D.E.

Ques:- $y = \cos x + c \longrightarrow y' + \sin x = 0$

Que:- $\boxed{y = \sqrt{1+x^2}}$; $\longrightarrow$ $\boxed{y' = \dfrac{xy}{1+x^2}}$

Diff. $\rightarrow$ $y' = \dfrac{1}{\cancel{2}\sqrt{1+x^2}} \times \cancel{2}x$

$$\boxed{y' = \frac{x}{\sqrt{1+x^2}}} \times \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} \Rightarrow \boxed{y' = \frac{x \times y}{(1+x^2)}}$$

G.S.

Ques: $y = Ax$; $xy' = y$

Ques:- $y = x\sin x$; $xy' = y + x\sqrt{x^2 - y^2}$

$y' = x.\cos x + \sin x$

$\Rightarrow$ from DE $\rightarrow$ $x \times [x\cos x + \sin x] = x\sin x + x\sqrt{x^2 - (x^2 \sin^2 x)}$

$x^2\cos x + \cancel{x\sin x} = \cancel{x\sin x} + x\sqrt{x^2(1-\sin^2 x)}$

$x^2\cancel{\cos x} = x^2\cancel{\cos x}$ H.P.

H.W

Q. $xy = \log y + c$; $y' = \dfrac{y^2}{1-xy}$

Ans:- $y - \cos y = x$; $(y\sin y + \cos y + x)y' = y$

Sol^n

$y' + \sin y \cdot y' = 1$

$y'(1+\sin y) = 1$

$y' = \dfrac{1}{1+\sin y}$

from DE

$[y\sin y + \cos y + y - \cos y] \cdot \dfrac{1}{1+\sin y} = y$

$y(\sin y + 1) \cdot \dfrac{1}{1+\sin y} = \dfrac{y}{y}$

$1 = 1$

H.P.

Ans: $\boxed{x+y=\tan^{-1}(y)}$

Soln:

$$1+y' = \frac{1}{1+y^2}\times y'$$

$$1+y' - \frac{y'}{1+y^2} = 0$$

$$1+y'\left(1-\frac{1}{1+y^2}\right)=0$$

$$y'\left(\frac{\cancel{1}+y^2-\cancel{1}}{1+y^2}\right)=-1$$

$$y'\left(\frac{y^2}{1+y^2}\right)=-1$$

$$\boxed{y'=\frac{-(1+y^2)}{y^2}}$$

$$y'y^2+y^2+1=0$$

; $\boxed{y^2 y'+y^2+1=0}$

$$\Rightarrow \cancel{y^2}\left[\frac{-(1+y^2)}{\cancel{y^2}}\right]+y^2+1=0$$

$$\Rightarrow \cancel{-1}-\cancel{y^2}+\cancel{y^2}+\cancel{1}=0$$

$$0=0$$

Q. $y = \sqrt{a^2 - x^2}$ $\qquad \left[x + y \cdot \frac{dy}{dx} = 0 \right]$

$y \to f(x) \to$ Create $\to$ D.E.

Diff. $\to$

arbitrary Constant

remove

$y = a\cos x \to$ D.E.

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Ques- Form a D.E. representing the given family of curves by eliminating arbitrary constant a & b.

Q. $\left[\frac{x}{a} + \frac{y}{b} = 1\right] \longrightarrow$ Diff. $\longrightarrow \frac{1}{a} + \frac{1}{b} \cdot y' = 0$

Diff again $\rightarrow 0 + \frac{1}{b} \cdot y'' = 0$

$\Rightarrow \frac{y''}{b} = 0$

$\Rightarrow \boxed{y'' = 0}$ Ans

Ans: $\left[y = ae^{3x} + be^{-2x}\right]$ — (1)

Diff. $\rightarrow$ $y' = a \cdot e^{3x} \cdot 3 + b \cdot e^{-2x} \times (-2)$

$\rightarrow \left[y' = 3ae^{3x} - 2be^{-2x}\right]$ — (2)

Diff again $\Rightarrow$ $y'' = 3a \cdot e^{3x} \times 3 - 2b\,e^{-2x} \times (-2)$

$\left[y'' = 9ae^{3x} + 4be^{-2x}\right]$ — (3)

(3) − (2) $\Rightarrow$

$$\begin{array}{l} y'' = 9ae^{3x} + 4be^{-2x} \\ \underline{-y' = -3ae^{3x} + 2be^{-2x}} \\ y'' - y' = 6ae^{3x} + 6be^{-2x} \end{array}$$

$y'' - y' = 6\left(\boxed{ae^{3x} + be^{-2x}}\right)$

$y'' - y' = 6y$ Ans

Ans: H.W $y = e^{2x}(a + bx)$

Ans:- H.W $y = e^{3x}[a\cos x + b\sin x] \xrightarrow{DE} [y'' - 2y' + 2y = 0]$

Statement

Read

Curve find → equation

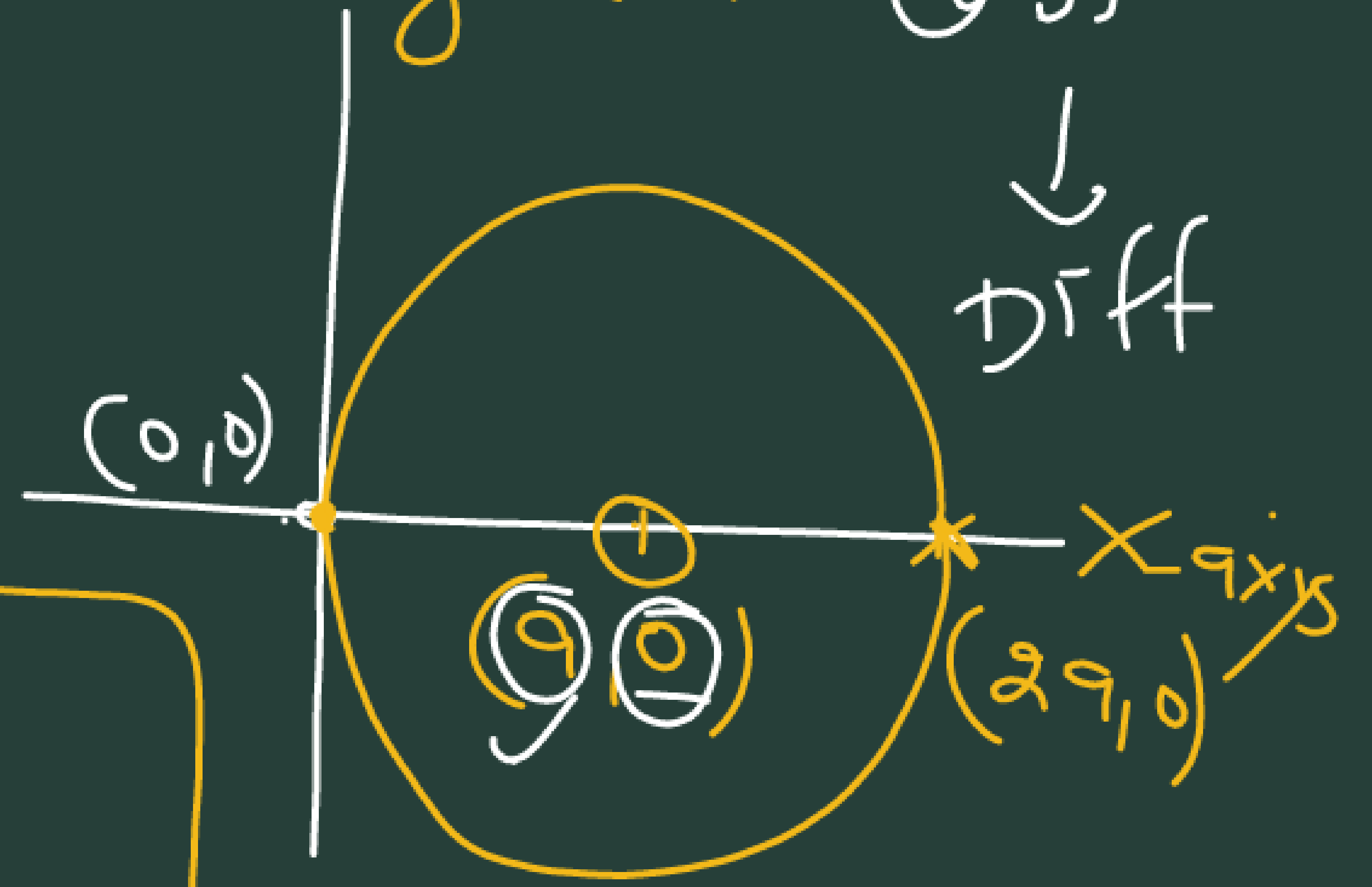
Convert

D.E.

Ques: Form the D.E. of the family of circles touching the y-axis at origin. a & b

Sol^n:- From fig.

Constant
↓
Diff



eq. of circle touching y-axis at origin

$$\Rightarrow \left[(x-a)^2 + y^2 = a^2\right] \quad \text{—(1)}$$

Diff $\Rightarrow$ $2(x-a)\times 1 + 2y\cdot y' = 0$

$$2\left[(x-a) + y\cdot y'\right] = 0$$

$$x - a + y\cdot y' = 0$$

$$\Rightarrow \left[a = x + yy'\right]$$

$$\Rightarrow \left[\cancel{x} - (\cancel{x} + y\cdot y')\right]^2 + y^2 = \left[x + yy'\right]^2$$

$$\Rightarrow \cancel{y^2(y')^2} + y^2 = x^2 + \cancel{y^2(y')^2} + 2xy\cdot y'$$

$$\Rightarrow \boxed{x^2 - y^2 + 2xy\cdot y' = 0}$$

Ans! form $\boxed{\text{D.E}}$ of family of parabolas having vertex at origin & axis along $\boxed{+\text{ive } y\text{-axis}}$.

sol^n! - from fig. → parabola eq.

$$\Rightarrow \quad x^2 = 4ay$$

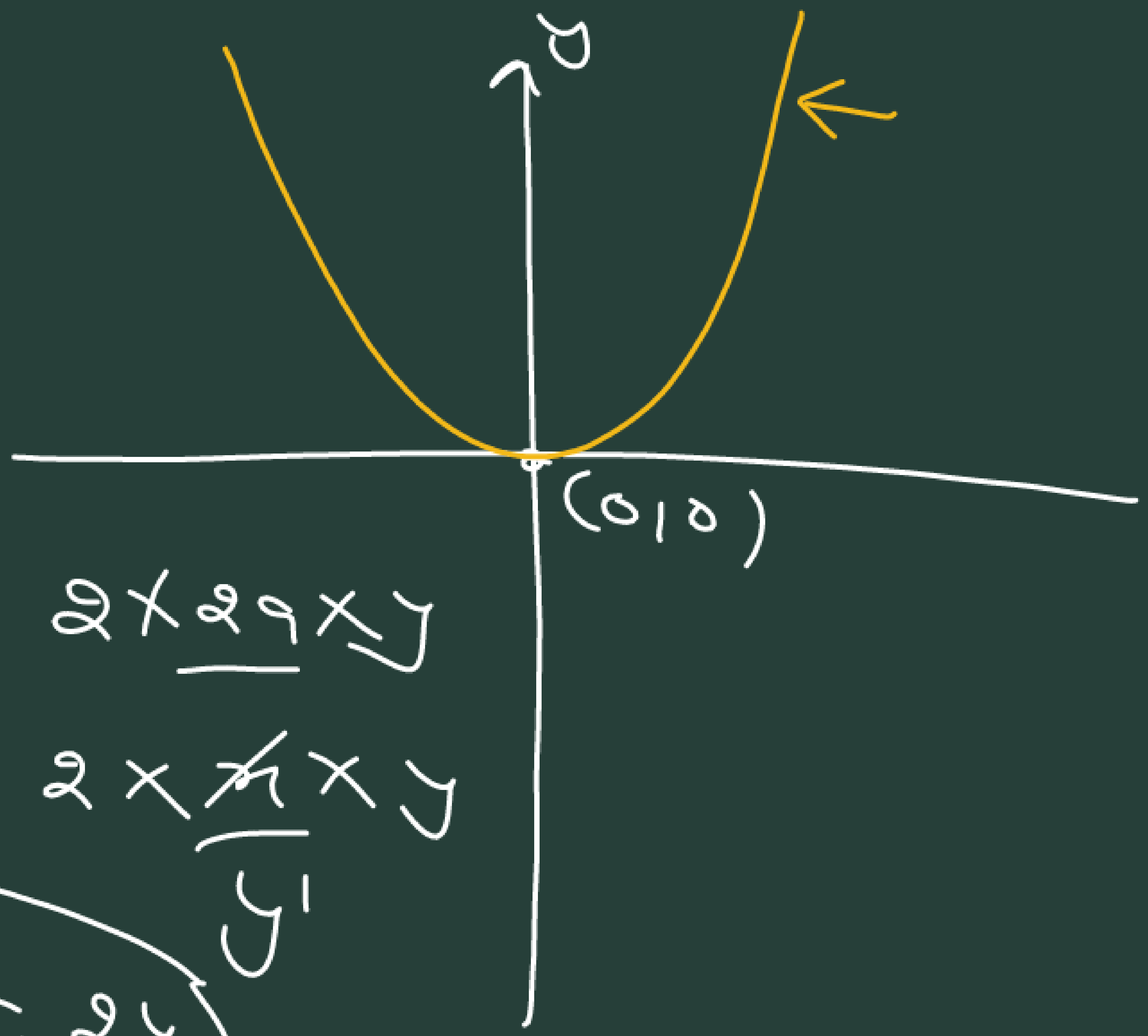
Diff.

$$2x(1) = 4a \cdot y'$$

$$\cancel{2}x = \cancel{4}a \cdot y$$

$$x = 2ay'$$

$$\boxed{2a = \frac{x}{y'}}$$



$$x^2 = 2 \times 2a \times y$$

$$x^2 = 2 \times \frac{\cancel{x}}{y'} \times y$$

$$\boxed{xy' = 2y}$$

Ans