rightarrow \log P = \frac{r}{100} \cdot t + c$$

$$\Rightarrow P = e^{\left(\frac{r}{100}t + c\right)}$$

$$\boxed{P = e^{\left(\frac{r}{100}t + c\right)}} = P = e^{\frac{r}{100}t} \cdot e^{c}$$

given that :- When $t = 0 \rightarrow \boxed{P = 100}$

$$\therefore \Rightarrow 100 = e^{\left(\frac{r}{100} \times 0 + c\right)} \Rightarrow 100 = e^{0} \cdot e^{c} \Rightarrow \boxed{e^{c} = 100}$$

So! $\boxed{P = 100\, e^{\frac{r}{100}t}}$

& When $\boxed{t = 10}$ year $\Rightarrow \boxed{P = 200}$

$$\therefore \cancel{200} = \cancel{100} \cdot e^{\left(\frac{r}{\cancel{100}} \times 1\cancel{0}\right)}$$

$$\Rightarrow 2 = e^{\frac{r}{10}}$$

$$\Rightarrow \boxed{\log_e 2} = \frac{r}{10} \Rightarrow 0.699 = \frac{r}{10} \Rightarrow r = 6.99\%$$

Que. a bactria count is 1,00000/-. The no. increase by 10% in 2 hours in how many Hours will the count reach 200000/- if rate of growth of bacteria is proportinoal to the no. present.

Sol^n:- Initial bacteria $\Rightarrow y_0 = 100000/-$

final —11 $\Rightarrow y = 200000/-$

$\Rightarrow$ in 2 hours $\to \uparrow 10\%$

given $\Rightarrow$ $\frac{dy}{dt} \propto y \Rightarrow \left[\frac{dy}{dt} = Ky\right] \Rightarrow \int \frac{dy}{y} = \int K\,dt$

$\Rightarrow \log y = Kt + C$

$\Rightarrow \boxed{\log y = Kt + C}$

$\Rightarrow$ ∴ in starting :- $(y \Rightarrow y_0) \Rightarrow (t = 0)$

100 ₹

$100 + 100 \times 10\%$

$100 + 10 = \underline{110}$

So :- $\log y_0 = K(0) + C \Rightarrow \boxed{C = \log y_0}$

∴ $\log y = Kt + \log y_0$

$\log y - \log y_0 = Kt$

$$\boxed{\log\left(\frac{y}{y_0}\right) = Kt}$$

$$\boxed{\frac{y}{y_0} = e^{Kt}} \quad (1)$$

∴ Bac. ↑ by $\underline{10\%}$ in 2 Hours

∴ $y = y_0 + 10\%$ of y_0

$\underline{y =} \frac{11\cancel{0}}{10\cancel{0}} \times y_0$

$$\boxed{\frac{y}{y_0} = \frac{11}{10}} \quad (2)$$

So: from ① & ②:-

$$\frac{y}{y_0} = e^{kt} \quad \& \quad \frac{y}{y_0} = \frac{11}{10}$$

So: $e^{kt} = \frac{11}{10}$ [at $\Rightarrow t = 2$]

C

K

$\Rightarrow k2 = \log\left(\frac{11}{10}\right)$

$\Rightarrow \boxed{k = \frac{1}{2}\log\left(\frac{11}{10}\right)}$

So:-

$\log\left(\frac{y}{y_0}\right) = kt$

$\log\left(\frac{y}{y_0}\right) = \frac{1}{2}\log\left(\frac{11}{10}\right)\times t$

$\therefore t = ?$

When $y = 20000$

& $y_0 = 10000$

So: $\log\left(\frac{20000}{10000}\right) = \frac{1}{2}\log\left(\frac{11}{10}\right)\times t$

$\log 2 = \frac{1}{2}\log\left(\frac{11}{10}\right)\times t$

$$\boxed{\frac{2\log 2}{\log\left(\frac{11}{10}\right)} = t}$$

Ans

Qus:- Show that the D.E. is Homogeneous & solve it.

$\Rightarrow \sqrt{}\,[x^2+xy]\cdot dy = (x^2+y^2)\cdot dx$

Let:- $\boxed{y = vx}$ — (2)

Diff.

$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$ — (3)

Solⁿ:-

$$\boxed{\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}} = F(x,y) \quad \text{— (1)}$$

Let $F(\lambda x, \lambda y) \Rightarrow \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)} = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda^2 x^2 + \lambda^2 xy}$

$\frac{dx}{dy} =$

Let $\boxed{x = vy}$

$$F(\lambda x, \lambda y) = \frac{\cancel{\lambda^2}[x^2+y^2]}{\cancel{\lambda^2}[x^2+xy]} = F(x,y)$$

$\therefore$ it's Homogenous.

$$\boxed{\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}} \Rightarrow \left[v + x\cdot\frac{dv}{dx} = \frac{x^2+(vx)^2}{x^2+x\cdot(vx)} \right]$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+x^2v} = \frac{\cancel{x^2}}{\cancel{x^2}}\left[\frac{1+v^2}{1+v}\right]$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow x\cdot\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$x\cdot\frac{dv}{dx} = \frac{1+\cancel{v^2}-v-\cancel{v^2}}{1+v}$$

$$\boxed{x\cdot\frac{dv}{dx} = \frac{1-v}{1+v}}$$

$$\int \frac{(1+v)}{(1-v)}\,dv = \int \frac{dx}{x}$$

$$\int \frac{2-1+v}{1-v}\cdot dv = \log x + \log c$$

$$\int \frac{2-1(1-v)}{1-v}\,dx = \log x\cdot c$$

$$\int \frac{2}{1-v}\cdot dv - \int \left(\frac{\cancel{1-v}}{\cancel{1-v}}\right)\cdot dv = \log x \cdot c$$

$$y = vx$$
$$v = y/x$$

$$\Rightarrow -2\int \frac{1}{v-1}\cdot dv - \int 1\cdot dv = \log x\cdot c$$

$$\Rightarrow \underline{-2}\log(v-1) - v = \log xc$$

$$\Rightarrow -v = 2\log(v-1) + \log xc$$

$$= \log(v-1)^2 + \log xc$$

$$\Rightarrow \boxed{-v = \log\left[(v-1)^2\cdot xc\right]}$$

put $\Rightarrow v = \frac{y}{x} \Rightarrow \frac{-y}{x} = \log\left[\left(\frac{y}{x}-1\right)^2\cdot x\cdot c\right]$ Ans