Q. $I = \int_0^{\pi} \frac{x}{1+\sin x} \cdot dx$ — (1)

By pro. (1) :-

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} \cdot dx$$

$$I = \int_0^{\pi} \frac{\pi - x}{1+\sin x} \cdot dx \quad \text{— (2)}$$

$$(1)+(2) \Rightarrow 2I = \int_0^{\pi} \frac{\cancel{x}+\pi-\cancel{x}}{1+\sin x} \cdot dx$$

$$\left[2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} \cdot dx \right]$$

$$2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} \cdot dx$$

$$2I = \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} \cdot dx$$

$$2I = \pi \left[\int_0^{\pi} \sec^2 x \cdot dx - \int_0^{\pi} \tan x \sec x \cdot dx \right]$$

$$2I = \pi \left[\tan x \right]_0^{\pi} - \left[\sec x \right]_0^{\pi}$$

$$2I = \pi \cdot (\tan \pi - 0) - (\sec \pi - \sec 0]$$

Q. $I = \int_{-\pi/2}^{\pi/2} \sin^7 x \cdot dx$

$\because$ $f(x) = \sin^7 x$

$f(-x) = \sin^7(-x)$

$= \{\sin(-x)\}^7$

$= \{-\sin x\}^7$

$f(-x) = -\sin^7 x$

$f(-x) = -f(x)$

$\therefore f(x)$ is odd

$\Rightarrow I = 0$

$\int_0^a f(x) \cdot dx = \int_0^a f(a-x) \cdot dx$

$\int_0^{2a} f(x) \cdot dx = \quad f(2a-x) = f(x)$

$\downarrow$

$2\int_0^a f(x) \cdot dx$

$\int_0^{2a} f(x)\cdot dx = \boxed{2\int_0^a f(x)\cdot dx \rightarrow \text{when } f(2a-x) = f(x)}$
$= 0 \rightarrow f(2a-x) = \boxed{-f(x)}$

$\int_{-a}^{a} f(x)\cdot dx = 2\int_0^a f(x)\cdot dx \rightarrow f(x)$ even
$= 0 \rightarrow f(x) = \text{odd}$

Q. $\int_0^{2\pi} \cos^5 x\cdot dx \xrightarrow{\text{By prop.}}$ $\because f(x) = \cos^5 x$

$f(2\pi - x) = \cos^5(2\pi - x) = \cos^5 x = f(x)$

So: $\int_0^{2\pi} \cos^5 x\cdot dx = 2\int_0^{\pi} \cos^5 x\cdot dx$

By prop again:- $f(x) = \cos^5 x \Rightarrow \boxed{f(\pi - x)} = \cos^5(\pi - x) = -\cos^5 x = \boxed{-f(x)}$

0 Ans

Q. $I = \int_0^{\pi} \log(1+\cos x)\, dx$ — (1)

By proo (1)

$I = \int_0^{\pi} \log[1+\cos(\pi-x)].\, dx$

$I = \int_0^{\pi} \log[1-\cos x].\, dx$ — (2)

(1) + (2)

$2I = \int_0^{\pi} \log[(1+\cos x][1-\cos x]).\, dx$

$2I = \int_0^{\pi} \log(\sin^2 x).\, dx$

$\cancel{2}I = \cancel{2}\int_0^{\pi} \log(\sin x).\, dx$ — (3)

$I = \int_0^{\pi} \log(\sin x).\, dx$

by proo. → $\int_0^{a} f(x)\, dx = 2\int_0^{a/2} f(x)\, dx$

When $f(a-x) = f(x)$

$\log(\sin(\pi-x)) = \log \sin x$

then

$I = 2\int_0^{\pi/2} \log(\sin x).\, dx$ — (4)

By proo (1) :-

$I = 2\int_0^{\pi/2} \log[\sin(\pi/2 - x)].\, dx$

$I = 2\int_0^{\pi/2} \log \cos x.\, dx$ — (5)

add eq (4) + (5)

$\cancel{2}I = \cancel{2}\int_0^{\pi/2} [\log(\sin x.\cos x).\, dx]$

$$I = \int_0^{\pi/2} \log(\sin x \cdot \cos x) \cdot dx = \int_0^{\pi/2} \left[\log(\sin x \cdot \cos x) + \log 2 - \log 2\right]$$

$$I - \frac{I}{2} =$$

$$I = \int_0^{\pi/2} \left[\log(2 \cdot \sin x \cdot \cos x) - \log 2\right] \cdot dx$$

$$I = \int_0^{\pi/2} \left[\log(\sin 2x) - \log 2\right] \cdot dx$$

$$I = \underbrace{\int_0^{\pi/2} \log(\sin 2x)\, dx}_{I_1} - \int_0^{\pi/2} \log 2 \cdot dx$$

$\therefore$ in $I_1 \Rightarrow \boxed{2x = t} \Rightarrow 2 \cdot dx = dt$

$dx = dt/2$

$x = 0 \Rightarrow t = 0$

$x = \pi/2 \Rightarrow t = \pi$

$$\text{so } I_1 = \int_0^{\pi} \log \sin t \cdot \frac{dt}{2}$$

$$I_1 = \frac{1}{2}\left[\int_0^{\pi} \log \sin t \cdot dt\right] = \frac{1}{2}\left[\int_0^{\pi} \log \sin x \cdot dx\right]$$

$$\boxed{I_1 = \frac{1}{2} I}$$

$$I = \frac{I}{2} - \log 2 [x]_0^{\pi/2}$$

$$\frac{I}{\cancel{2}} = -\frac{\log 2 \cdot \pi}{\cancel{2}}$$

$$\boxed{I = -\pi \log 2}$$

$$\left[\int_a^b f(x)\cdot dx = \int_a^b f(t)\cdot dt\right]$$

H.W $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} \cdot dx$

H.W $= \int_0^4 |x-1| \cdot dx$

Q. I $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cdot \cos x} \cdot dx$ — (1)

H.W. $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1) \cdot dx$

Sol^n:- by pro. (1)

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \cdot \sin x} \cdot dx \quad \text{— (2)}$$

$$2I = \int_0^{\pi/2} \frac{0}{1+\sin x \cdot \cos x} \cdot dx = 0 \quad \text{Ans}$$