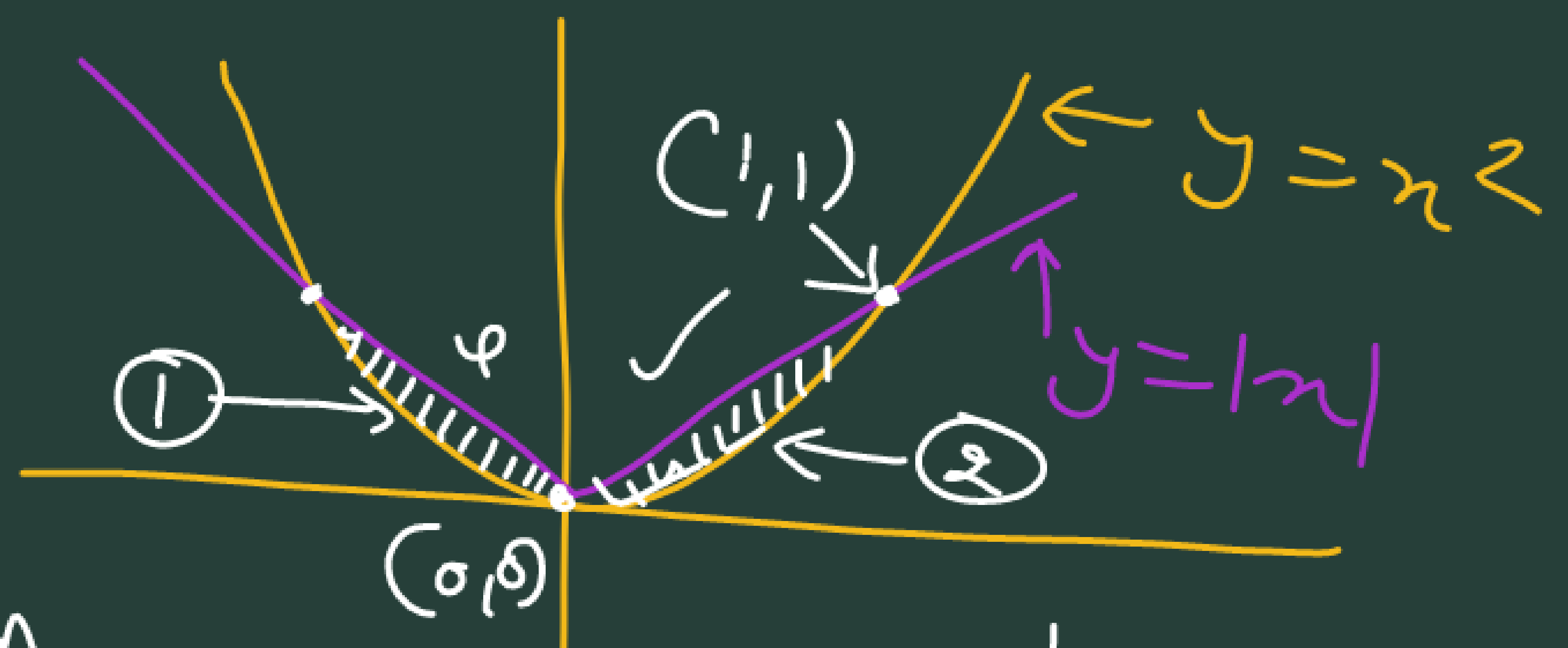
Q. Find the area of region bounded by parabola $\boxed{y = x^2}$ & the line $\boxed{y = |x|}$



Sol^n:- parabola $\rightarrow$ $y = x^2$ — ①

line $\rightarrow$ $y = |x|$ $\Rightarrow$ $\boxed{y = x}$ — ⑪

$\because$ intersection point:- $y = x$ in eq ①

$\Rightarrow x = x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\Rightarrow$ $x = 0$ $x = 1$ $\Rightarrow$ when $x = 1$, $y = 1$

So Area $= 2\left[\int_0^1 \text{line} - \int_0^1 \text{parabola}\right]$

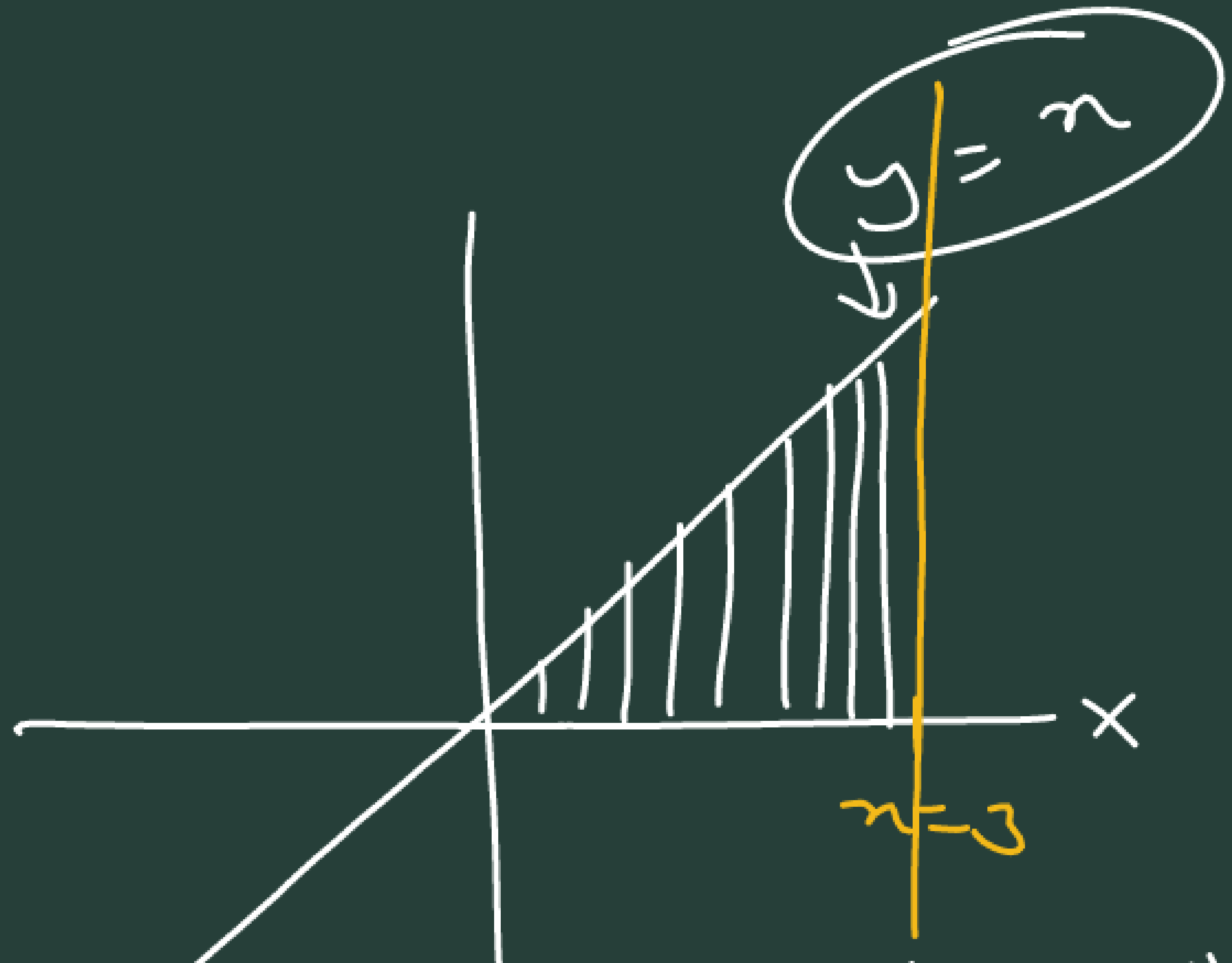
$A = 2\left[\int_0^1 y\cdot dx_{(\text{line})} - \int_0^1 y\cdot dx_{(\text{P.})}\right]$

$A = 2\left[\int_0^1 x\cdot dx - \int_0^1 x^2\cdot dx\right]$

$A = 2\left[\left(\frac{x^2}{2}\right)_0^1 - \left(\frac{x^3}{3}\right)_0^1\right]$

$A = \frac{\cancel{2}}{\cancel{2}}[1-0] - \frac{2}{3}(1-0)$

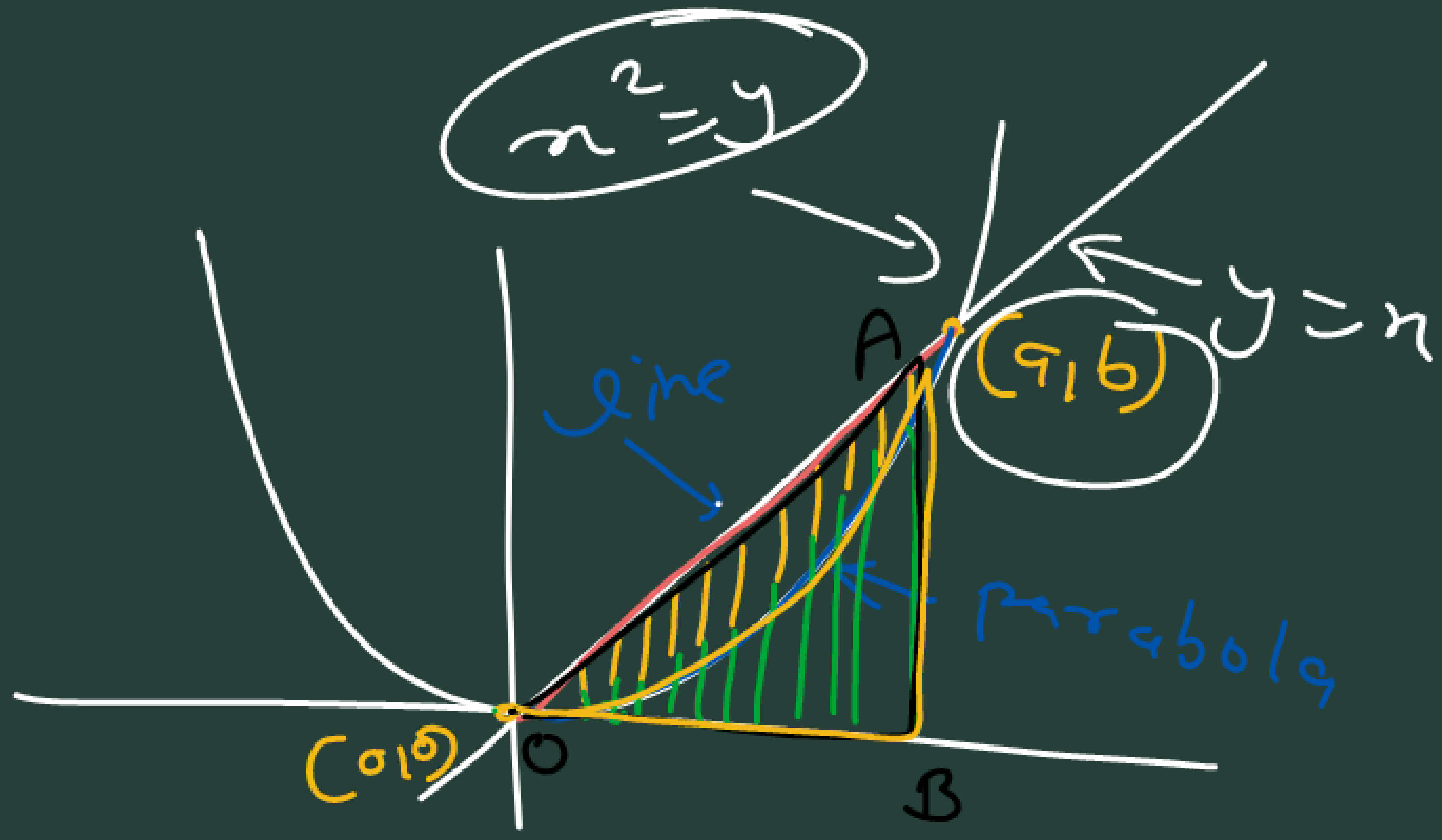
$A = 1 - \frac{2}{3} = \frac{1}{3}$ Ans



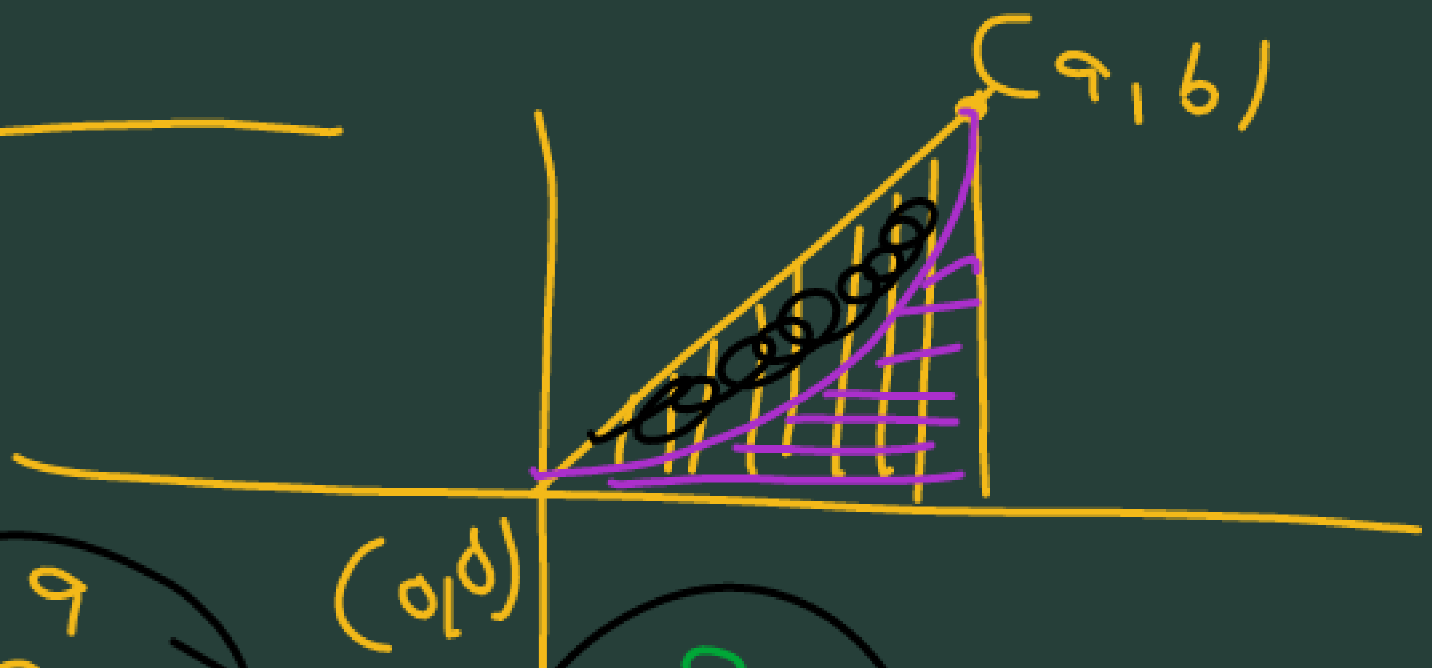
x-axis above

$y = x$

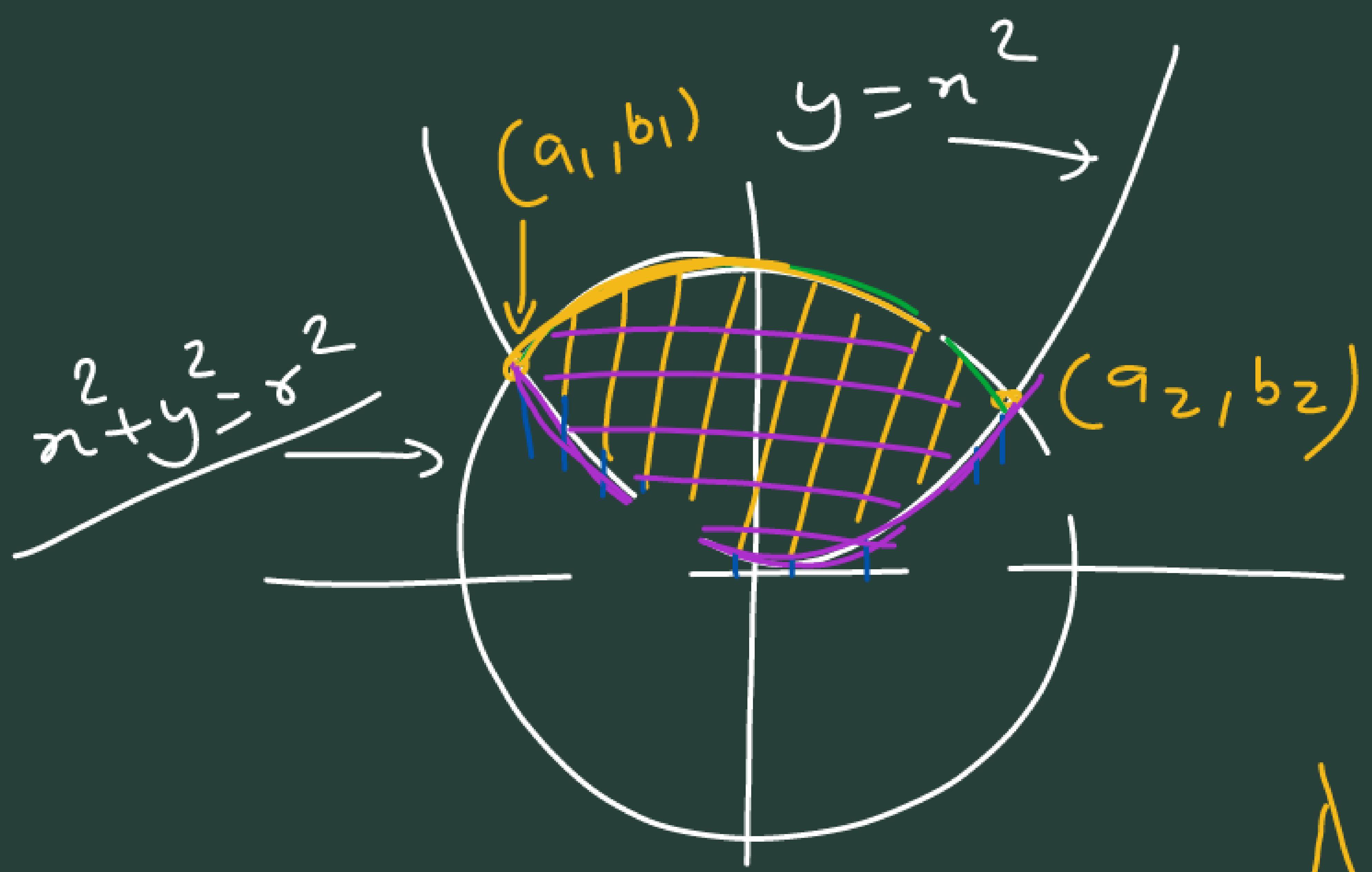
& $x = 3$



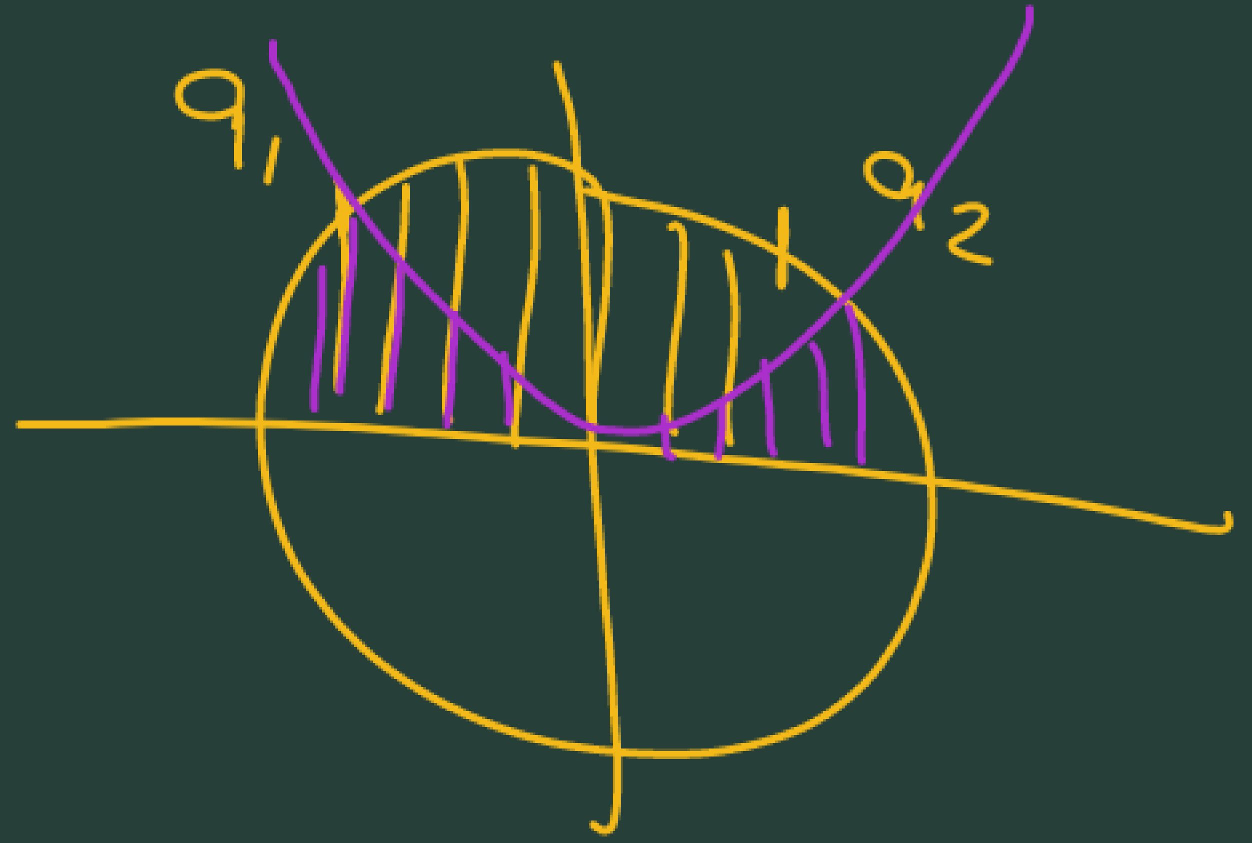
OAB –



$\int_0^a y \cdot dx$ (line) $-$ $\int_0^a y \cdot dx$ (parabola)



Circle Area — Parabola Area



Q Find Area bounded by $x^2 = 4y$ & line $x = 4y - 2$

Sol^n:- $\because$ $x^2 = 4y$ — (1) & line $x = 4y - 2$ — (2)

$y = x^2/4$ $\qquad$ $y = \frac{x+2}{4}$

$x = 0 \Rightarrow y = 1/2$

$y = 0 \Rightarrow x = -2$

→ So intersection point:-

⇒ from (1) & (2) ...

→ $(4y-2)^2 = 4y \Rightarrow 16y^2 + 4 - \underline{2 \times 4y \times 2} = 4y$

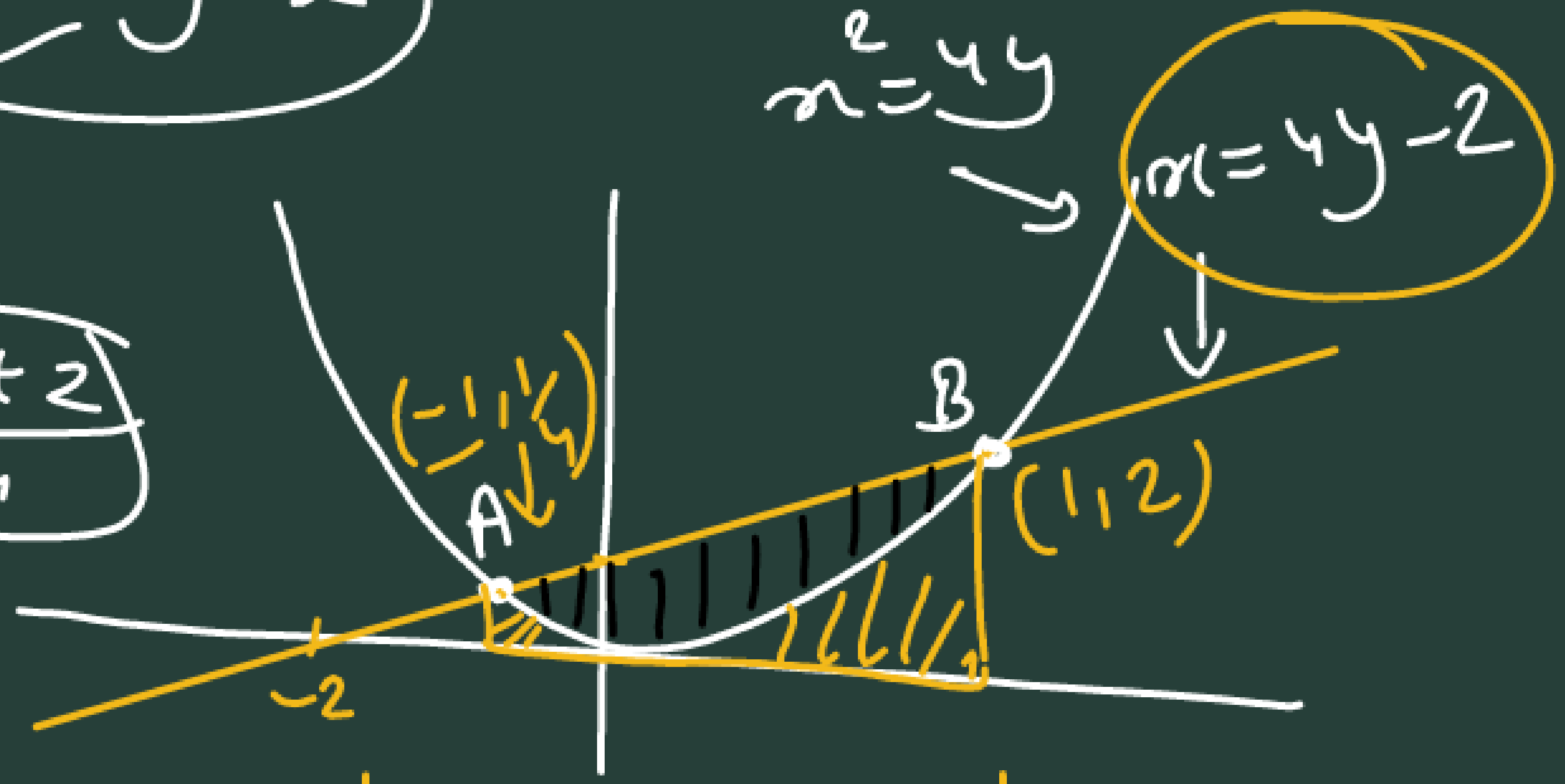
→ $16y^2 - 16y - 4y + 4 = 0 \Rightarrow 16y^2 - 20y + 4 = 0$

⇒ $4y^2 - 5y + 1 = 0 \Rightarrow \underline{4y^2 - 4y} - \underline{y + 1} = 0$

⇒ $4y(y-1) - 1(y-1) = 0$

$y = 1$ & $y = 1/4$

So ⇒ $x = 2$ & $x = -1$



So Area $= \int_{-1}^{1} \text{ar(line)} - \int_{-1}^{1} \text{area(parabola)}$

$A = \int_{-1}^{1} y \cdot dx_{\text{(line)}} - \int_{-1}^{1} y \cdot dx_{\text{(parabola)}}$

$A = \int_{-1}^{1} \left(\frac{x+2}{4}\right) \cdot dx - \int_{-1}^{1} \frac{x^2}{4} \cdot dx$

$A =$

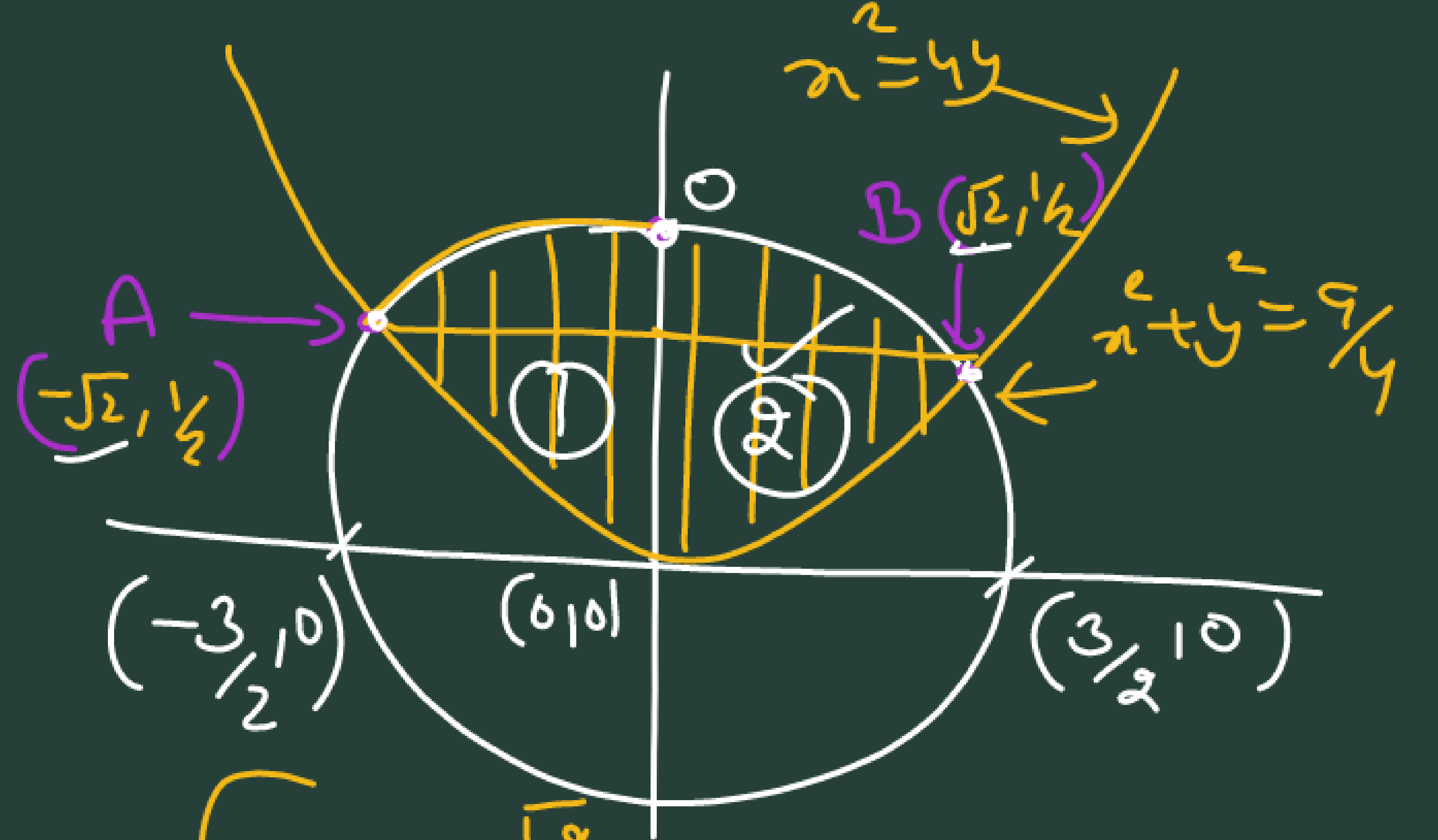
Ques:- Find area of circle $4x^2+4y^2=9$, which is interior to the $x^2=4y$.

Soln.:- circle:- $4x^2+4y^2=9$ — (1)

$y=\sqrt{\frac{9}{4}-x^2} \leftarrow x^2+y^2=9/4$ — (II)

$y=\sqrt{\frac{9-4x^2}{4}}$ So: $\boxed{r=3/2}$

& parabola:- $\boxed{x^2=4y}$ — (3)



→ so intersection point:- from (1) & (2):-

$\Rightarrow 4(4y)+4y^2=9 \Rightarrow 4y^2+16y-9=0$

$\Rightarrow 4y^2+18y-2y-9=0 \Rightarrow 2y(2y+9)-1(2y+9)$

$(2y+9)(2y-1)=0$

$y=\frac{-9}{2} \Rightarrow x=\varphi$ $\quad y=\frac{1}{2} \rightarrow x^2=4\times\frac{1}{2}=2$

$\therefore \left[x=\pm\sqrt{2}\right]$

So $A=(-\sqrt{2},1/2)$

$B=(\sqrt{2},1/2)$

So $\Rightarrow A = \left[\int_{-\sqrt{2}}^{\sqrt{2}} \text{cir.} - \int_{-\sqrt{2}}^{\sqrt{2}} \text{parabola}\right]$

Area $= 2\left[\int_0^{\sqrt{2}} y \cdot dx_{(cir.)} - \int_0^{\sqrt{2}} y \cdot dx_{(para.)}\right]$

$x^2 = 4y$

$\frac{9}{2}\sin^{-1}\left(-\frac{2\sqrt{2}}{3}\right)$

$$A = 2\left[\int_0^{\sqrt{2}} \left(\frac{1}{2}\sqrt{9-4x^2}\right) dx - \int_0^{\sqrt{2}} \frac{x^2}{4} \cdot dx\right]$$

$$A = 2\left[\frac{1}{2}\left[\left(\frac{2x}{2}\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right)_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}\right]\right]$$

$$= \cancel{2} \times \frac{1}{\cancel{2}}\left[\sqrt{2}(1) + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right) - \frac{2}{12}\left[(\sqrt{2})^3 - 0\right]$$

$\frac{2\sqrt{2}}{3} +$

$$\Rightarrow \sqrt{2} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{1}{\cancel{6}\,3}\cancel{2}\sqrt{2} \Rightarrow \left(\sqrt{2} - \frac{\sqrt{2}}{3}\right) + \frac{9}{2}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$