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ii (True) $\displaystyle\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx = \frac{22}{7}-\pi$ $\left[\frac{132}{42}-\pi\right] = \boxed{\frac{22}{7}-\pi}$ A

168, 42, 6, 216, 84, 132, 42 ; 56, 28, 84

Soln:- $(1-x)^4 = \left[(1-x)^2\right]^2 = (1+x^2-2x)^2 = (1+x^2-2x)(1+x^2-2x)$

$(1-x)^4 = 1+x^2-2x+x^2+x^4-2x^3-2x-2x^3+4x^2 = x^4-4x^3+6x^2-4x+1$

So $I = \displaystyle\int_0^1 \frac{x^4\left[x^4-4x^3+6x^2-4x+1\right]}{1+x^2}\,dx = \int_0^1 \frac{x^8-4x^7+6x^6-4x^5+x^4}{1+x^2}\,dx$

So:

$$
\begin{array}{l}
1+x^2\,\big)\; x^8-4x^7+6x^6-4x^5+x^4 \;\big(\, x^6-4x^5+5x^4-4x^2+4 \\
\quad \cancel{x^8} \qquad\qquad +x^6 \\
\quad - \qquad\qquad\quad - \\
\hline
\quad -4x^7+5x^6-4x^5+x^4 \\
\quad -4x^7 \qquad\quad -4x^5 \\
\quad + \qquad\qquad\quad + \\
\hline
\quad 5x^6+x^4 \\
\quad 5x^6+5x^4 \\
\quad - \qquad - \\
\hline
\quad -4x^4 \\
\quad -4x^4 \quad -4x^2 \\
\quad + \qquad\quad + \\
\hline
\quad 4x^2 \\
\quad 4x^2+4 \\
\quad - \quad - \\
\hline
\quad -4
\end{array}
$$

So: $\displaystyle\int_0^1 \left(x^6-4x^5+5x^4-4x^2+4\right)+\frac{(-4)}{1+x^2}\,dx$

$= \left[\frac{x^7}{7}-\frac{4x^6}{6}+\frac{5x^5}{5}-\frac{4x^3}{3}+4x\right]_0^1 - 4\left(\tan^{-1}x\right)_0^1$

$= \left[\frac{1}{7}-\frac{4}{6}+1-\frac{4}{3}+4\right]-4\left[\tan^{-1}1-\tan^{-1}0\right]$

$= \frac{6-28+42-56+168}{42} - 4\cdot\frac{\pi}{4}$

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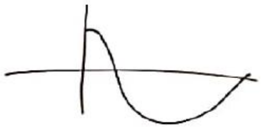
(iii) $\int_{-1}^{1}\frac{|x+2|}{x+2}\,dx = 2 \Rightarrow \because \underline{-1\le x\le 1}$ in which $\rightarrow |x+2| = (x+2)$

$I=\int_{-1}^{1}\frac{\cancel{x+2}}{\cancel{x+2}}\,dx = [x]_{-1}^{1} = [1-(-1)] = 2$ H.P.



Ques:- The no. of Triplets (x, y, z) satisfying $\sin^{-1}x+\sin^{-1}y+\cos^{-1}z = 2\pi$ is

(A) 1 (B) 0 (C) 2 (D) ∞

Sol:- $\underline{\sin^{-1}x} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \in \sin^{-1}y$ $\qquad \underbrace{\frac{\pi}{2}+\frac{\pi}{2}}_{\pi}+\pi = 2\pi$

$\cos^{-1}z \in [0, \pi]$ $\quad \therefore \boxed{x=1}$ & $\boxed{y=1}$ & $\boxed{z=-1}$

Q. Range of $\boxed{\sin^{-1}x-\cos^{-1}x} = ?$ $\Rightarrow \left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$

(A) $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left[-\frac{5\pi}{3}, \frac{\pi}{3}\right]$ (C) $\left[-\frac{3\pi}{2}, \pi\right]$ (D) $[0, \pi]$

Sol:- $\boxed{\sin^{-1}x+\cos^{-1}x} - 2\cos^{-1}x \Rightarrow \frac{\pi}{2} - 2\cos^{-1}x = \frac{\pi}{2} - 2(0) = \frac{\pi}{2}$ | $\frac{\pi}{2} - 2(\pi) = \frac{\pi}{2} - \frac{2\pi}{1} = -\frac{3\pi}{2}$

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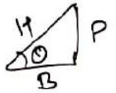
Q. The value of x where $x > 0$ → $\left[\tan\left(\sec^{-1}\frac{1}{x}\right) = \sin(\tan^{-1}2)\right]$ is



(A) $\sqrt{5}$ (B) $\sqrt{5}/3$ (C) 1 (D) $2/3$

Sol → $\tan\left(\sec^{-1}\frac{1}{x}\right) = \sin(\tan^{-1}2)$

$\left[\because \sec^{-1}\frac{1}{x} = t \Rightarrow \sec t = \frac{1}{x} = \frac{H}{B} \Rightarrow P = \sqrt{1-x^2}\right.$

$\left.\tan t = \frac{P}{B} = \frac{\sqrt{1-x^2}}{x} \Rightarrow t = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right]$

$\Rightarrow \cancel{\tan}\left[\cancel{\tan^{-1}}\left(\frac{\sqrt{1-x^2}}{x}\right)\right] = \cancel{\sin}\left[\cancel{\sin^{-1}}\frac{2}{\sqrt{5}}\right]$

$\left[\because \tan^{-1}2 \Rightarrow t \Rightarrow \tan t = \frac{2}{1} = \frac{P}{B} \Rightarrow H = \sqrt{4+1} = \sqrt{5}\right.$

$\left.\sin t = \frac{2}{\sqrt{5}} \Rightarrow t = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right]$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$

$\Rightarrow \frac{1}{x^2} - 1 = \frac{4}{5} \Rightarrow \frac{1}{x^2} = \frac{4}{5} + 1 = \frac{9}{5} \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}$

Que if $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$; $0 \le x \le 1$, then the smallest interval in which θ lies is given by:

(A) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ B = $\left[-\frac{\pi}{4}, 0\right]$ (C) $\left[0, \frac{\pi}{4}\right]$ (D) $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

Solⁿ: $\sin^{-1}x + \cos^{-1}x - \tan^{-1}x \Rightarrow \frac{\pi}{2} - \tan^{-1}x$

$\left[\because 0 \le x \le 1 \Rightarrow \tan^{-1}(0) \,|\, \tan^{-1}(1)\right.$ $\left(0, \frac{\pi}{4}\right)$

so $\tan^{-1}x \Rightarrow \left(0, \frac{\pi}{4}\right) \Rightarrow \theta = \left(\frac{\pi}{2} - 0\right)\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

$\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

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Ques: if $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi/2$ then $x+y+z = ?$

Soln: $\cot^{-1}x + \cot^{-1}y = \pi/2 - \cot^{-1}z$

$[\tan^{-1}\theta + \cot^{-1}\theta = \pi/2]$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}z$

$\Rightarrow \cancel{\tan^{-1}}\left[\frac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{x}\cdot\frac{1}{y}}\right] = \cancel{\tan^{-1}}z$

$\Rightarrow \frac{\frac{x+y}{xy}}{\frac{xy-1}{xy}} = z \Rightarrow x+y = z(xy-1) \Rightarrow x+y = xyz - z$

$\Rightarrow x+y+z = xyz$ Ans

Ques: The eq. $\boxed{2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}}$ has.

(A) No Sol. (B) One Sol. (C) Two Sol. (D) Three Solutions.

$\Rightarrow 2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6} \Rightarrow \cos^{-1}x + \cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6} \Rightarrow \cos^{-1}x = \frac{11\pi}{6} - \frac{\pi}{2} = \frac{11\pi - 3\pi}{6} = \frac{8\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{8\pi}{6} = \frac{4\pi}{3} \rightarrow (0, \pi)$