$$\int x^{51}\left(\tan^{-1}x+\cot^{-1}x\right)dx = \int x^{51}\cdot\frac{\pi}{2}\cdot dx$$

$\frac{\pi}{2}$

(a) $\dfrac{x^{52}}{52}\left(\tan^{-1}x+\cot^{-1}x\right)+c$

$\dfrac{x^{52}}{52}\cdot\dfrac{\pi}{2}+C$

(b) $\dfrac{x^{52}}{52}\left(\tan^{-1}x-\cot^{-1}x\right)+c$

(c) $\dfrac{\pi x^{52}}{104}+\cancel{\dfrac{\pi}{2}}+c$

$\dfrac{x^{52}\cdot\pi}{104}+C$

(d) $\dfrac{x^{52}}{52}+\cancel{\dfrac{\pi}{2}}+c$

Evaluate: $\int 2^{2^{2^x}}\, 2^{2^x}\, 2^x\, dx$

$$\int \frac{dt}{(\log 2)^3} = \frac{1}{(\log 2)^3}\, t$$

(a) $\frac{1}{(\log 2)^3} 2^{2^{2^x}} + C$ (b) $\frac{1}{(\log 2)^3} 2^{2^x} + C$

(c) $\frac{1}{(\log 2)^2} 2^{2^x} + C$ (d) $\frac{1}{(\log 2)^4} 2^{2^{2x}} + C$

$$t = 2^{2^{2^x}}$$

$$dt = 2^{2^{2^x}} \times \log 2 \times 2^{2^x} \times \log 2 \times 2^x \times \log 2 \times 1 \cdot dx$$

$$dt = \left[2^{2^{2^x}} \times 2^{2^x} \times 2^x \times (\log 2)^3\right] \cdot dx$$

$f(x) + f'(x)$ $\quad 1+\cos x = 2\cos^2 \frac{x}{2}$

If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + C$, then f(x)

is equal to

$$\frac{1 + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

$$\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}$$

$f(x)$

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$$\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx \text{ is equal to}$$

(a) $\frac{1}{8}(x^2-1)+k$

(b) $\frac{1}{2}x^2+k$

(c) $\frac{1}{2}x+k$

(d) None of these

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$x = \cos 2\theta$

$dx = -2\sin 2\theta \cdot d\theta$

$\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}} = \tan\theta$

$\int \cos\{2\tan^{-1}\tan\theta\}\,(-2\sin 2\theta \cdot d\theta)$

$= -\int 2\,\underbrace{\sin 2\theta}_{t} \cdot \cos 2\theta \cdot d\theta = -\int t \cdot dt = -\frac{t^2}{2} + C$

$-\frac{t^2}{2} + C$

$\sqrt{1-\cos^2 2\theta}$

$\sqrt{1-x^2}$

$-\frac{(\sin 2\theta)^2}{2} + C$

$-\frac{\left(\sqrt{1-x^2}\right)^2}{2} + C$

$-\frac{1-x^2}{2} + C$

$\frac{x^2}{2} - \frac{1}{2} + C$

$\boxed{\frac{x^2}{2} + k}$

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx \text{ equals}$$

$\int \frac{dt}{\cos^2 t} = \sec^2 t$ $\tan(xe^x)+C$

$t \Rightarrow e^x + xe^x \Rightarrow e^x(1+x)$

(a) $-\cot(ex^x)+C$ (b) $\tan(xe^x)+C$

(c) $\tan(e^x)+C$ (d) $\cot(e^x)+C$

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The value of $\int \sqrt{\frac{a-x}{a+x}}\,dx$ is

$\sqrt{\frac{a-x}{a-x}} = \frac{(a-x)}{\sqrt{a^2-x^2}}$

(a) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{x^2-a^2}+C$

(b) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

(c) $a\sin^{-1}\left(\frac{a}{x}\right)+\sqrt{x^2-a^2}+C$

(d) None of these

$+\frac{1}{2}\frac{t^{1/2}}{1/2} = \sqrt{a^2-x^2}$

$x\,dx = dt/2$

$a\int\frac{1}{\sqrt{a^2-x^2}}\cdot dx - \int\frac{x\cdot dx}{\sqrt{a^2-x^2}}$ $a^2-x^2 \to t$

$a\cdot\sin^{-1}x/a + \frac{1}{2}\int\frac{1}{\sqrt{t}}\cdot dt$

$\int \sin 2x.\log \cos x\, dx$ is equal to

(a) $\cos^2 x\left(\frac{1}{2} + \log \cos x\right) + k$

(b) $\cos^2 x.\log \cos x + k$

(c) $\cos^2 x\left(\frac{1}{2} - \log \cos x\right) + k$

(d) None of these.

ILATE

$\log(\cos x) = t$

$\frac{-\sin x}{\cos x} dx = dt$

$\cos x = e^t$

$\tan x\, dx = dt$

$dx = \frac{dt}{-\tan x}$

$-\log(\cos x)$

$\int 2\sin x.\cos x . t \times \frac{dt}{-\tan x} \Rightarrow -\int 2\cos x . t \times dt\, \cos x$

$-2\int \cos^2 x . t . dt = -2\int e^{2t}.t.dt$

$-2\left[t.\frac{e^{2t}}{2} - \int \frac{e^{2t}}{2}.dt\right]$

$-2\left[-\frac{te^{2t}}{2} + \frac{e^{2t}}{4} + C\right]$

$e^{2t}\left[\frac{1}{2} - t\right] + C \rightarrow \cos^2 x\left[\frac{1}{2} - \log \cos x\right] + C$

Match the following integrals in column-I with their corresponding solutions in column-II.

Column - I	Column - II
A. $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$	1. $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec^2 2x + C$
B. $\int \tan^3 2x \sec 2x \, dx$	2. $\tan x + C$
C. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$	3. $\frac{-1}{\sin x + \cos x} + C$
D. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$	4. $\sec x - \operatorname{cosec} x + C$

tan x sec x
+ cot x . cosec x

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	3	4	1	2
(d)	2	1	4	3

$\frac{2\cos^2 x - 1}{\cos^2 x} + 2\tan^2 x$

$2 - \sec^2 x + 2(\sec^2 x - 1)dx$

$\cancel{2} + \sec^2 x - \cancel{2}$

If $\int \cos^n x \sin x \, dx = -\frac{\cos^6 x}{6} + C$, then $n =$

(a) 0 t (b) 1

(c) 2 $-\int t^n \, dt = -\frac{t^{n+1}}{n+1} + C$ (d) 5

$n = 5$

Q. if $f(a+b-x) = f(x)$ then $\int_a^b x \cdot f(x) \cdot dx$

$I = \int_a^b x \cdot f(x) \cdot dx$

$I = \int_a^b (a+b-x) \cdot f(a+b-x) \cdot dx$

$I = \int_a^b (a+b-x) \cdot f(x) \cdot dx = \int_a^b (a+b) f(x) \cdot dx - \int_a^b x \cdot f(x)\, dx$

$I = a+b \int_a^b f(x)\, dx - I \rightarrow 2I = \frac{(a+b)}{2} \int_a^b f(x)\, dx$

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If $\int \frac{3x+1}{(x-3)(x-5)}dx = \int \frac{-5}{(x-3)}dx + \int \frac{B}{(x-5)}dx$,

then the value of B is

(a) 3

(b) 4

(c) 6

(d) 8

$x=5$

$(3x+1) = -5(x-5) + B(x-3)$

$3\times5+1 = 0 + B(5-3)$

$B=8$

Q. $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) \cdot dx$

$\frac{x-(1-x)}{1+x\cdot(1-x)}$

$\to \int_0^1 \tan^{-1}\left[\frac{2x-1}{1+x(1-x)}\right] = \int_0^1 \tan^{-1}\left[\frac{x-(1-x)}{1+x(1-x)}\right]$

$\left[\tan^{-1}\left[\frac{A-B}{1+A\cdot B}\right] = \tan^{-1}A - \tan^{-1}B\right]$

$I \to \int_0^1 \left[\tan^{-1}x - \tan^{-1}(1-x)\right] dx \to$ property (1)

$I \to \int_0^1 \tan^{-1}(1-x) - \tan^{-1}[1-(1-x)] \cdot dx$

$2I = \int_0^1 0 \cdot dx = 0$ Ans

1	2	3	4	5	6	7	8	9	10
A	A	C	B	B	B	C	B	D	D