App. of Integration

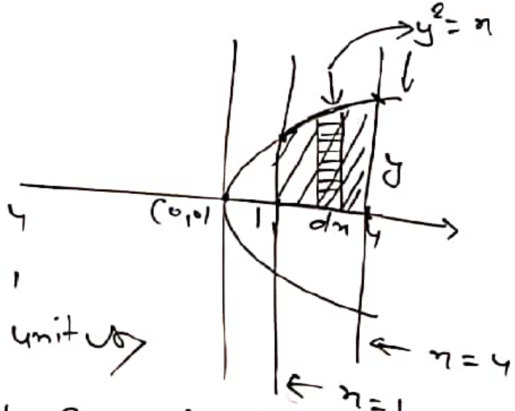
Ques: Find area of region bounded by $y^2 = x$, lines $x = 1$, $x = 4$ & x-axis in I^{st} Quar.

Sol^n:- $y^2 = x$ — (1) $\quad x = 1$ — (2)

$x = 4$ — (3)

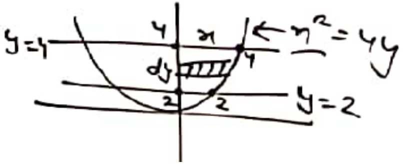
So area $= \int_1^4 \underset{(\text{curve})}{y} \cdot dx = \int_1^4 \sqrt{x} \cdot dx = \left[x^{3/2} \times \frac{2}{3} \right]_1^4$

$= \frac{2}{3}\left[(4)^{3/2} - (1)^{3/2} \right] = \frac{2}{3} \times [8 - 1] = 14/3$ sq. units



Q. Find area of region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ & y axis in I^{st} Qua.

$\int_2^4 x \cdot dy$.

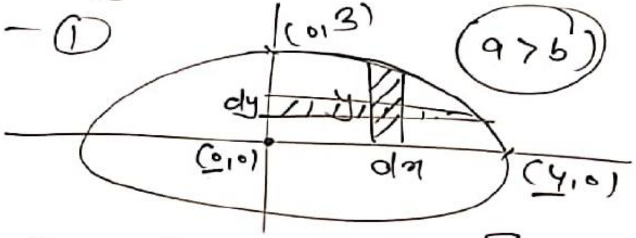


App. of Integration

Ques:- find area of region bounded by ellipse $\left(\dfrac{x^2}{16}+\dfrac{y^2}{9}=1\right)$, $\left[\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\right]$

Sol^n:- $\because$ the given ellipse is:- $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$ — ①

so the area bounded by ellipse is:—



$$A \Rightarrow 4\times\int_0^4 y\cdot dx$$

from eq ①:- $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1 \Rightarrow \dfrac{y^2}{9}=1-\dfrac{x^2}{16}=\dfrac{16-x^2}{16} \Rightarrow y^2=\dfrac{9}{16}(16-x^2) \Rightarrow \left[y=\dfrac{3}{4}\sqrt{16-x^2}\right]$

$$\text{so } A=\cancel{4}\times\int_0^4\frac{3}{\cancel{4}}\sqrt{16-x^2}\cdot dx = 3\int_0^4\sqrt{(4)^2-x^2}\cdot dx = 3\left(\frac{x}{4}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right)_0^4$$

$$A=3\left[0+\cancel{8}^{4}\times\frac{\pi}{\cancel{2}}-(0+0)\right]=3\times4\pi=\underline{12\pi} \text{ sq. unit}$$

$$\int_0^3 x\cdot dy$$

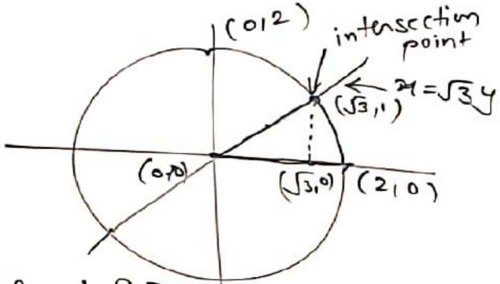
App. of Integration

Qus:- Find the area of Region in I^{st} Qua. enclosed by x-axis, line $x=\sqrt{3}y$ & the circle $x^2+y^2=4$.

Sol^n:- Circle : $x^2+y^2=4$ — ① $\Rightarrow \therefore \boxed{r=2}$

$\Rightarrow y^2 = 4-x^2 \Rightarrow \boxed{y=\sqrt{4-x^2}}$

line :- $\boxed{x=\sqrt{3}y}$ — ② & x-axis.



→ for intersection point :- from eq^n ① & ② :-

$x^2+y^2=4$, $x=\sqrt{3}y \Rightarrow (\sqrt{3}y)^2+y^2=4 \Rightarrow 3y^2+y^2=4$

$\Rightarrow y^2=1 \Rightarrow \boxed{y=\pm1} \rightarrow I^{st}$ Qua $\rightarrow y=1$

put $y=1$ in eq^n ② :- $x=\sqrt{3}\times1 \Rightarrow x=\sqrt{3}$

So Required Area from Graph = area under the line + area under the circle

$= \int_0^{\sqrt{3}} y\cdot dx\,(\text{line}) + \int_{\sqrt{3}}^{2} y\cdot dx\,(\text{Circle}) \Rightarrow A = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx + \int_{\sqrt{3}}^{2}\sqrt{4-x^2}\cdot dx$

$A = \left[\frac{1}{\sqrt{3}}\cdot\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$

$A = \frac{1}{2\sqrt{3}}[3] + \left[0+\cancel{2}\cdot\frac{\pi}{\cancel{2}} - \left(\frac{\sqrt{3}}{2}\cdot + 2\cdot\frac{\pi}{3}\right)\right]$

$A = \frac{\sqrt{3}}{2} + \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$

$A = \pi - \frac{2\pi}{3} = \pi/3$ sq.

App. of Integration

Ques:- Find the area of smaller part of circle $x^2+y^2=a^2$ cut off by line :- $x=\frac{a}{\sqrt{2}}$.

Soln:- Circle :- $x^2+y^2=a^2 \Rightarrow \therefore r=a$

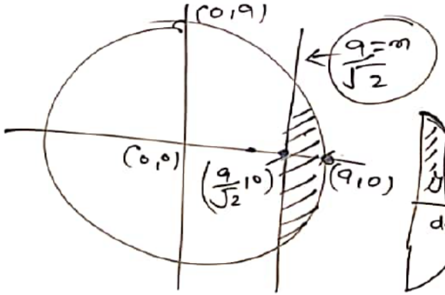
$y=\sqrt{a^2-x^2}$

→ line :- $x=\frac{a}{\sqrt{2}}$

$\frac{a^2\pi}{4}-\frac{a^2\pi}{8}$

$a^2\pi\left[\frac{1}{4}-\frac{1}{8}\right]$

$\frac{2-1}{8}=\frac{1}{8}$



So! Smaller area bounded by line & circle is,

$\sqrt{a^2-\frac{a^2}{2}}=\sqrt{\frac{a^2}{2}}$ Area $= 2\int_{a/\sqrt{2}}^{a} y\cdot dx = 2\int_{a/\sqrt{2}}^{a}\sqrt{a^2-x^2}\cdot dx$

$A=2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\cdot\sin^{-1}\frac{x}{a}\right]_{a/\sqrt{2}}^{a} \Rightarrow 2\left[0+\frac{a^2}{2}\cdot\frac{\pi}{2}-\left(\frac{a}{2\sqrt{2}}\cdot\frac{a}{\sqrt{2}}+\frac{a^2}{2}\cdot\frac{\pi}{4}\right)\right]$

$\Rightarrow 2\left(\frac{a^2\pi}{4}-\frac{a^2}{4}-\frac{a^2\pi}{8}\right) \Rightarrow 2\left(\frac{1}{8}a^2\pi-\frac{a^2}{4}\right)$

$\Rightarrow \frac{2}{8}a^2\left[\pi-2\right]=\frac{1}{4}a^2(\pi-2)$ Ans

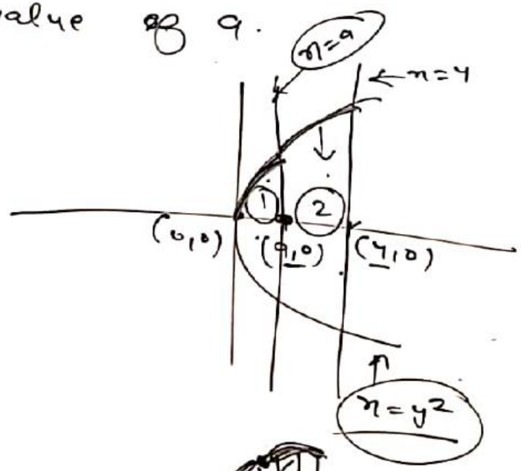
$\Rightarrow \frac{1}{2}a^2\left(\frac{\pi}{2}-1\right)$ Ans

App. of Integration

Ques:- The area b/w $x = y^2$ & $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

Sol^n:- $x = y^2 \rightarrow$ parabola

$x = 4 \rightarrow$ line & $x = a$



from given fig. $\rightarrow$ area of (I) part = area of (II) part.

$$2\int_0^a y \cdot dx = 2\int_a^4 y \cdot dx$$

$$\left[2\int_0^a \sqrt{x} \cdot dx = 2\int_a^4 \sqrt{x} \cdot dx\right]$$

$$a = (4)^{2/3}$$