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Q. $\int \frac{1}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}} \cdot dx = \int \frac{1}{\sqrt{\sin^3 x \cdot [\sin x \cdot \cos\alpha + \cos x \cdot \sin\alpha]}} dx = \int \frac{1}{\sin^2 x \sqrt{\cos\alpha + \cot x \cdot \sin\alpha}} \cdot dx$

$\int \frac{\operatorname{cosec}^2 x \cdot dx}{\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}} \to t$ (Const, Const)

$\cos\alpha + \cot x \cdot \sin\alpha = t$

Diff $\Rightarrow 0 + (-\operatorname{cosec}^2 x) dx \cdot \sin\alpha = dt \Rightarrow \operatorname{cosec}^2 x \cdot dx = \frac{dt}{-\sin\alpha}$

$\Rightarrow -\int \frac{1}{\sqrt{t}} \cdot \frac{dt}{\sin\alpha}$

$\sin^{-1}\theta = \pi/2 - \cos^{-1}\theta$

Q. $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} \cdot dx \Rightarrow \sin^{-1}\theta + \cos^{-1}\theta = \pi/2 \Rightarrow \int \frac{\frac{\pi}{2} - 2\cos^{-1}\sqrt{x}}{\pi/2} \cdot dx = \frac{2}{\pi}\left[\int \frac{\pi}{2} \cdot dx - 2\int \cos^{-1}\sqrt{x} \cdot dx\right]$

$\Rightarrow I = \frac{2}{\pi}\left[\frac{\pi}{2} \cdot x - I_1\right] \Rightarrow I_1 = \int \cos^{-1}\sqrt{x} \, dx \to \sqrt{x} = t \to \frac{1}{2\sqrt{x}} \cdot dx = dt \Rightarrow dx = 2\sqrt{x}\, dt = 2t \cdot dt$

ILATE

$2\int \underset{I}{\cos^{-1} t} \cdot \underset{II}{t} \cdot dt$

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Q. $\int \frac{\sqrt{x^2+1}\cdot[\log(x^2+1)-2\log x]}{x^4}\,dx$

$\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\quad \tan^{-1}x = \frac{1}{1+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$\Rightarrow \log(x^2+1)-\log x^2 \Rightarrow \log\left(\frac{x^2+1}{x^2}\right) = \log\left[1+\frac{1}{x^2}\right]$

$\int \frac{\sqrt{x^2+1}\cdot\log\left(\left(1+\frac{1}{x^2}\right)\right)}{x^4}\,dx = \int \frac{1}{x^4}\cdot\sqrt{x^2+1}\cdot\log\left[1+\frac{1}{x^2}\right] = \boxed{\int\frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)}$

$\frac{x^2+1}{x^2}$

$\int \frac{1}{x^3}\underbrace{\sqrt{1+\frac{1}{x^2}}}_{t}\cdot\log\underbrace{\left(1+\frac{1}{x^2}\right)}_{t}\cdot dx \quad 1+\frac{1}{x^2}=t \xrightarrow{\text{Diff}} 0-2x^{-3}\cdot dx = dt \Rightarrow -\frac{2}{x^3}\cdot dx = dt \Rightarrow \frac{dx}{x^3} = \frac{-dt}{2}$

$\sqrt{\frac{x^2+1}{x^2}} = \frac{1}{x}\sqrt{x^2+1} = \sqrt{\frac{x^2+1}{x^2}}$

$\int \sqrt{t}\cdot\log t\cdot -\frac{dt}{2} \Rightarrow -\frac{1}{2}\int \log t\cdot\sqrt{t}\cdot dt$

Q. $\int_0^{\pi/4} \frac{\sin x\cdot\cos x}{\cos^4 x+\sin^4 x}\,dx = \int_0^{\pi/4}\frac{\frac{\sin x\cdot\cos x}{\cos^4 x}}{1+\tan^4 x}\,dx = \int_0^{\pi/4}\frac{\tan x\cdot\sec^2 x\cdot dx}{1+(\underbrace{\tan^2 x}_{t})^2} = \int_0^1 \frac{1}{1+t^2}\cdot\frac{dt}{2}$

$2\tan x\cdot\sec^2 x\cdot dx = dt$

$x=0 \Rightarrow \tan^2 0 = 0 = t$

$x=\pi/4 \Rightarrow \tan^2\pi/4 = 1 = t$

$\Rightarrow \frac{1}{2}\left[\tan^{-1}t\right]_0^1 = \frac{1}{2}\left[\cancel{\tan^{-1}}\cdot\tan\frac{\pi}{4}-0\right]$

$= \pi/8$ Ans

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$$\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\cdot dx \Rightarrow \int_0^{\pi/2} \frac{1}{1+4\cdot\tan^2 x}\cdot dx = \int_0^{\pi/2} \frac{1}{1+(2\tan x)^2}\cdot dx$$

$$\int \frac{1}{a^2+x^2}\cdot dx \Rightarrow \left[\frac{1}{1}\cdot \tan^{-1}(2\tan x)\right]_0^{\pi/2}$$

$$\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\cdot dx = \int \frac{\cos^2 x}{\cos^2 x + 4(1-\cos^2 x)}\cdot dx = \int \frac{\cos^2 x}{4-3\cos^2 x}\cdot dx = \frac{-1}{3}\int \frac{-3\cos^2 x}{4-3\cos^2 x}\cdot dx$$

$$I \Rightarrow -\frac{1}{3}\left[\int \frac{4-3\cos^2 x}{4-3\cos^2 x}\cdot dx - \underbrace{\int \frac{4}{4-3\cos^2 x}\cdot dx}_{I_1}\right] \Rightarrow I_1 = -4\int \frac{1}{4-3\cos^2 x}\cdot dx = \int \frac{1}{4-3\cdot\frac{1}{\sec^2 x}}\cdot dx$$

$$I_1 = \int \frac{\sec^2 x\cdot dx}{4\sec^2 x - 3} = \int \frac{\sec^2 x\cdot dx}{4(1+\tan^2 x)-3}\cdot dx$$

$$I_1 \Rightarrow \int \frac{\sec^2 x\cdot dx}{4\tan^2 x + 1} \Rightarrow \int \frac{dt}{4t^2+1} = \int \frac{dt}{(2t)^2+(1)^2}$$

$\tan x = t \rightarrow$

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$$\int_0^{\pi/4} \frac{\boxed{(\sin x + \cos x)\, dx}}{9 + 16 \sin 2x} \longrightarrow dt$$

$x = 0 \Rightarrow -\cos 0 + \sin 0 = t = -1$

$x = \pi/4 \Rightarrow -\cos \pi/4 + \sin \pi/4 = t = 0$

let $\Rightarrow [-\cos x + \sin x = t] \longrightarrow$ square $\longrightarrow (\sin x - \cos x)^2 = t^2$ — (1)

Diff $\Rightarrow [-(\sin x) + \cos x)]\, dx = dt$

$\rightarrow [\sin x + \cos x]\, dx = dt$ — (II)

$\rightarrow \sin^2 x + \cos^2 x - 2 \sin x \cdot \cos x = t^2$

$\Rightarrow 1 - \sin 2x = t^2$

$\Rightarrow \boxed{\sin 2x = 1 - t^2}$ — (III)

$$\rightarrow \int_{-1}^{0} \frac{dt}{9 + 16(1 - t^2)} = \int_{-1}^{0} \frac{dt}{9 + 16 - 16t^2}$$

$$\Rightarrow \int_{-1}^{0} \frac{dt}{25 - 16t^2} = \int_{-1}^{0} \frac{dt}{(5)^2 - (4t)^2}$$