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Ques:- if $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x-1}\right) = \pi/4$ → find the value x.?

$\left[\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta\right]$

Sol^n:- $\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x-1}}{\frac{1}{1}-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x-1}\right)}\right] = \pi/4 \Rightarrow \left[\frac{\frac{(x-1)^2+(x+1)(x-2)}{\cancel{(x-2)}(x-1)}}{\frac{(x-2)(x-1)-(x-1)(x+1)}{\cancel{(x-2)}(x-1)}}\right] = \tan \pi/4$

$\Rightarrow \left[\frac{x^2+1-2x+x^2-2x+x-2}{x^2-x-2x+2-(\cancel{x^2}-1)}\right] = 1 \Rightarrow \left[\frac{2x^2-3x-1}{-3x+3} = 1\right] \Rightarrow 2x^2-\cancel{3x}-1 = -\cancel{3x}+3$

$\Rightarrow 2x^2 = 3+1 \Rightarrow 2x^2 = 4$

$x^2 = 2 \Rightarrow \boxed{x = \pm\sqrt{2}}$ Ans

Ques:- prove: $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \pi/4$

Ques:- $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ ⟶ RHS $\Rightarrow \frac{1}{2}\cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] = \frac{1}{2}\cancel{\cos^{-1}}(\cancel{\cos}\,2\theta) = \frac{1}{\cancel{2}}\times\cancel{2}\theta$

put $x = \tan^2\theta \Rightarrow \tan\theta = \sqrt{x} \Rightarrow [\theta = \tan^{-1}\sqrt{x}]$ $\Rightarrow$ $\theta = \tan^{-1}\sqrt{x}$ = LHS

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$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Ques:- prove $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \dfrac{x}{2}$

$1+\cos x$

$1-\cos x$

$\sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$ (a^2, b^2, $2ab$)

$\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}$

$(a+b)^2$

$\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}$

$\Rightarrow \cot^{-1}\left[\dfrac{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}+\sqrt{\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}-\left(\sqrt{\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}\right)}\right] = \dfrac{x}{2}$

$\Rightarrow \cot^{-1}\left[\dfrac{\cancel{2}\cos\frac{x}{2}}{\cancel{2}\sin\frac{x}{2}}\right] = \dfrac{x}{2} \Rightarrow \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2} \Rightarrow \frac{x}{2} = \frac{x}{2}$ H.P.

Q. $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$

Put $x = \cos\theta \Rightarrow \left[\dfrac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right] = \dfrac{\sqrt{2\cos^2\theta/2}-\sqrt{2\sin^2\theta/2}}{2\cos\theta/2 + 2\sin\theta/2} = \dfrac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$

$\boxed{\cos^{-1}x} = \theta$

$\cos\theta/2 \rightarrow$ Divide

LHS

$\Rightarrow \tan^{-1}\left[\dfrac{1-\tan\theta/2}{1+1\cdot\tan\theta/2}\right] = \tan^{-1}(1) - \tan^{-1}(\tan\theta/2)$

$= \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$ H.P.

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H.W. $\left[\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x\right]$

prove:- $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

H.W → $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{(x-y)}{(x+y)}=?$

LHS ⇒ $\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right]=\frac{9}{4}\times\left[\cos^{-1}\frac{1}{3}\right]=\frac{9}{4}\times\sin^{-1}\frac{2\sqrt{2}}{3}$ = RHS

let :- $\boxed{\cos^{-1}\frac{1}{3}\Rightarrow x}\Rightarrow\cos x=\frac{1}{3}\Rightarrow\sin x=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

$\Rightarrow x=\sin^{-1}\frac{2\sqrt{2}}{3}$

Solve:-

Q. $\boxed{2\tan^{-1}(\cos x)}=\tan^{-1}(2\operatorname{cosec}x)$

LHS → ∵ $2\tan^{-1}x\Rightarrow\tan^{-1}\frac{2x}{1-x^2}\Rightarrow 2\tan^{-1}(\cos x)=\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}\left[\frac{2\cos x}{\sin^2 x}\right]$

so:- $\cancel{\tan}^{-1}\left[\frac{\cancel{2}\cos x}{\sin^2 x}\right]=\cancel{\tan}^{-1}(\cancel{2}\operatorname{cosec}x)\Rightarrow\frac{\cos x}{\sin^{\cancel{2}} x}=\frac{1}{\cancel{\sin x}}\Rightarrow\frac{\cos x}{\sin x}=1\Rightarrow\cot x=1$

$x=\pi/4$ Ans

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Ques:- $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$ → find $x = ?$ $= \frac{1}{2}$

Sol^n:- $-2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x) = \cos^{-1}(1-x)$

$\Rightarrow \left[-2\sin^{-1}x = \cos^{-1}(1-x)\right]$ — (1)

let $\boxed{\sin^{-1}x} = t \Rightarrow [\sin t = x]$ — (2) $\Rightarrow$ so $\cos t = \sqrt{1-\sin^2 t} = \sqrt{1-x^2} \Rightarrow \boxed{t = \cos^{-1}(\sqrt{1-x^2})}$

$\Rightarrow \left[\sin^{-1}x = \cos^{-1}(\sqrt{1-x^2})\right]$ — (3)

from eq. (1) (2) & (3):-

$\Rightarrow -2\cos^{-1}(\sqrt{1-\sin^2 t}) = \cos^{-1}(1-\sin t)$

$\Rightarrow -2\,\cancel{\cos^{-1}}(\cancel{\cos} t) = \cos^{-1}(1-\sin t)$

$\Rightarrow -2t = \cos^{-1}(1-\sin t)$

$\Rightarrow \cos(-2t) = 1-\sin t$

$= \cos 2t = 1-\sin t$

$\cancel{1} - 2\sin^2 t = \cancel{1} - \sin t$

$\Rightarrow 2\sin^2 t - \sin t = 0$

$\Rightarrow \sin t[2\sin t - 1] = 0$

$\sin t = 0$ | $2\sin t = 1$

$t = 0$ | $\sin t = \frac{1}{2} = x$

| $t = \frac{\pi}{6}$

so: $x = \sin 0 = 0$

$x = 0$