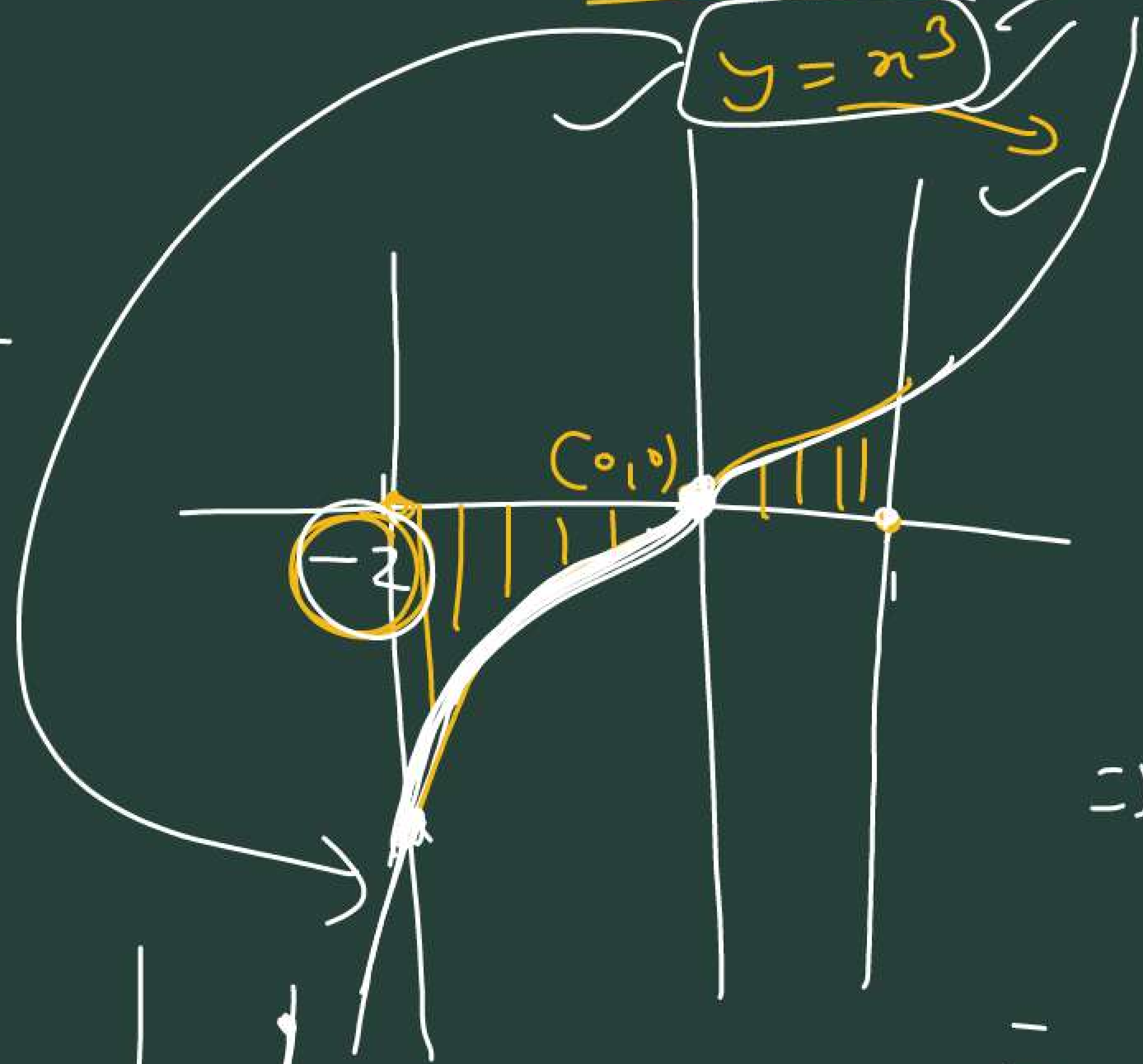
AOI

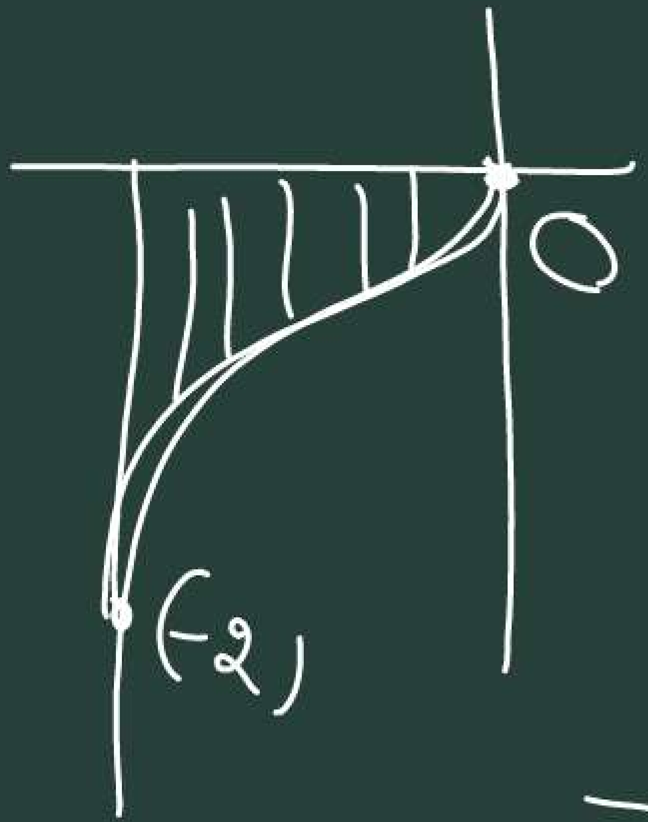
$y = x^3$

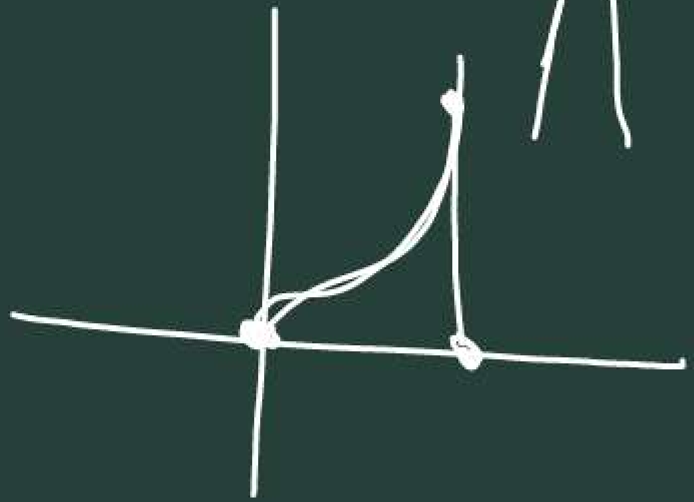
$x = -2$

$x = 1$

$x = -1 \Rightarrow \boxed{y = -1}$







$$\text{Area} = \int_{-2}^{0} x^3 \cdot dx + \int_{0}^{1} x^3 \cdot dx$$

$$\Rightarrow \left[\frac{x^4}{4}\right]_{0}^{-2} + \left[\frac{x^4}{4}\right]_{0}^{1}$$

$$\Rightarrow \left(\frac{(-2)^4 - 0}{4}\right) + \left(\frac{(1)^4}{4} - 0\right)$$

$$= \frac{16}{4} + \frac{1}{4} = \frac{1}{4} + \frac{16}{4} = \frac{17}{4}$$

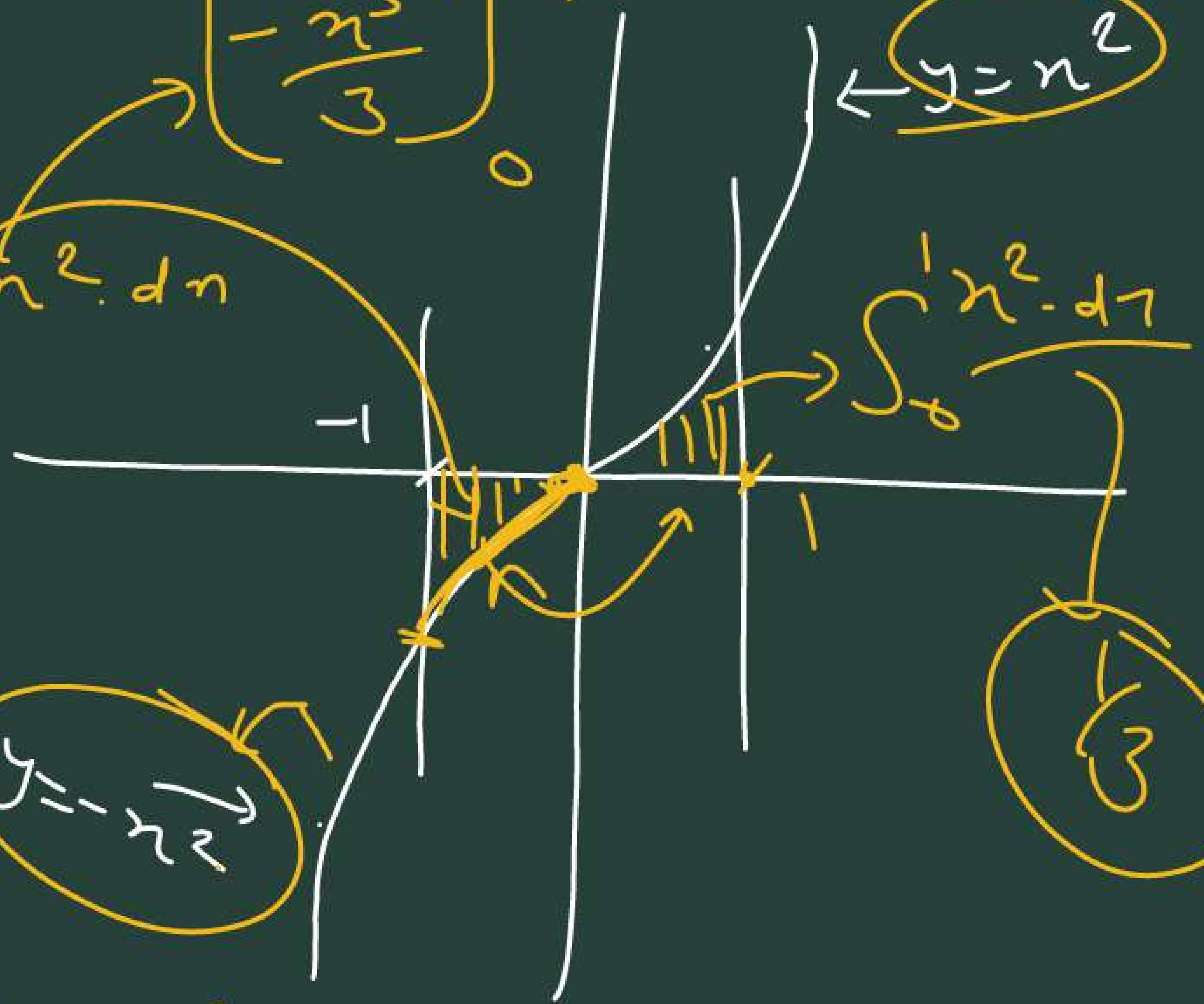
Q. The area bounded by curve $\boxed{y = x|x|}$, x-axis & ordinates $x=-1$ & $x=1$ is given by.

Solⁿ:- $y = x|x|$

$$y = \begin{cases} x \cdot x = x^2 , & x > 0 \\ x\cdot(-x) = -x^2 ; & x < 0 \end{cases}$$

$\int_0^{-1} -x^2 \cdot dx$ $\quad \left[-\frac{x^3}{3}\right]_0^{-1}$

$\int_0^1 x^2 \cdot dx$ $\quad \frac{1}{3}$



So:- $\text{Area} = 2\int_0^1 x^2 \cdot dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}\left((1)^3 - 0\right) = \frac{2}{3}$ Ans

$\int_0^1 x^2 \cdot dx + \int_0^{-1} -x^2 \cdot dx$

$\left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3}\right]_0^{-1} = \frac{(-1)^3}{3} = \frac{+1}{3}$

$\frac{1}{3} + \frac{1}{3} = 2/3$ Ans

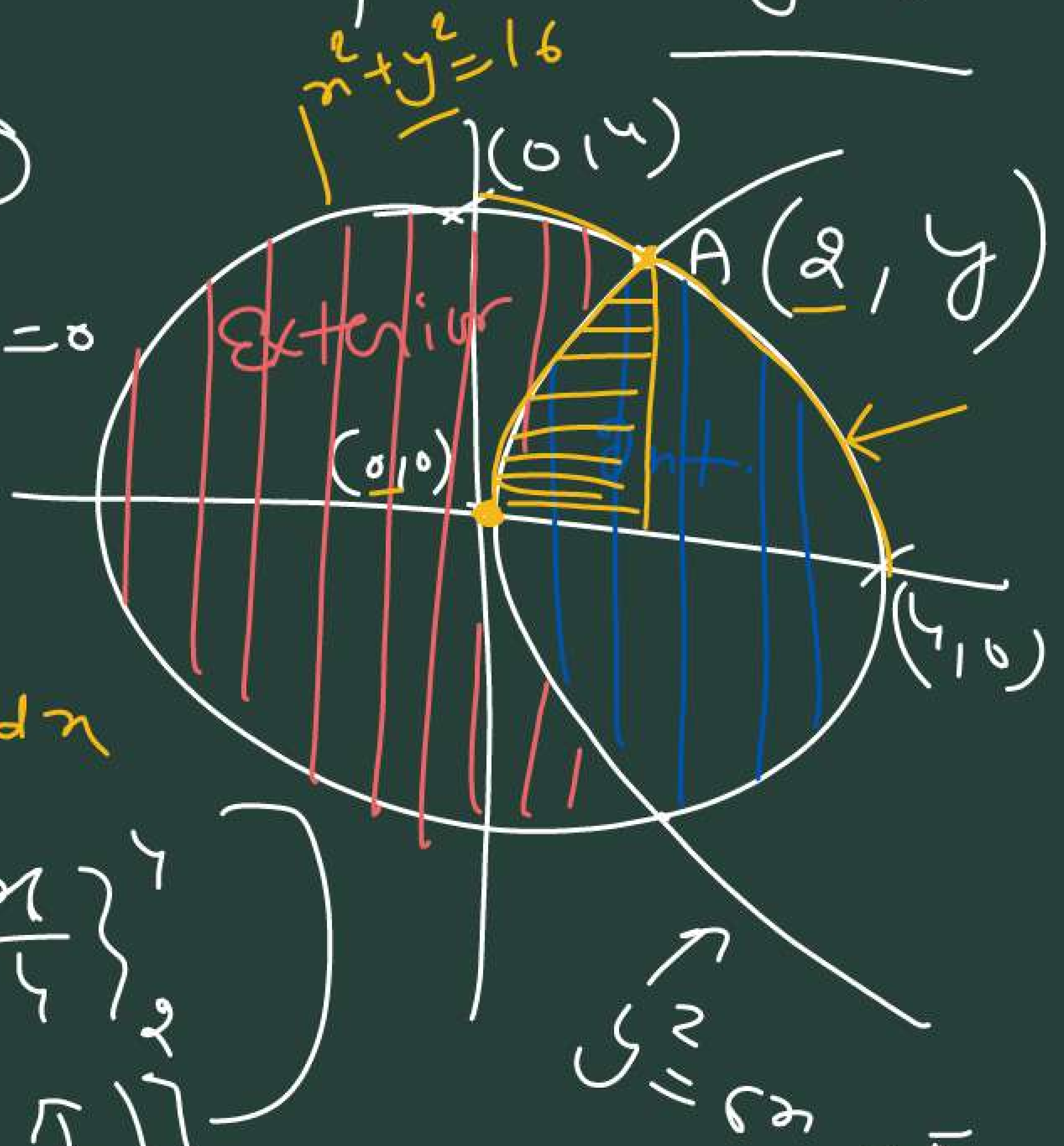
Q. area of circle $x^2+y^2=16$ exterior to the parabola $y^2=6x$.

Sol:- $x^2+y^2=16$ — ① $\Rightarrow r=4$, $y^2=6x$ — ②

So! intersection :→ $x^2+6x=16 \Rightarrow x^2+6x-16=0$

$\to x^2+8x-2x-16=0 \Rightarrow x(x+8)-2(x+8)$

$x=-8$ $x=2$



interior part area $= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$

$\Rightarrow 2\left[\sqrt6\left(\frac{2}{3}x^{3/2}\right)_0^2 + \left\{\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_2^4\right]$

$\sqrt{12}$

$\sqrt{4\times3}$ $\Rightarrow 2\left[\sqrt6\times\frac{2}{3}\left(2\sqrt2\right) + \left\{0+8\times\frac{\pi}{2} - \left(\sqrt{12}+8\cdot\frac{\pi}{6}\right)\right\}\right]$

$2\sqrt3$ $= 2\left(2\sqrt3\times\frac{4}{3} + 4\pi - \sqrt{12} - \frac{4}{3}\pi\right) = 2\left(\frac{8}{\sqrt3} - \sqrt{12} + \frac{8\pi}{3}\right)$

$4\pi - \frac{4}{3}\pi = \frac{8\pi}{3}$

interior Area $= 2\left(\frac{8}{\sqrt{3}} - \sqrt{12} + \frac{8\pi}{3}\right) = 2\left[\frac{8}{\sqrt{3}} - 2\sqrt{3} + \frac{8\pi}{3}\right]$

$\Rightarrow 2\left[\frac{8-6}{\sqrt{3}} + \frac{8\pi}{3}\right] = 2\times\frac{2}{\sqrt{3}} + \frac{16\pi}{3}$

$= \left(\frac{4}{\sqrt{3}} + \frac{16\pi}{3}\right)$

So: Exterior area $= \pi r^2 -$ interior part area

$= 16\pi - \left(\frac{4}{\sqrt{3}} + \frac{16\pi}{3}\right)$

$= 16\pi - \frac{16\pi}{3} - \frac{4}{\sqrt{3}} = \frac{32\pi}{3} - \frac{4}{\sqrt{3}}$ Ans

Ques: Find area bounded by y-axis, $y = \cos x$, $y = \sin x$ when $\boxed{0 \le x \le \pi/2}$.

Sol^n:- y-axis

① — $y = \cos x \Rightarrow x = \cos^{-1} y$

② → $y = \sin x \Rightarrow x = \sin^{-1} y$

$$\text{So area} = \int_0^{1/\sqrt{2}} \frac{x \cdot dy}{\sin^{-1} y} + \int_{1/\sqrt{2}}^{1} \frac{x \cdot dy}{\cos^{-1} y}$$

$$= \int_0^{1/\sqrt{2}} \sin^{-1} y \cdot dy + \int_{1/\sqrt{2}}^{1} \cos^{-1} y \cdot dy.$$

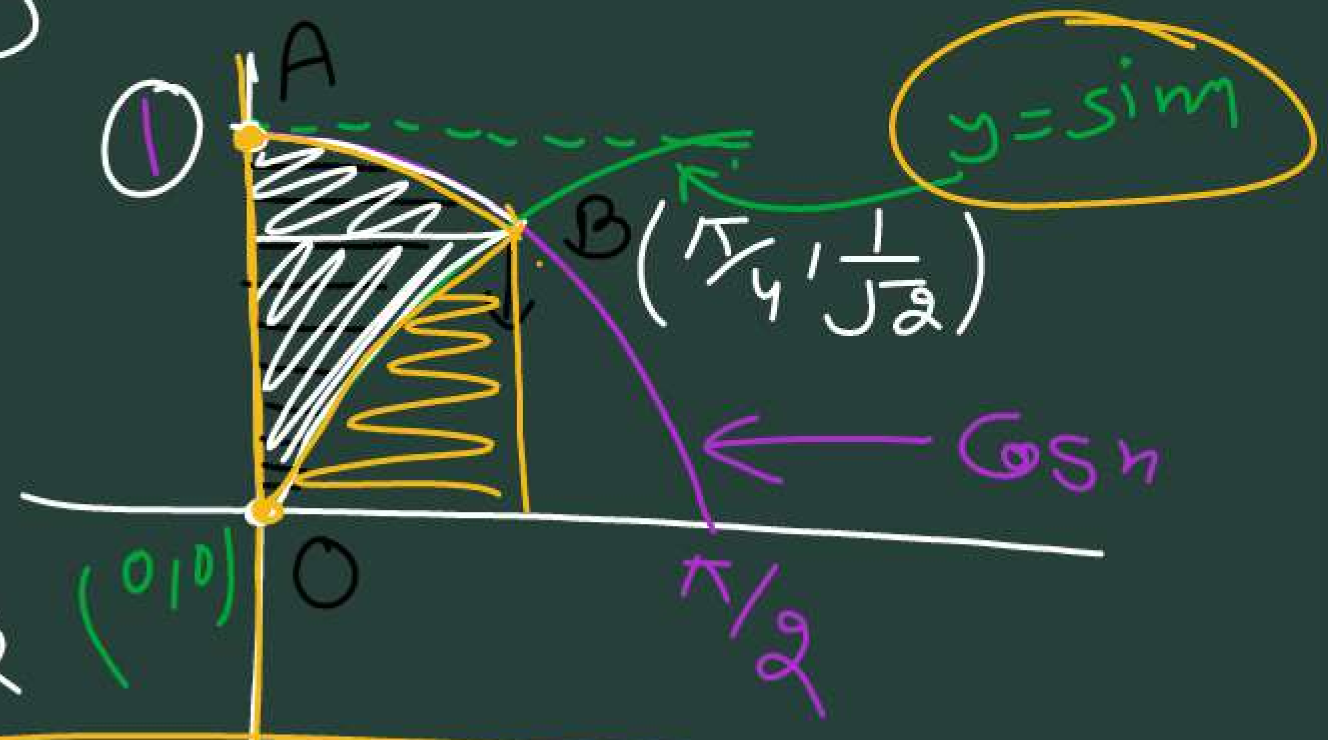
from ① & ②

$\sin x = \cos x$

$\tan x = 1$

$\boxed{x = \pi/4}$

$y = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$



$$\text{Area} = \int_0^{\pi/4} \cos x \, dx - \int_0^{\pi/4} \sin x \, dx$$

$$= \left[\sin x\right]_0^{\pi/4} - \left[-\cos x\right]_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}} - \left[-\cos \frac{\pi}{4} + \cos 0\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = \sqrt{2} - 1$$