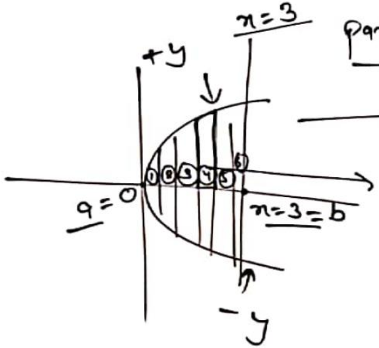
Application of Integration

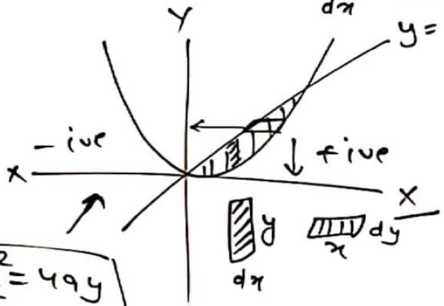


parabola:-

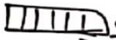
$y^2 = 4ax$

$y \cdot dx$

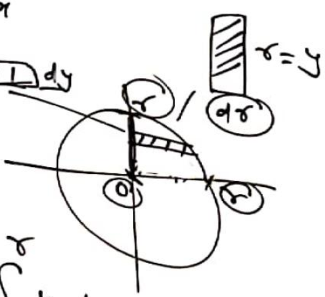
$$\text{Area} \Rightarrow \int_a^b y \cdot dx$$



$\boxed{x^2 = 4ay}$



$\Rightarrow x^2 + y^2 = r^2$



$\int_0^r x\, dy$

$\int_0^r y \cdot dx =$

$\int_0^r x \cdot dx$

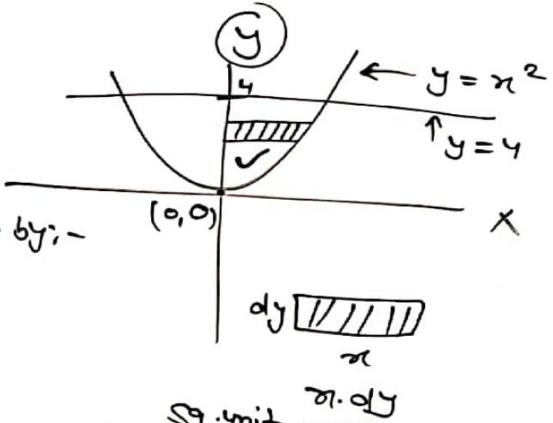
Application of Integration

Ques: Find the area of Region bounded by the curve $\boxed{y=x^2}$ & the line $y=4$.

Soln:- ∵ parabola :- $\boxed{y=x^2} \longleftrightarrow x^2=4ay$

$\boxed{x=\sqrt{y}}$

& line → $y=4$



∵ Area bounded by curve $y=x^2$ & $y=4$ is given by:-

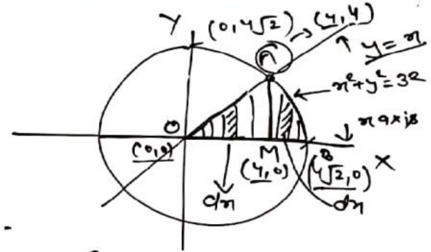
$$A = 2\int_0^4 x\,dy = 2\int_0^4 \sqrt{y}\,dy = 2\left[\frac{y^{1/2+1}}{1/2+1}\right]_0^4$$

$$A = 2\times\frac{2}{3}\times\left[y^{3/2}\right]_0^4 = \frac{4}{3}\times\left((4)^{3/2}-0\right) = \frac{4}{3}\times 8 = \frac{32}{3} \text{ sq. unit}$$

Ans //

Application of Integration

Ques: Find the area of region in I^{st} quar. enclosed by $\boxed{x\text{-axis}}$, $\boxed{\text{line } y=x}$ & the circle $\boxed{x^2+y^2=32.}$



Sol^n:- $\because$ circle $\rightarrow$ $x^2+y^2=32 \Rightarrow \boxed{r=4\sqrt{2}}$

$\Rightarrow y^2=32-x^2 \Rightarrow \boxed{y=\sqrt{32-x^2}}$

& line: $\boxed{y=x}$

So, area bounded by x-axis, $y=x$ & circle $x^2+y^2=32$ is:-

$A=$ area(OAMO) + area(MABM)

$\because$ intersection point of circle & $y=x$.

$x^2+y^2=32$, $y=x$

$x^2+x^2=32$

$2x^2=32 \Rightarrow x^2=16$

$x=\pm 4$

$$A=\underbrace{\int_0^4 y\,dx}_{\text{line}}+\underbrace{\int_4^{4\sqrt{2}} y\,dx}_{\text{circle}} \Rightarrow \left[A=\int_0^4 x\,dx+\int_4^{4\sqrt{2}}\overset{\sqrt{a^2-x^2}}{\sqrt{32-x^2}}\right]$$

$$A=\left[\frac{x^2}{2}\right]_0^4+\left[\frac{x}{2}\sqrt{32-x^2}+\frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}} \Rightarrow 8+\left[\left(0+16\cdot\frac{\pi}{2}\right)-\left(\frac{4}{2}\times 4+\frac{32}{2}\cdot\frac{\pi}{4}\right)\right]$$

$$=\cancel{8}+8\pi-\cancel{8}-4\pi=4\pi \text{ Ans}$$