

Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}} dx = \frac{2}{3} g \circ f(x) + C$, then

(a) $f(x) = \sqrt{x}$

$x^3 = (x^{3/2})^2$

~~(b)~~ $f(x) = x^{3/2}$ and $g(x) = \sin^{-1}x$

(c) $f(x) = x^{2/3}$

(d) None of these

$x^{3/2} = t$
 $\frac{3}{2} x^{1/2} dx = dt \cdot \frac{2}{3}$

$\Rightarrow I = \int \frac{dt x^{2/3}}{\sqrt{1-t^2}} \Rightarrow \left(\frac{2}{3} \right) \frac{\sin^{-1}(x^{3/2})}{x^{3/2}} + C$
 $f(x) = x^{3/2}$
 $g(x) = \sin^{-1}x$

$\int_{\log 1/2}^{\log 2} \sin \left\{ \frac{e^x - 1}{e^x + 1} \right\} dx$ equals $f(x)$

(a) $\cos \frac{1}{3}$

(b) $\sin \frac{1}{2}$

(c) $2 \cos 2$

~~(d) 0~~

Odd function

$\Rightarrow f(-x) = \sin \left(\frac{e^{-x} - 1}{e^{-x} + 1} \right)$

$= \sin \left(\frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1} \right) = \sin \left(\frac{1 - e^x}{1 + e^x} \right) = - \sin \left(\frac{e^x - 1}{1 + e^x} \right)$

$f(-x) = -f(x) \Rightarrow \int = 0$ ✓

Evaluate: $\int_0^{\pi/2} \frac{\cos x}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^3} dx$

$\cos \frac{\pi}{2} + \sin \frac{\pi}{2} = t$
 $n=0 \Rightarrow t=1 \Rightarrow n=\frac{\pi}{2} \Rightarrow t=\sqrt{2}$
 $\int_1^{\sqrt{2}} \left[-\frac{2}{t}\right] dt = -2 \left[\frac{1}{\sqrt{2}} - \frac{1}{1}\right]$

- (a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
 (c) $3 + \sqrt{3}$ (d) $3 - \sqrt{3}$

$\frac{(\cos^2 \frac{n}{2}) - (\sin^2 \frac{n}{2})}{\left(\cos \frac{n}{2} + \sin \frac{n}{2}\right)^2} = \frac{\cos^2 \frac{n}{2} - \sin^2 \frac{n}{2}}{\left(\cos \frac{n}{2} + \sin \frac{n}{2}\right)^2} \cdot dn$

$\Rightarrow \int \frac{1}{t^2} \cdot 2 dt = 2 \frac{t^{-2+1}}{-2+1} = \frac{2 \cdot \frac{1}{t}}{-1} = \frac{-2}{\cos \frac{n}{2} + \sin \frac{n}{2}}$

$-\frac{2}{\sqrt{2}} + 2$
 $2 - \sqrt{2}$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$ is

(a) $1/2$

~~(b) $3/2$~~

(c) 2

(d) 1

$$\rightarrow \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-(6-x)} + \sqrt{9-x}} dx = \int_3^6 \frac{\sqrt{9-x} + \sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx$$

$$\Rightarrow [x]_3^6$$

$$6 - 3 = \frac{3}{2}$$

$\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$ equal to

$n^3 = \frac{a^3}{(a^{3/2})^2}$

(a) $\frac{1}{3} \sin^{-1} \sqrt{\frac{x^3}{a^3}} + C$

(b) $\frac{2}{3} \sin^{-1} \sqrt{\frac{x^3}{a^3}} + C$

(c) $\frac{2}{3} \sin^{-1} \sqrt{\frac{x}{a}} + C$

(d) None of these

\Rightarrow Let $(x^{3/2})^2 \Rightarrow x^{3/2} = t = \frac{1}{2} \frac{dx}{dt} = \frac{dx}{2}$

$\frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} \rightarrow \frac{2}{3} \left[\sin^{-1} \frac{t}{a^{3/2}} \right]$

If $\int \sin^3 x \cos^5 x \, dx = A \sin^4 x + B \sin^6 x +$

$C \sin^8 x + D$. Then

$\int \sin^3 x \cos^5 x \cdot \cos x \cdot dx$
 $\sin x = t \rightarrow \cos x \cdot dx = dt$

$A = 1/4$ $C = 1/8$

(a) $A = \frac{1}{4}, B = -\frac{1}{3}, C = \frac{1}{8}, D \in \mathbb{R}$

$B = -1/3$

(b) $A = \frac{1}{8}, B = \frac{1}{4}, C = \frac{1}{3}, D \in \mathbb{R}$

(c) $A = 0, B = -\frac{1}{6}, C = \frac{1}{8}, D \in \mathbb{R}$

(d) None of these.

$\int t^3 \cdot (1+t-2t^2) dt$
 $\frac{t^{3+1}}{4} + \frac{t^{7+1}}{8} - 2 \cdot \frac{t^{5+1}}{6} = \frac{\sin^4 x}{4} + \frac{\sin^8 x}{8} - \frac{6t}{3}$

Value of $\int_0^4 \frac{1}{\sqrt{x^2 + 2x + 3}}$ dx is \rightarrow Simplify

(a) $\log\left(\frac{1 + \sqrt{3}}{5 + 3\sqrt{3}}\right)$

(b) $\log\left(\frac{5 - 3\sqrt{3}}{1 - \sqrt{3}}\right)$

~~(c) $\log\left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}}\right)$~~

(d) None of these

$$\frac{(x^2 + 2x + 1) - (1)^2 + 3}{(x+1)^2 + (\sqrt{2})^2 + (\sqrt{2})^2}$$

Consider the following statements

Statement-I :

The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is $\frac{22}{7} - \pi$.

Statement-II :

The value of integral $\int_{-1}^1 \frac{|x+2|}{x+2} dx$ is 2.

- (a) Statement I is true
- (b) Statement II is true
- (c) Both statements are true
- (d) Both statements are false

Handwritten red annotations and calculations:

For Statement I, the integral is transformed using the substitution $x = \frac{n-1}{n}$. The integrand becomes $\frac{(n-1)^4 (1-(n-1)/n)^4}{1+(n-1)^2/n^2}$. The denominator is simplified to $\frac{n^2 - 2n + 1 + n^2}{n^2} = \frac{2n^2 - 2n + 1}{n^2}$. The numerator is $\frac{(n-1)^4 (1/n)^4}{n^2} = \frac{(n-1)^4}{n^6}$. The integral becomes $\int_0^1 \frac{(n-1)^4}{n^6} \cdot \frac{n^2}{2n^2 - 2n + 1} \cdot \frac{1}{n} dn$. This is further simplified to $\int_0^1 \frac{(n^2-1)((1-n)^2)^2 + \frac{(1-2n+n^2)^2}{1+n^2}}{1+n^2} dn$.

For Statement II, the integral is split into two parts: $\int_{-1}^0 \frac{|x+2|}{x+2} dx + \int_0^1 \frac{|x+2|}{x+2} dx$. The first part is $\int_{-1}^0 \frac{-(x+2)}{x+2} dx = \int_{-1}^0 -1 dx = 1$. The second part is $\int_0^1 \frac{x+2}{x+2} dx = \int_0^1 1 dx = 1$. The total value is 2.

If m is an integer, then $\int_0^\pi \frac{\sin(2m\pi x) \cos A \sin B}{\sin x} dx$ is $\sin(2m\pi - 2m\pi)$

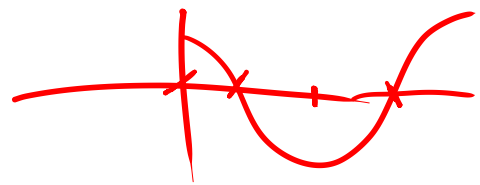
equal to :

- (a) 1
- (b) 2
- (c) 0
- (d) π

$$\int_0^\pi \frac{\sin(2m\pi) - \sin(2m\pi)}{\sin x} = \int_0^\pi \frac{0 - \sin(2m\pi)}{\sin x} = \frac{0 - \sin(2m\pi)}{-\sin(2m\pi)}$$

$$I = -I = 2I = 0$$

$I = 0$



1	2	3	4	5	6	7	8	9	10
B	D	A	B	B	A	C	C	A	C