Differential Equations

For ex → $x + y = 1$

(1) order

$x^2 + x + 2 = 0$

(2) Degree

$\Rightarrow$ $y = f(x)$ $\Rightarrow$ $\frac{dy}{dx} = y'$ & y'' & y'''

Ex:- $y'' + 2y' + y = 0$ → D.E.

Ex: $(y')^2 + 2\sin y = 0$

② $y = f(x)$ ⟶ Check ⟶ D.E. ⟶

Solve $x^2 - 1 = 0$

$x = \pm 1$

③ $y = f(x)$ → Create D.E.

roots of eq. given ↓ eq. create

$x^2 - x$ (add roots) + M.U.H. (roots) $= 0$

Ex :- $y = a \sin x$ → D.E. → y'

a ↓ Constant

Ex: $y = a \sin x + b \cos x$ → D.E. → $y' \& y''$

order :-

Ex :→ $y' + 2 = 0$ → Degree :- 1 → order = 1

Ex :- $y'' + 2y' + \sin y = 0$ → Degree = 1 → order = 2

$[(y''')^2 + y' = 0]$ → [order = 3]

Ex :- $\frac{d^2y}{dx^2} = 0$

Degree :-

⇒ D.E → polynomial in Derivative

Ex :- $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$

Degree → Not Define

order → 2

Order

↓

highest derivative

Q y

Ex :- $y' \rightarrow 1$

Ex: $\frac{d^2y}{dx^2} \rightarrow 2$

Ex $\frac{d^3y}{dx^3} \Big| y''' \rightarrow 3$

Degree → Polynomial

↓

power of highest order Derivative

Ex :- $(y''')^1 + y = 0$

order = 3, Deg → 1

Ex: $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^5 + 2 = 0$

order = 3

Degree = 2

Q.1 $\frac{d^4y}{dx^4} + \sin(y''') = 0$ → Order = 4, Degree → not define

Derivation

2. $\left(\frac{ds}{dt}\right)^4 + 3s\left(\frac{d^2s}{dt^2}\right)^1 = 0$ → Order = 2, Degree = 1

3. $y''' + 2y'' + y' = 0$ → Order = 3, Degree = 1

4. $y'' + 2y' + \log y' = 0$ → Order = 2, Degree = 1

General Solution of D.E. → (indefinate Integ.)

Constant — C

Particular Sol. of D.E. → (definate Integ.)

Constant → value of C find

Q. Verify that the function $y = e^{-3x}$ is a sol. of D.E.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

Soln:- $\because y = e^{-3x}$

Diff. $\rightarrow \frac{dy}{dx} = e^{-3x} \times (-3)$

Diff again $\rightarrow \frac{d^2y}{dx^2} = -3 \times e^{-3x} \times -3$

$$= 9 \cdot e^{-3x}$$

D.E. $\rightarrow m^2 - 1 = 0$

$m = 1$

$\therefore$ D.E. $\rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

$\Rightarrow 9e^{-3x} + (-3e^{-3x}) - 6e^{-3x} = 0$

$= 9e^{-3x} - 9e^{-3x} = 0$

$0 = 0$

Q. Verify the Fun. is a Sol. of D.E. eq.

$y = Ax$ — (1) & D.E. → $xy' = y$ — (2)

Sol:- $\therefore y = Ax$

Diff. → $y' = A \cdot (1)$ ✓

Sol: D.E. → $xy' = y$

$\Rightarrow x \cdot (A) = y$

$y = y$

H.P.

Q.no:- derivative of $\log_7(\log x)$

$$\Rightarrow \log_7(\log x) = \frac{\log_e(\log x)}{\log_e 7}$$

So:- $\frac{d}{dx}\left\{\frac{\log_e(\log x)}{\log_e 7}\right\} = \frac{1}{\log 7}\frac{d}{dx}\{\log(\log x)\}$

Constant $= \frac{1}{\log 7} \times \frac{1}{\log x} \times \frac{1}{x}$

Q. $y = \cos x + c \longrightarrow$ D.E. $\rightarrow \boxed{y' + \sin x = 0}$

Q. $y = x \sin x \rightarrow xy' = y + x\sqrt{x^2 - y^2}$

Q. $x + y = \tan^{-1} y \Rightarrow$ D.E. $\rightarrow y^2 y' + y^2 + 1 = 0$