properties :-

$$\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$$

(1) $$\int_0^a f(x)\cdot dx = \int_0^a f(a-x)\cdot dx$$

Ex:

$$\int_0^{\pi/2} \sin x\cdot dx = \left[-\cos x\right]_0^{\pi/2} = -\cos\frac{\pi}{2} + \cos 0 = 1$$

$$\int_0^{\pi/2} \sin\left(\frac{\pi}{2}-x\right)\cdot dx = \int_0^{\pi/2} \cos x\cdot dx = \left[\sin x\right]_0^{\pi/2}$$

$$= \sin\frac{\pi}{2} - \sin 0 = 1$$

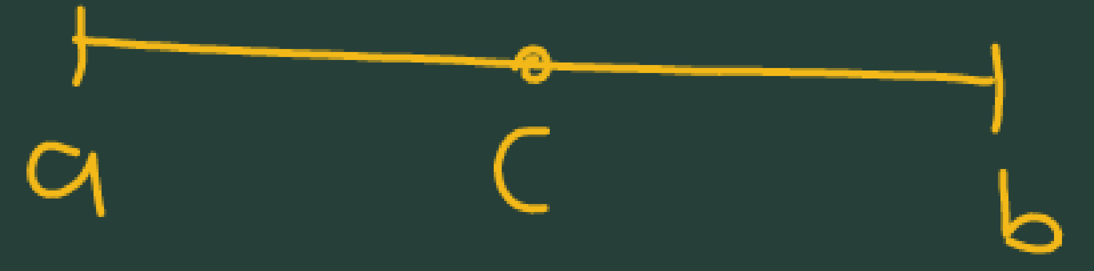
(2) $$\int_a^b f(x)\cdot dx = \int_a^b f(t)\cdot dt$$

$f(x) = \cos x$

$f(-x) = \cos(-x)$

$f(x) = \cos x$

(3) $$\int_a^b f(x)\cdot dx = \int_a^c f(x)\cdot dx + \int_c^b f(x)\cdot dx$$



(4) $$\int_{-a}^{a} f(x)\cdot dx = 0$$; when $f(x)$ is odd function.

odd fun. $\rightarrow$ $[f(-x) = -f(x)]$

(5) $$\int_{-a}^{a} f(x)\cdot dx = 2\int_0^a f(x)\cdot dx$$; when $f(x)$ is even fun.

even fun. $\rightarrow$ $[f(-x) = f(x)]$

Q. $\boxed{\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx} = I$ — (1)

By pro. (1)

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}\, dx = \boxed{\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx = I} \text{ — (2)}$$

add eq^n (1) & (2) :-

$$\underline{I + I} = \int_0^{\pi/2} \cancel{\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}}\, dx \Rightarrow 2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = [x]_0^{\pi/2} = 2I = \frac{\pi}{2}$$

$$I = \pi/4 \text{ Ans}$$

Q. $$\int_0^{\pi/2} \frac{\cos^5 x}{\boxed{\sin^5 x + \cos^5 x}} \cdot dx = I \quad \text{— (1)}$$

H.W $$\int_0^{\pi/2} \frac{\sin^{3/2} x \, dx}{\sin^{3/2} x + \cos^{3/2} x}$$

By Pro.

$$\rightarrow I = \int_0^{\pi/2} \frac{\cos^5(\pi/2 - x)}{\sin^5(\pi/2 - x) + \cos^5(\pi/2 - x)} \cdot dx = \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} \cdot dx = I \quad \text{— (2)}$$

$$\Rightarrow 2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \pi/2$$

$$I = \pi/4 \quad \text{Ans}$$

Q. $\int_{-5}^{5} |x+2| \cdot dx$

By using pr.

$$\left\{ |x+2| \longrightarrow x+2 \; ; \; x > -2 \right.$$

$$x+2=0 \longrightarrow -(x+2) \; ; \; x < -2$$

$$\boxed{x=-2} \longrightarrow \frac{> -2}{< -2}$$

$$\Rightarrow \int_{-5}^{5} |x+2| \cdot dx = \int_{-5}^{-2} -(x+2) \cdot dx + \int_{-2}^{5} (x+2) \cdot dx$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left(\frac{4}{2} - 4 - \left\{\frac{25}{2} - 10\right\}\right) + \left[\frac{25}{2} + 10 - \left\{\frac{4}{2} - 4\right\}\right]$$

Q. $\int_2^8 |x-5| \cdot dx$

$$\int_2^5 \text{-ive} \;/\; \int_5^8 +.$$

$x - 5 = 0$

$\boxed{x = 5}$

$\boxed{x > 5}$

Q. $\int_0^1 x(1-x)^n \cdot dx = I$

By pro. $\rightarrow$ 1

$$I = \int_0^1 (1-x)\left[1-(1-x)\right]^n \cdot dx = \int_0^1 (1-x)\left[x\right]^n \cdot dx$$

$$I = \int_0^1 \left[x^n - x^n \cdot x^1\right] \cdot dx = \int_0^1 x^n \cdot dx - \int_0^1 x^{n+1} \cdot dx$$

$$= \left[\frac{x^{n+1}}{n+1}\right]_0^1 - \left[\frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

Q. $\displaystyle\int_0^{\pi/4} \log(1+\tan x)\,.dx = I$

$$\Rightarrow I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right].dx$$

$$\Rightarrow I = \int_0^{\pi/4} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\cdot\tan x}\right].dx$$

$$\Rightarrow I = \int_0^{\pi/4} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right].dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan x} + 1 - \cancel{\tan x}}{1+\tan x}\right].dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right).dx$$

$$I = \int_0^{\pi/4} \log 2\,dx - \boxed{\int_0^{\pi/4} \log(1+\tan x)\,dx}$$

$$I = \int_0^{\pi/4} \log 2 \,.\, dx - I$$

$$2I = \log 2 \int_0^{\pi/4} 1\,.dx$$

$$2I = \log 2\,.\,[x]_0^{\pi/4}$$

$$\Rightarrow 2I = \log 2\,.\,\frac{\pi}{4}$$

$$I = \frac{\pi}{8}\,.\log 2$$

Q. $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx$

$I = \int_0^{\pi/2} [2\log\sin x - \log(2\sin x\cos x)]\,dx$ M. N. P

$I = \int_0^{\pi/2} [2\log\sin x - \log 2 - \log\sin x - \log\cos x]\,dx$

$I = \int_0^{\pi/2} [\log\sin x - \log\cos x - \log 2]\,dx$ — (1)

By pro. $I = \int_0^{\pi/2} \left[\log\sin\left(\frac{\pi}{2}-x\right) - \log\cos\left(\frac{\pi}{2}-x\right) - \log 2\right]dx$

$I = \int_0^{\pi/2} [\log\cos x - \log\sin x - \log 2]\,dx$ — (2)

add eq (1) + (2) $\Rightarrow$ $I + I = \int_0^{\pi/2} -2\log 2\,dx$

$\cancel{2}I = -\cancel{2}\log 2\,[x]_0^{\pi/2}$

$I = \boxed{-\log 2} \cdot \frac{\pi}{2}$

$I = \log\frac{1}{2} \cdot \frac{\pi}{2}$

$I = \log(2)^{-1}$

$= \log\frac{1}{2}$

H.Q

Q. $$\int_0^2 x\cdot\sqrt{2-x}\,dx$$

Q. $$\int_0^{\pi} \frac{x}{1+\sin x}\cdot dx \longrightarrow 1$$

Q. $$\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x\cdot\cos x}$$