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ITF

Ques:- find value of $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$

$\because \sin^{-1}\left(\frac{2x}{1+x^2}\right) \Rightarrow$ let $x = \tan\theta$ $\Rightarrow \sin^{-1}\left(\frac{2\cdot\tan\theta}{1+\tan^2\theta}\right)$

$\theta = \tan^{-1}x$

$\Rightarrow \sin^{-1}(\sin 2\theta) = 2\theta = \boxed{2\cdot\tan^{-1}x}$

$\because \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) \Rightarrow$ let $y = \tan\phi$ $\Rightarrow \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right) = \cancel{\cos^{-1}}(\cancel{\cos}2\phi)$

$\phi = \tan^{-1}y$

$\Rightarrow 2\phi = \boxed{2\cdot\tan^{-1}y}$

$\therefore \tan\frac{1}{2}\left[2\tan^{-1}x + 2\tan^{-1}y\right] = \tan\left[\tan^{-1}x + \tan^{-1}y\right] = \cancel{\tan}\left[\cancel{\tan^{-1}}\left(\frac{x+y}{1-xy}\right)\right]$

$\left(\frac{x+y}{1-xy}\right)$ Ans

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Q. if $\sin\left(\underset{A}{\sin^{-1}\frac{1}{5}} + \underset{B}{\cos^{-1}x}\right) = 1$ Find x

$\because \sin(A+B) = \sin A\cos B + \cos A\cdot\sin B$

$\therefore \cancel{\sin}\left(\cancel{\sin^{-1}}\frac{1}{5}\right)\cdot\cancel{\cos}\left(\cancel{\cos^{-1}}x\right) + \cos\left(\sin^{-1}\frac{1}{5}\right)\cdot\sin\left(\cos^{-1}x\right) = 1$

$\Rightarrow \frac{1}{5}\times x + \boxed{\cos\left(\sin^{-1}\frac{1}{5}\right)}\cdot\boxed{\sin\left(\cos^{-1}x\right)} = 1 \Rightarrow$ (1)

$\because \cos\left(\sin^{-1}\frac{1}{5}\right) \Rightarrow$ let $\boxed{\sin^{-1}\frac{1}{5} = y} \Rightarrow \sin y = \frac{1}{5} \Rightarrow \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\frac{1}{25}} = \sqrt{\frac{24}{25}}$

$\Rightarrow \cos y = \frac{\sqrt{24}}{5} \Rightarrow \boxed{y = \cos^{-1}\left(\frac{\sqrt{24}}{5}\right)} \Rightarrow \therefore \boxed{y = \sin^{-1}\frac{1}{5} = \cos^{-1}\frac{\sqrt{24}}{5}}$

So: $\cos\left(\sin^{-1}\frac{1}{5}\right) = \cancel{\cos}\left(\cancel{\cos^{-1}}\frac{\sqrt{24}}{5}\right) = \boxed{\frac{\sqrt{24}}{5}}$ — (2)

Now $\sin(\cos^{-1}x) \Rightarrow$ let $\boxed{\cos^{-1}x = t} \Rightarrow \cos t = x \Rightarrow \therefore \sin t = \sqrt{1-\cos^2 t} = \sqrt{1-x^2}$

So $\boxed{t = \sin^{-1}\left(\sqrt{1-x^2}\right)} \Rightarrow \therefore t = \cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right)$

So: $\sin(\cos^{-1}x) = \cancel{\sin}\left(\cancel{\sin^{-1}}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$ — (3)

Put value from eq (2) & (3) in eq. (1)

$\Rightarrow \frac{x}{5} + \frac{\sqrt{24}}{5}\times\sqrt{1-x^2} = 1$

$\Rightarrow \sqrt{24}\cdot\sqrt{1-x^2} = \left(1-\frac{x}{5}\right)5$

$\Rightarrow \left(\sqrt{24}\cdot\sqrt{1-x^2}\right) = 5-x$

Square :-

$24(1-x^2) = (5-x)^2$

$24 - 24x^2 = 25 + x^2 - 10x$

$\Rightarrow +24x^2 + x^2 - 10x + 25 - 24 = 0$

$\Rightarrow 25x^2 - 10x + 1 = 0$

$\Rightarrow 25x^2 - 5x - 5x + 1 = 0$

$5x(5x-1) - 1(5x-1) = 0$

$(5x-1)(5x-1) = 0$

$\boxed{x = 1/5}$ Ans

ITF

Ques:- if $\tan^{-1}\frac{(x-1)}{(x-2)} + \tan^{-1}\frac{(x+1)}{(x-1)} = \pi/4$, find value of x.

Ques:- $\sin^{-1}\left(\sin 2\pi/3\right) = ?$ $= 2\pi/3$ $\longrightarrow$ $\sin^{-1} \longrightarrow (-\pi/2, \pi/2)$

$\Rightarrow \sin^{-1}\left[\sin\left(\pi - \pi/3\right)\right] \Rightarrow \sin^{-1}\left[\sin \underset{120^\circ}{\pi/3}\right] = \pi/3$ Ans

II

Ques: $\tan^{-1}\left(\tan 3\pi/4\right)$

$\frac{3\times 180}{4}^{45} = 135^\circ$ ← $180-45$

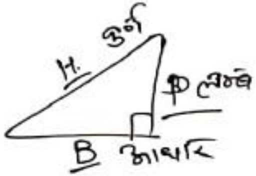
$\pi - \pi/4$

$\tan^{-1}\left[\tan\left(\pi - \pi/4\right)\right]$

$\tan^{-1}\left[-\tan\left(\pi/4\right)\right] = \tan^{-1}\left[\tan\left(-\pi/4\right)\right] = -\pi/4$ Ans

ITF

Q. $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$



Sol^n:- let $\boxed{\sin^{-1}\frac{3}{5}=x} \Rightarrow \sin x = \frac{3}{5}\frac{P}{H} \Rightarrow \cos x = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = 4/5 \Rightarrow \tan x = 3/4$

OR:- $\therefore B = \sqrt{H^2-P^2} = \sqrt{25-9} = \sqrt{16} = 4 \Rightarrow \therefore \tan x = \frac{P}{B} = 3/4$

so: $x = \tan^{-1}(3/4) = \sin^{-1}(3/5)$

Now let $\cot^{-1}\frac{3}{2} = y \Rightarrow \cos y = 3/2 \Rightarrow \tan y = 2/3 \Rightarrow y = \tan^{-1}(2/3) = \cot^{-1}(3/2)$

so: $\tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right] \Rightarrow \cancel{\tan}\left(\cancel{\tan^{-1}}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right) = \frac{\frac{9+8}{\cancel{12}}}{\frac{12-6}{\cancel{12}}} = \frac{17}{6}$ Ans

Q H.W $\rightarrow \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

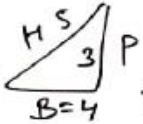
Q. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right] \Rightarrow \because \sin^{-1}\left(-\frac{1}{2}\right) = x \Rightarrow \sin x = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right) \Rightarrow x = -\frac{\pi}{6}$

$\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1$ Ans

ITF

Q H.W $\cos^{-1}\left[\cos\left(13\pi/6\right)\right]$

Q. $\tan^{-1}\left(\tan 7\pi/6\right)$



P $\sin\theta = P/H$

$\tan\theta = P/B$

Q. $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$ $\Rightarrow$ LHS $\Rightarrow$ $\boxed{\sin^{-1}\frac{3}{5} = t}$ $\Rightarrow \sin t = 3/5 \Rightarrow$ $\boxed{\tan t = \frac{3}{4}}$

$t = \tan^{-1}\frac{3}{4}$ $\Rightarrow$ so LHS $\Rightarrow 2\cdot t = \boxed{2\cdot\tan^{-1}\frac{3}{4}}$

Now LHS $\rightarrow$ $2\tan^{-1}\frac{3}{4} = \tan^{-1}\left(\frac{2\times 3/4}{1-(3/4)^2}\right) = \tan^{-1}\left[\frac{6/4}{1-9/16}\right] = \tan^{-1}\left[\frac{6/4}{7/16}\right]$

$\begin{array}{r} 289 \\ 64 \\ \hline 225 \end{array}$

$\tan^{-1}\left[\frac{6}{\cancel{4}}\times\frac{\cancel{16}\,4}{7}\right] = \tan^{-1}\left(24/7\right) =$ RHS

a

Q. $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

H.P.

H.W $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

$\because \sin^{-1}\frac{8}{17} = x$

$\sin x = \frac{8}{17} \Rightarrow \boxed{\tan x = \frac{8}{15}}$

$\sin^{-1}\frac{3}{5} = y$

$\sin y = \frac{3}{5} \Rightarrow \tan y = \frac{3}{4}$

$\Rightarrow \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4} =$