Inverse Trigonometry Function

$[\sin x = f(x)] \Rightarrow$ $y = \sin x$ → Domain (Angle) / Input

Result → Range → Value

$\sin x = f(x)$ → $y = \sin x$

Ex:- $\sin\left(\frac{\pi}{4}\right) = y$ (I/P, O/P)

$y = \frac{1}{\sqrt{2}}$

$\sin^{-1}(y) = x$ → angle

input (Domain) → Value

Result (Range)

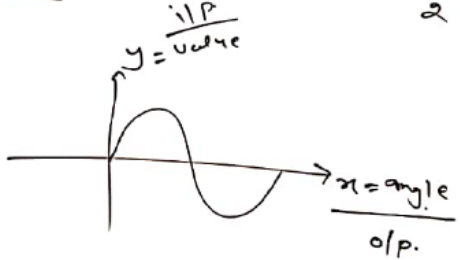
Ex:- $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = ?$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y \Rightarrow \frac{1}{\sqrt{2}} = \sin y$

$\sin\frac{\pi}{4} = \sin y \Rightarrow y = \frac{\pi}{4}$ ∴ $\sin^{-1}\frac{1}{\sqrt{2}}$

$2q = 1$

$q = \frac{1}{2} = 2^{-1}$



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Inverse Trigonometry Function

(1) $\sin^{-1}(\sin x) = \sin(\sin^{-1}x) = x$ $\{\sin^2 x = (\sin x)^2\}$

(2) $\sin^{-1}x = (\sin x)^{-1} = \frac{1}{\sin x} = \operatorname{Cosec} x$

(3) $\sin^{-1}(-x) = -\sin^{-1}x$

(4) $\operatorname{Cosec}^{-1}(-x) = -\operatorname{Cosec}^{-1}x$

(5) $\tan^{-1}(-x) = -\tan^{-1}x$

$\Rightarrow [\sin^{-1}(-x) = y = -\sin^{-1}x]$

$\Rightarrow [\sin y = -x]$

$\sin(-\theta) = -\sin\theta \Rightarrow -\sin y = x$

$\Rightarrow \sin(-y) = x$

$-y = \sin^{-1}x$

$y = -\sin^{-1}x$

(6) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

(7) $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

(8) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

(9) $\sin^{-1}x + \cos^{-1}x = \pi/2$

(10) $\tan^{-1}x + \cot^{-1}x = \pi/2$

(11) $\sec^{-1}x + \operatorname{Cosec}^{-1}x = \pi/2$

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Inverse Trigonometry Function

#12) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}(x)$

13) $\cos^{-1}(x) = \sec^{-1}(x) = \cot^{-1}(x)$

15) $\operatorname{cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1}x$

16) $\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}(x)$

17) $\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(x)$

$$1x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$$

$$\therefore \tan^{-1}y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$$

$$x - \tan^{-1}y = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$$

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Inverse Trigonometry Function

(Principle value)

Q. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$ (let) $= \pi/4$ Ans.

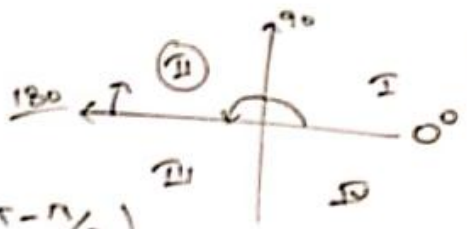
$\frac{1}{\sqrt{2}} = \sin y \Rightarrow \sin(\pi/4) = \sin y \Rightarrow y = \pi/4$

Q. $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$ (let)

$\cot y = -\frac{1}{\sqrt{3}}$

$\cot y = -\cot(\pi/3) = \cot(\pi - \pi/3)$

$\cot y = \cot(2\pi/3) \Rightarrow y = 2\pi/3$ Ans.



$\sin^{-1}x \rightarrow [-\pi/2, \pi/2]$

$\cos^{-1}x \rightarrow [0, \pi]$

[$\sin^{-1}$ $\operatorname{cosec}^{-1}$ $\tan^{-1}$

[$\cos^{-1}$ $\cot^{-1}$ $\sec^{-1}$

Inverse Trigonometry Function

Q. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

CSC $\rightarrow [0, \pi]$

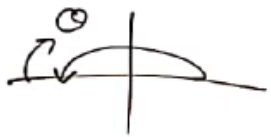
M.W $\operatorname{Cosec}^{-1}(2)$

Q. $\sin^{-1}\left(\frac{-1}{2}\right) = y$

SCT $\rightarrow \left[-\pi/2, \pi/2\right]$

$\cos(-\theta) = \cos\theta$

$\sin y = -\frac{1}{2} = -\sin(\pi/6) = \sin(-\pi/6)$



$y = -\pi/6 \rightarrow \in \left(-\frac{\pi}{2}, \pi/2\right)$

Q. $\tan^{-1}(-\sqrt{3}) = y$

$\tan y = -\sqrt{3} = -\tan\pi/3 = \tan(-\pi/3)$

Q. $\cos^{-1}\left(-\frac{1}{2}\right) = \theta = y \Rightarrow \cos y = -\frac{1}{2} = -\cos\pi/3 = \cos(\pi - \pi/3)$