ITF

find value :- $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$\tan^{-1}(\tan \pi/4) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}(\sin \pi/6)$

$\frac{\pi}{4} + \frac{\pi}{6} + \frac{2\pi}{3} = \frac{3\pi + 2\pi + 8\pi}{12} = \frac{13\pi}{12}$ Ans

$\because \cos^{-1}\left(-\frac{1}{2}\right) = y \Rightarrow \cos y = -\frac{1}{2} = -\cos\pi/3 = \cos(\pi - \pi/3) = \cos 2\pi/3$

$y = 2\pi/3$

H.W $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Q. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) \rightarrow$ let $\sec^{-1}(-2) = y$

$\sec y = -2 = -\sec\frac{\pi}{3} = \sec(\pi - \pi/3)$

$\sec y = \sec 2\pi/3 \Rightarrow y = 2\pi/3$

Solⁿ :- $\tan^{-1}(\tan \pi/3)$

$\pi/3 - 2\pi/3 = -\pi/3$ Ans

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prove :-

Q. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

$[4\cos^3\theta - 3\cos\theta = \cos 3\theta]$

$[3\sin\theta - 4\sin^3\theta = \sin 3\theta]$

let from RHS $\rightarrow \cos^{-1}[4x^3 - 3x]$

let $x = \cos\theta \longrightarrow \cos^{-1}[4\cos^3\theta - 3\cos\theta] \Rightarrow \cos^{-1}[\cos 3\theta] = 3\theta$

$[\theta = \cos^{-1}x]$ $\Rightarrow \boxed{3\times\cos^{-1}x} = LHS$ H.P.

Q H.W $\rightarrow$ $3\sin^{-1}x = \sin^{-1}[3x - 4x^3]$

Ques: $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

$\left\{ \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right) \right.$

$\rightarrow$ LHS $\rightarrow$ $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right)$

$\Rightarrow \tan^{-1}\left[\dfrac{\frac{2}{11} + \frac{7}{24}}{\frac{1}{1} - \frac{2}{11}\times\frac{7}{24}}\right] \Rightarrow \tan^{-1}\left[\dfrac{\frac{48+77}{11\times 24}}{\frac{11\times 24 - 14}{11\times 24}}\right] = \tan^{-1}\left[\frac{125}{250}\right] = \tan^{-1}\left(\frac{1}{2}\right) = RHS$

H.P.

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Q. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$ $\Rightarrow \because\ 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $\therefore\ 2\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}$

$= \tan^{-1}\left[\frac{1}{3/4}\right]$

$= \tan^{-1}\left[\frac{4}{3}\right]$

So:- $\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$\Rightarrow \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right]$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta/2 = 1 - \cos\theta$

Q. write the fun. in simple form:

$\left[\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}\right]$ $\Rightarrow$ put $\boxed{x = \tan\theta} \rightarrow \left[\theta = \tan^{-1}x\right]$

$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right] = \tan^{-1}\left[\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right]$

$\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right] \Rightarrow \tan^{-1}\left[\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2\cdot\cos\theta/2}\right] \Rightarrow \cancel{\tan^{-1}}\left[\cancel{\tan}\ \theta/2\right] = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

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Q. write the simplest form :- $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ $1+x^2$

$\left\{\tan^{-1}\left[\frac{A-B}{1+AB}\right] = \tan^{-1}A - \tan^{-1}B\right\}$

Q. $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}}\right] = \tan^{-1}\left[\frac{\overset{A}{1} - \overset{B}{\tan x}}{\underbrace{1+1\cdot\tan x}_{A\cdot B}}\right] = \tan^{-1}(1) - \tan^{-1}(\tan x)$$

$$= \boxed{\frac{\pi}{4} - x} \text{ Ans}$$

Q. $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$ let $\boxed{x = a\sin\theta} \longrightarrow \sin\theta = \frac{x}{a} \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \tan^{-1}\left[\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right] = \tan^{-1}\left[\frac{a\sin\theta}{a\cdot\sqrt{\cos^2\theta}}\right] = \tan^{-1}\left[\frac{\cancel{a}\sin\theta}{\cancel{a}\cos\theta}\right] \Rightarrow \cancel{\tan^{-1}}\cancel{\tan}\theta$$

$$\Rightarrow \theta = \sin^{-1}\frac{x}{a} \text{ Ans}$$

ABLES KOTA

ITF

H.W. Q → $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

let $\boxed{x = \operatorname{cosec}\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

Q. $\tan^{-1}\left[\frac{3a^2x - x^3}{a^3 - 3ax^2}\right]$

$a^2x = \tan\theta$

$x^3 = \left(\frac{\tan\theta}{a^2}\right)^3$

$\left\{\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \tan 3\theta\right]$

let ⇒ $[x = a\tan\theta]$

$\tan^{-1}\left[\frac{3\cdot a^2\cdot a\tan\theta - a^3\cdot\tan^3\theta}{a^3 - 3\cdot a\cdot a^2\tan^2\theta}\right] = \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right] = 3\theta \neq 3\times\tan^{-1}x/a$

Q. $\cot\left[\tan^{-1}a + \cot^{-1}a\right] = \cot\pi/2 = 0$

Ques: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ ⇒ $2\cdot\sin^{-1}\left(\frac{1}{2}\right) = 2\sin^{-1}\left[\sin\pi/6\right] = \pi/3$

$\tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[\cancel{2}\cdot\frac{1}{\cancel{2}}\right]$

$= \tan^{-1}(1) = \tan^{-1}(\tan\pi/4) = \frac{\pi}{4}$ Ans