Ques:- find area bounded by curve

circle:> $\boxed{(x-1)^2 + y^2 = 1}$ — 1 & $x^2 + y^2 = 1$ — 2

$\therefore \boxed{(x-a)^2 + (y-b)^2 = r^2} \longrightarrow (a, b) \rightarrow$ Centre of circle

$r \rightarrow$ radius of circle.

from eq. 1 :- [Centre → (1, 0) & $r = 1$]

from eq. 2 :- [Centre → (0, 0) & $r = 1$]

So intersection point:-

$$(x-1)^2 + y^2 = 1$$
$$\underline{-x^2 \;\; \underline{+} y^2 = 1}$$
$$(x-1)^2 - x^2 = 0$$

Sub eq. 1 from 2 :-

$\cancel{x^2} - 2x + 1 - \cancel{x^2} = 0$

$2x = 1$

$x = \frac{1}{2}$

So area bounded is, $\left[\frac{\pi}{2} - \frac{\pi}{6}\right]$

$$\Rightarrow 2\left[\int_0^{1/2} y.\text{circle } ① + \int_{1/2}^{1} y.\text{circle } ②\right] dx$$

$x^2+y^2=1$

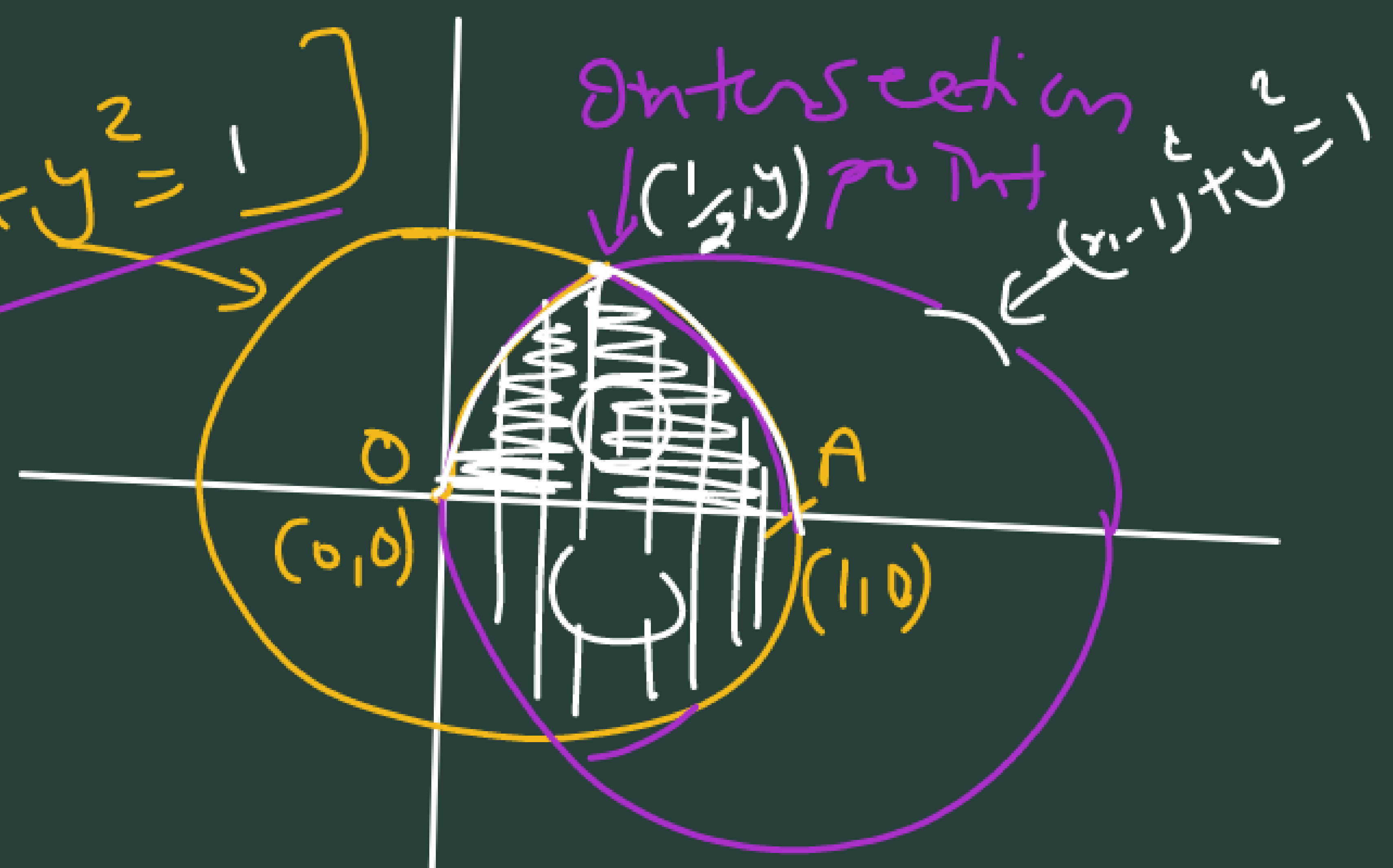
$$\Rightarrow 2\left[\int_0^{1/2} \sqrt{1-(x-1)^2} + \int_{1/2}^{1} \sqrt{1-x^2}\right]$$

$$\Rightarrow 2\left[\left\{\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\frac{x-1}{1}\right\}_0^{1/2} + \left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right\}_{1/2}^{1}\right]$$

$$\Rightarrow 2\left[\left\{-\frac{1}{2}\times\frac{1}{2}\sqrt{1-\frac{1}{4}} + \frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right) - \frac{1}{2}\sin^{-1}(-1)\right\} + \left\{0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$$

$$\Rightarrow 2\left[-\frac{1}{4}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\left(-\frac{\pi}{6}\right) - \frac{1}{2}\left(-\frac{\pi}{2}\right) + \frac{1}{2}\frac{\pi}{2} - \frac{1}{4}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\pi}{6}\right]$$

$$= 2\left[-\cancel{2}\times\frac{1}{4}\times\frac{\sqrt{3}}{\cancel{2}} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12}\right] = -\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\ \text{Ans}$$



Q. Find area bounded by $y = x^2 + 2$, $y = x$, $x = 0$, $x = 3$

Soln :-

eq. $\rightarrow$ $y = x^2 + 2 \rightarrow$ parabola

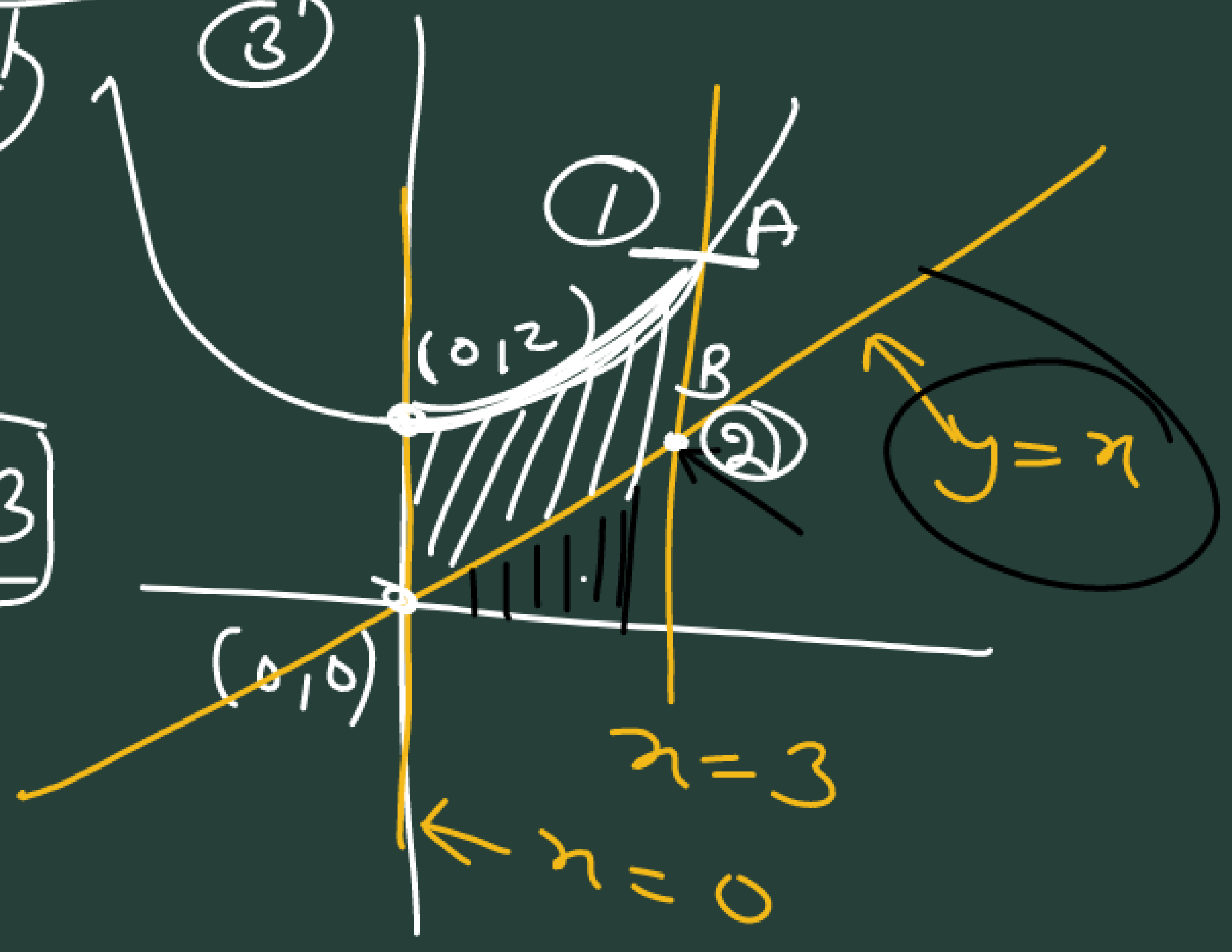
eq. $\rightarrow$ $y = x$, $x = 0$ & $x = 3$

$\rightarrow$ from 1 & 4 :- $(3, 11)$ A & from 2 & 4 :- $(3,3)$ B

$\Rightarrow$ area $\Rightarrow \int_0^3 y\,dx \text{ (para)} - \int_0^3 y\,dx \text{ (line)}$

$$A = \left[\int_0^3 x^2 + 2\,dx\right] - \int_0^3 x\,dx$$

$\frac{x^2}{2} = \frac{1}{2}(9)$



$\frac{1}{2} \times 3 \times 3$

Q. using Integration find area of region bounded by the triangle whose vertices are $(-1,0)$ $(1,3)$ & $(3,2)$

Sol^n:- $A\ (\overset{x_1}{-1}, \overset{y_1}{0})$

$B\ (1,3)$

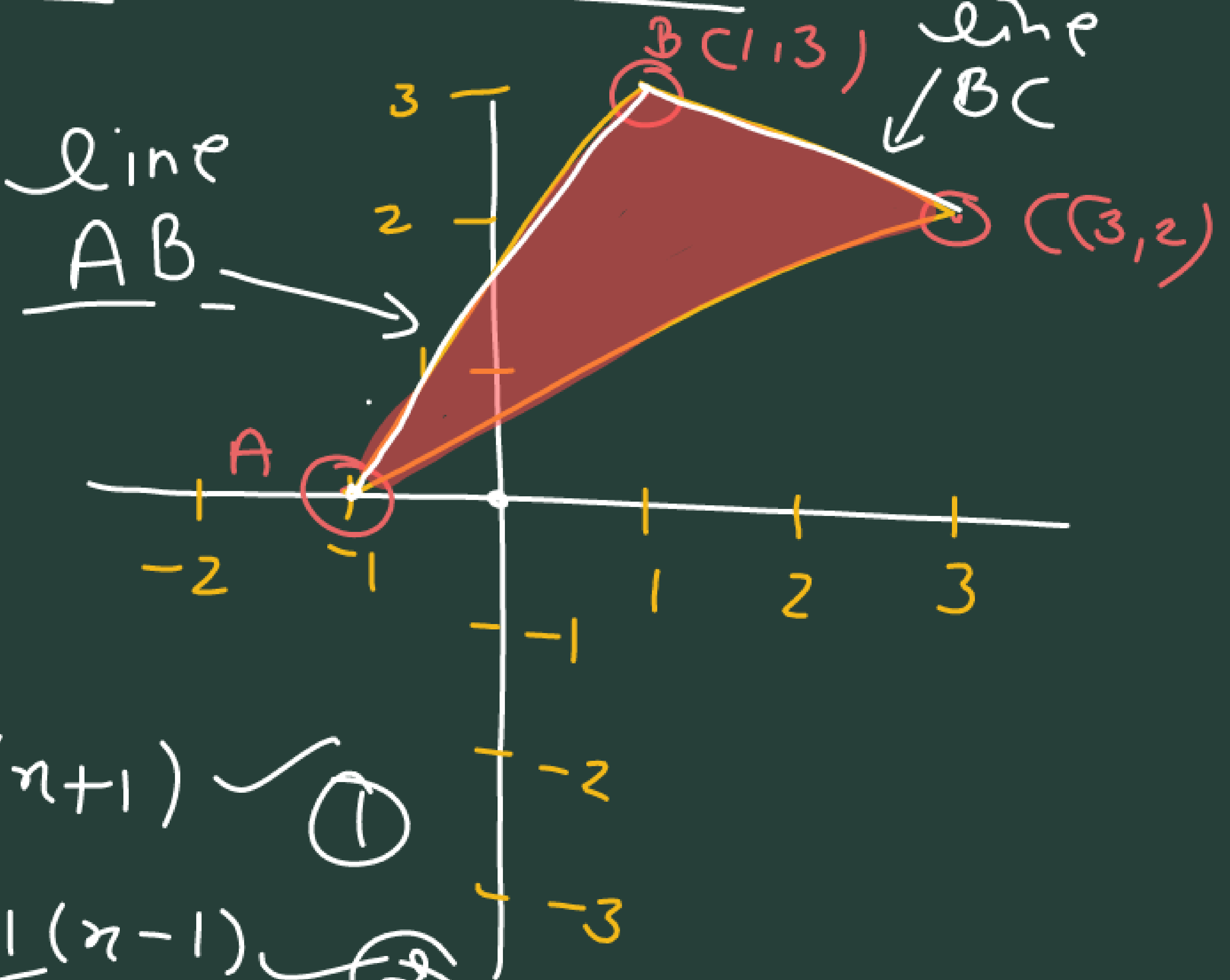
$C\ (3,2)$

$\because$ line AB $\Rightarrow \left[(y-y_1) = \overbrace{\frac{y_2-y_1}{x_2-x_1}}^{m}(x-x_1)\right]$

$AB = (y-0) = \frac{3-0}{1+1}(x+1) \Rightarrow y = \frac{3}{2}(x+1)$ — (1)

$\Rightarrow$ line BC $= (y-3) = \frac{2-3}{3-1}(x-1) \Rightarrow y = 3 - \frac{1}{2}(x-1)$ — (2)

$\rightarrow$ line AC $= (y-0) = \frac{2-0}{3+1}(x+1) \Rightarrow y = \frac{2}{4}(x+1)$ — (3)



So area bounded by Δ is:-

$\rightarrow$ area $\Rightarrow$ $\int_{-1}^{1}$ line AB $y \cdot dx$ + $\int_{+1}^{3}$ line BC $-$ $\int_{-1}^{3}$ line AC

$$A = \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3}\left(\frac{7}{2} - \frac{x}{2}\right)dx - \int_{-1}^{3} \frac{1}{2}(x+1) \cdot dx$$

$$A = \frac{3}{2}\left[\left(\frac{x^2}{2} + x\right]_{-1}^{1} + \left[\left(\frac{7}{2}x - \frac{x^2}{4}\right)_{1}^{3} - \frac{1}{2}\left(\frac{x^2}{2} + x\right)_{-1}^{3}$$

$$= \frac{3}{2}\left[\cancel{\frac{1}{2}} + 1 - \cancel{\frac{1}{2}} + 1\right] + \left(\frac{21}{2} - \frac{9}{4} - \frac{7}{2} + \frac{1}{4}\right) - \frac{1}{2}\left(\frac{9}{2} + 3 - \frac{1}{2} - 1\right)$$

$$= 3$$

Qus 1 – Using Integration find area of (Δ) region whose side have eq. $y = 2x + 1$ (1), $y = 3x + 1$, $x = 4$.

Sol^n :-

(1) → $y = 2x + 1$ →

x	y	
0	1	– A
1	3	– B

(2) :– $y = 3x + 1$

x	y	
0	1	C
1	4	→ D

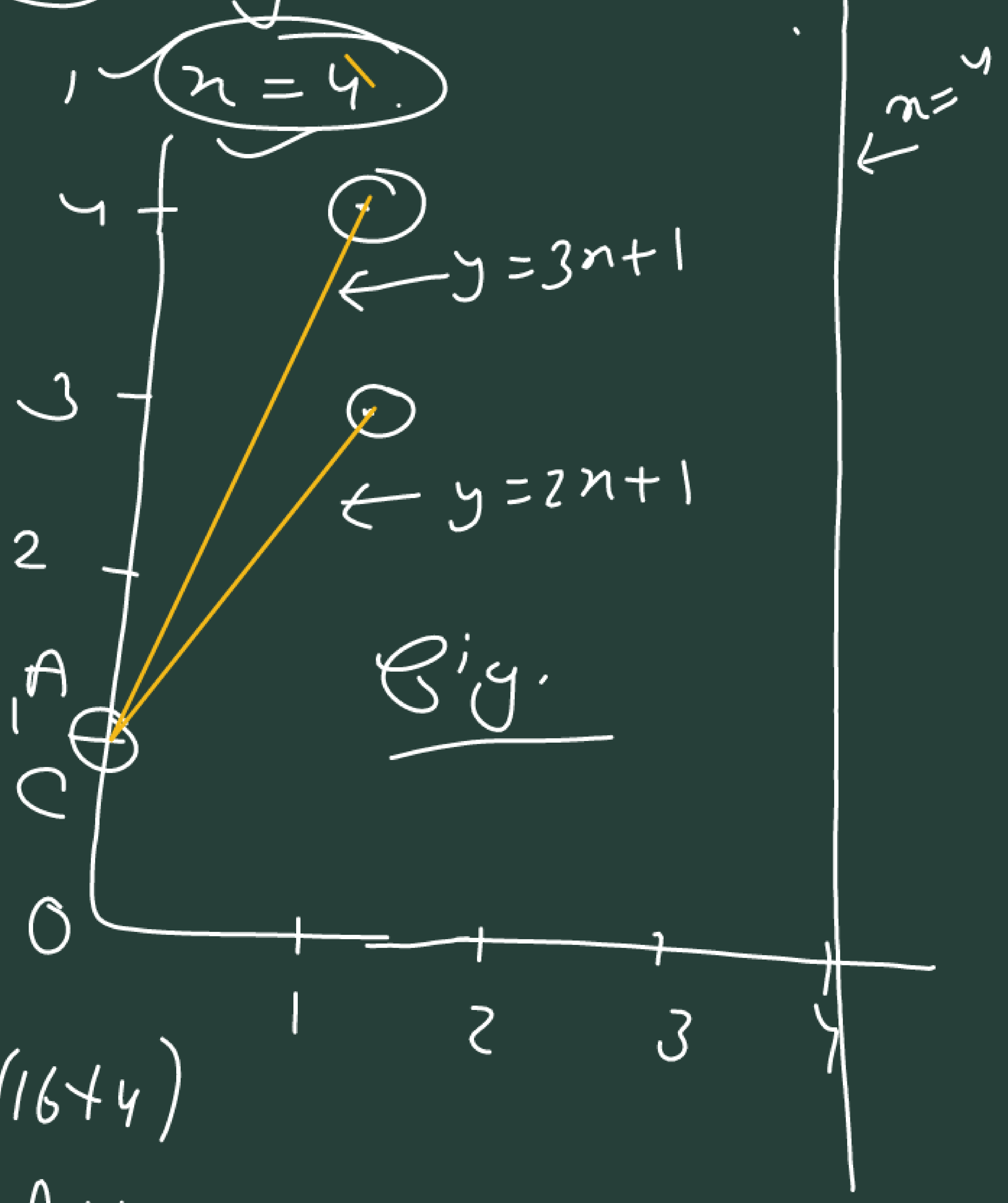


fig.

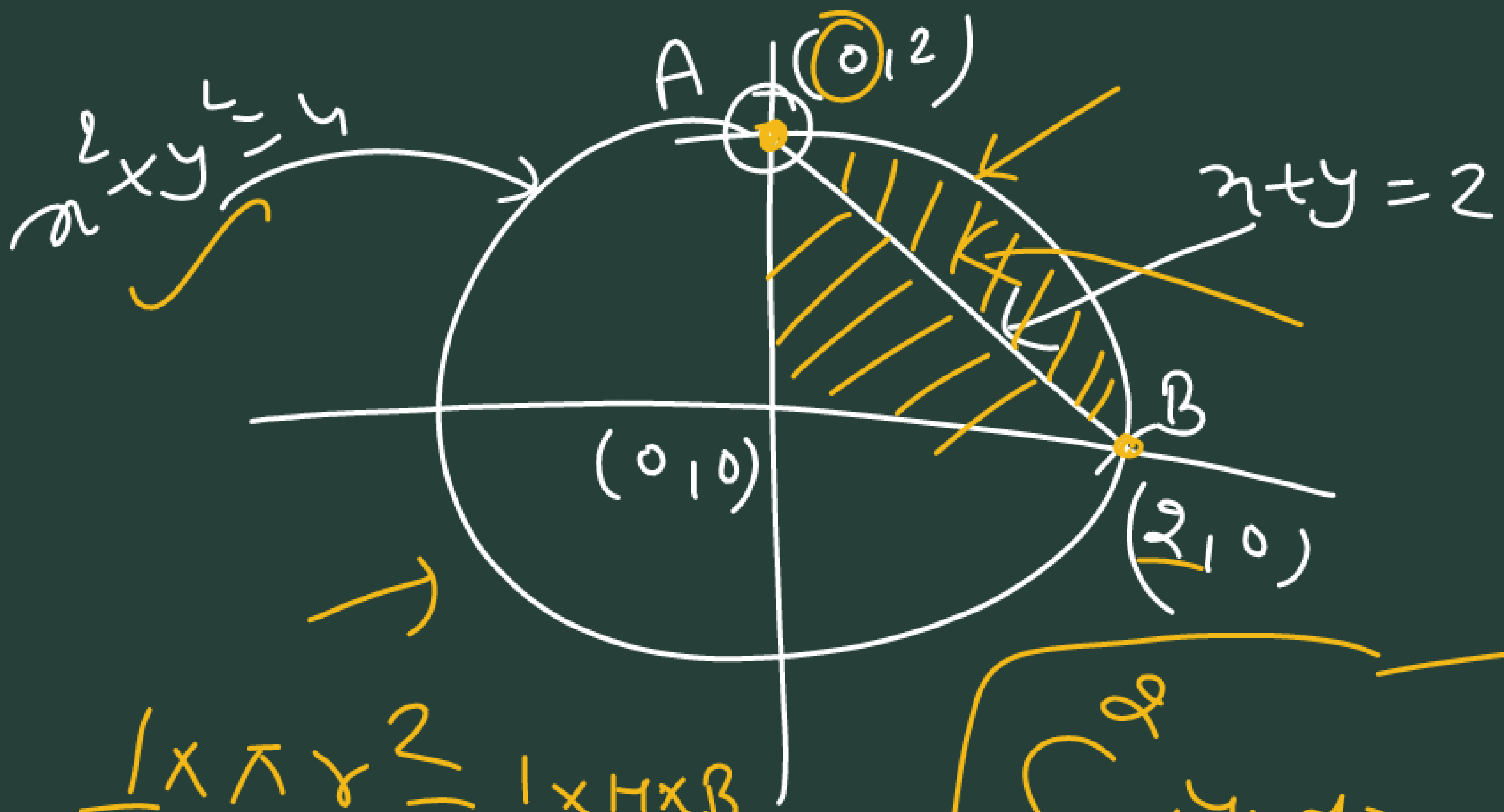
So : from fig → Area $= \int_0^4 \underset{eq.(2)}{y \cdot dx} - \int_0^4 \underset{eq(1)}{y\, dx}$

$$\Rightarrow \left[\int (3x+1) \cdot dx - \int (2x+1) \cdot dx\right]_0^4$$

$$\Rightarrow \left[\left(\frac{3x^2}{2} + x\right)_0^4 - \left(\frac{2x^2}{2} + x\right)_0^4 \Rightarrow (24+4) - (16+4)\right.$$

8 A//

Ques:- $x^2+y^2=4$ & line $x+y=2$



$x=0 \Rightarrow y=2 \rightarrow (0,2)$ A

$y=0 \rightarrow x=2 \rightarrow (2,0)$ B

$\frac{1}{4}\times\pi r^2 - \frac{1}{2}\times H\times B$

$\pi-2$

$$\int_0^2 y\cdot dx_{\text{circle}} - \int_0^2 \frac{y\cdot dx}{\text{line}}$$

Q. $y^2 = 4x$ & $y = 2x$

Q. Find area under the given curves & given lines:-

(1) $y = x^2$, $x = 1$, $x = 2$ & x-axis ✓

Q. find area b/w the curve $y = x$ & $y = x^2$

Q. Find area in I^{st} quar. & bounded by → $y = 4x^2$, $x = 0$, laying $y = 1$ & $y = 4$.

Sol^n:- $\boxed{y = 4x^2} \rightarrow x = \sqrt{\frac{y}{4}} = \frac{\sqrt{y}}{2}$

⇒ So from figure:-

$$\text{Area} = \int_1^4 x \cdot dy \rightarrow \text{(parabola)}$$

$$= \int_1^4 \frac{\sqrt{y}}{2} \cdot dy \Rightarrow \frac{1}{2} \times \frac{2}{3}\left[(y)^{3/2}\right]_1^4$$

$$= \frac{1}{3}(8-1) = \frac{7}{3}$$