Integrals

Q. $\int_{-\pi/2}^{\pi/2} (x^3 + x\cdot\cos x + \tan^5 x + 1)\,dx.$

$\Rightarrow x^3 \Rightarrow f(x) \Rightarrow f(-x) = (-x)^3 = -x^3 = -f(x) \rightarrow \therefore f(x) = x^3$ is odd fun.

$\Rightarrow x\cdot\cos x \Rightarrow f(x) \Rightarrow f(-x) = -x\cdot\cos(-x) = -x\cos x \rightarrow -f(x) \rightarrow x\cdot\cos x$ is odd.

$\rightarrow \tan^5 x \Rightarrow f(x) \Rightarrow f(-x) = \tan^5(-x) = -\tan^5 x = -f(x) \rightarrow \tan^5 x$ is odd.

So $I = \int_{-\pi/2}^{\pi/2} 0 + 0 + 0 + 1\cdot dx = [x]_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ A

Shortcut

① $I = \int_0^a f(x)\cdot g(x)\cdot dx = 2\int_0^a f(x)\cdot dx.$ — ① if f & g define as:- $\boxed{f(x) = f(a-x)}$ & $\boxed{g(x) + g(a-x) = 4}$

By pro. ①:- $I = \int_0^a f(a-x)\cdot g(a-x)\,dx = \int_0^a f(x)\cdot g(a-x)\cdot dx$ — ② $\rightarrow$ add eqⁿ ① & ②

$2I = \int_0^a f(x)[g(x) + g(a-x)]\,dx \Rightarrow \cancel{2}I = \int_0^a f(x)\cdot\frac{4}{\cancel{2}}\cdot dx \Rightarrow I = 2\int_0^a f(x)\cdot dx$ H.P.

Integrals

Q. $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right).dx$ — (1) $= \int_0^{\pi/2} \log\left[\frac{4+3\cos x}{4+3\sin x}\right].dx$ — (2) $\Rightarrow 2I = \int_0^{\pi/2} \log\left[\frac{4+\cancel{3\sin x}}{4+\cancel{3\cos x}} \times \frac{4+\cancel{3\cos x}}{4+3\sin x}\right].dx$ M

$\int_0^{\pi/2} \log 1 \cdot dx = 0$ Ans

Q. $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \cdot dx = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$

$\int \frac{\sqrt{x+a}-\sqrt{x+b}}{(\cancel{x}+a)-(\cancel{x}+b)} .dx = \frac{1}{a-b}\left[\int \sqrt{x+a}\cdot dx - \int \sqrt{x+b}\cdot dx\right]$

Q. $\int \frac{1}{x\sqrt{a(x)-x^2}} \cdot dx \Rightarrow$ let $\left[x = \frac{a}{t}\right]$ $\Rightarrow \int \frac{1}{\frac{a}{\cancel{t}}\sqrt{\frac{a^2}{t}-\frac{a^2}{t^2}}} \times \frac{-\cancel{a}.dt}{t^{\cancel{2}}} = -\int \frac{dt}{t\sqrt{\frac{a^2}{t}-\frac{a^2}{t^2}}} = -\frac{1}{a}\int \frac{dt}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}$

$dx = -\frac{a}{t^2}.dt$

$I = -\frac{1}{a}\int \frac{dt}{\sqrt{t-1}} = -\frac{1}{a}\int (t-1)^{-1/2}.dt$

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Integrals

Q. $\int \frac{1}{x^{1/3}+x^{1/2}} \cdot dx = \int \frac{1}{x^{1/3}\left[1+x^{1/6}\right]} \cdot dx$

$\frac{x^{1/2}}{x^{1/3}} = x^{1/2} \cdot x^{-1/3} = \frac{3-2}{6} = \frac{1}{6}$

Let $\boxed{x = t^6} \rightarrow \boxed{t = (x)^{1/6}}$

$\boxed{dx = 6t^5 \cdot dt}$

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$I = \int \frac{1}{(t^6)^{1/3}\left[1+(t^6)^{1/6}\right]} 6t^5 \cdot dt = 6\int \frac{t^5 \cdot dt}{t^2(1+t)} = 6\int \frac{t^3\, dt}{t+1} = 6\int \left\{\frac{t^3+(1)^3}{t+1} - \frac{1}{t+1}\right\} dt$

$I = 6\left[\int \frac{\cancel{(t+1)}(t^2+1-t)}{\cancel{(t+1)}} \cdot dt - \int \frac{1}{t+1} \cdot dt\right] = 6\left[\int t^2 \cdot dt + \int 1 \cdot dt - \int t \cdot dt - \log(t+1)\right]$

Q. $\int \frac{e^{5\log x^5} - e^{4\log x^4}}{e^{3\log x} - e^{2\log x}} \cdot dx = \frac{x^5-x^4}{x^3-x^2} = \frac{x^4(\cancel{x-1})}{x^2(\cancel{x-1})} = x^2 = \frac{x^3}{3}$ Ans $\left\{e^{\log} = 1\right\}$

Q. $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x \cdot \cos^2 x} \cdot dx = \frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{\sin^4 x+\cos^4 x - \sin^2 x \cdot \cos^2 x - \sin^2 x \cdot \cos^2 x} = \frac{(\sin^4 x+\cos^4 x)(1)(\sin^2 x-\cos^2 x)}{\sin^4 x+\cos^4 x}$

$= 1 - \int (\cos^2 x - \sin^2 x)\, dx = -\int \cos 2x \cdot dx$

Integrals #1

Q. $\int f'(ax+b)\,[f(ax+b)]^n\,dx \Rightarrow \int dt/a \times t^n = \frac{1}{a}\int t^n\,dt = \frac{1}{a}\cdot\frac{t^{n+1}}{n+1}$

$\Rightarrow \boxed{f(ax+b) = t} \Rightarrow f'(ax+b)\cdot a\cdot dx = dt$

$= \frac{1}{a}\cdot\frac{f(ax+b)^{n+1}}{n+1} + C$

Q. $\int \left(\frac{2+\sin 2x}{1+\cos 2x}\right)\cdot \boxed{e^x}\,dx = \int\left(\frac{\cancel{2}+\cancel{2}\sin x\cdot\cos x}{\cancel{2}\cos^2 x}\right)e^x dx = \int \frac{1+\sin x\cdot\cos x}{\cos^2 x}\,e^x dx$

$\underbrace{\frac{2+\sin 2x}{1+\cos 2x}}_{f(x)+f'(x)}$

$\rightarrow \int\left(\underset{f(x)}{\sec^2 x} + \underset{f(x)}{\tan x}\right)\cdot e^x\cdot dx = e^x\cdot\tan x + C$