Q $\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right).dx = -\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right).dx$

$= -\int_0^{\pi}\cos x.\,dx = -\left[\sin x\right]_0^{\pi} \Rightarrow -\left[\sin\pi - \sin 0\right] = 0$ Ans

Q $\int_0^{2}\frac{6x+3}{x^2+4}.dx = \int_0^{2}\overset{I_1}{\frac{6x}{x^2+4}}.dx + \int_0^{2}\overset{I_2}{\frac{3}{x^2+4}}.dx$

$I_1 = \int_0^{2}\frac{\boxed{6x\ .dx}}{x^2+4} \Rightarrow x^2+4=t$

$2x.dx = dt$

$x\,dx = dt/2$

$I_1 = \int_0^{2}\frac{3\,6\times dt/\cancel{2}}{t} = 3\left[\log(x^2+4)\right]_0^2$

$I_1 = 3\left[\log 8 - \log 4\right] = 3\log 2$

$I_2 = 3\int_0^{2}\frac{1}{x^2+(2)^2}.dx = 3.\frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2$

$I_2 = \frac{3}{2}\left[\tan^{-1}1 - \tan^{-1}0\right]$

$I_2 = \frac{3}{2}\times\frac{\pi}{4}$

$I = 3\log 2 + \frac{3}{8}\pi$ Ans

H.W. Q. $\int_{1}^{\sqrt{3}} \frac{1}{1+x^2} \cdot dx$

$I \Rightarrow 2 + \frac{\pi^4}{4^5} - \pi/2$ Ans

$4 \times 4 \times 4 \times 4 \times 4$

Q. $\int_{0}^{1} \left(x e^{x} + \sin \frac{\pi x}{4} \right) \cdot dx$

Q. $\int_{0}^{\pi/4} (2\sec^2 x + x^3 - 2) \cdot dx$

$\rightarrow \int_{0}^{\pi/4} 2 \cdot \sec^2 x \cdot dx + \int_{0}^{\pi/4} x^3 \cdot dx - \int_{0}^{\pi/4} 2 \cdot dx$

$\Rightarrow 2 \left[\tan x\right]_{0}^{\pi/4} + \left[\frac{x^4}{4}\right]_{0}^{\pi/4} - 2 \left[x\right]_{0}^{\pi/4}$

$\Rightarrow 2 \left[\tan \frac{\pi}{4} - \tan 0 \right] + \frac{1}{4} \left[\left(\frac{\pi}{4}\right)^4 - 0 \right] - \cancel{2} \left[\frac{\pi}{\cancel{4}} - 0 \right]$

Q. $\int_0^{\pi/2} \sqrt{\sin\phi}\cdot\cos^5\phi\cdot d\phi = \int_0^{\pi/2} \sqrt{\sin\phi}\cdot\cos^4\phi\cdot\cos\phi\cdot d\phi$ $\{(\cos^2\phi)^3$

$I \Rightarrow \int_0^{\pi/2} \sqrt{\sin\phi}\cdot(1-\sin^2\phi)^2\cdot\cos\phi\cdot d\phi$ — ①

let $\sin\phi = t$ — ②

Diff $\cos\phi\cdot d\phi = dt$ — ③

when $\phi = 0 \Rightarrow \sin 0 = t \Rightarrow t = 0$

when $\phi = \pi/2 \Rightarrow \sin\pi/2 = t \Rightarrow t = 1$

from eq ① ② ③ :- $I = \int_0^1 \sqrt{t}\cdot(1-t^2)^2\cdot dt$

$I = \int_0^1 t^{1/2}\left[1+t^4-2t^2\right]\cdot dt = \left[\int_0^1 t^{1/2}\cdot dt + \int_0^1 t^{9/2}\cdot dt - 2\int t^{5/2}\cdot dt\right]$

So $I = \left[t^{3/2}\times\frac{2}{3}\right]_0^1 + \left[t^{11/2}\times\frac{2}{11}\right]_0^1 - 2\left[t^{7/2}\times\frac{2}{7}\right]_0^1$

$I = \frac{2}{3}\left[(1)^{3/2}-0\right] + \frac{2}{11}\left[(1)^{11/2}-0\right] - 2\times\frac{2}{7}\left[(1)^{7/2}-0\right] = \frac{2}{3}+\frac{2}{11}-\frac{4}{7} = \frac{154+42-132}{77\times 3}$

$\frac{64}{231}$ $\Rightarrow \frac{64}{77\times 3}$

$\begin{array}{r} 196 \\ 132 \\ \hline 64 \end{array}$ $\begin{array}{r} 1 \\ 33 \\ 4 \\ \hline 132 \end{array}$

Q. $\int_0^2 x\sqrt{x+2}\cdot dx$

$\boxed{x+2=t} \to \boxed{x=t-2}$

$\boxed{dx=dt}$

$\to$ $x=0 \Rightarrow 0+2=t \Rightarrow \boxed{t=2}$

$x=2 \Rightarrow 2+2=t \Rightarrow \boxed{t=4}$

$$I=\int_2^4 (t-2)\sqrt{t}\cdot dt$$

$$I=\int_2^4 t^{3/2}\cdot dt-\int_2^4 2t^{1/2}\cdot dt$$

Q. $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\cdot dx$

Q. $\displaystyle\int_0^2 \frac{dx}{x+4-x^2}$ H.W.

Ans. $\dfrac{1}{\sqrt{17}} \log\left(\dfrac{21+5\sqrt{17}}{4}\right)$

Q. $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}\, dx$ $\qquad \boxed{2x = t}$ — ① $\Rightarrow \boxed{x = t/2}$ — ② $\qquad \boxed{x^2 = t^2/4}$ — ③

Sol^n: let $2x = t$

Diff $\quad 2\cdot dx = dt$

$\boxed{dx = dt/2}$ — ④

$\left[\begin{array}{l} \text{When } x = 1 \Rightarrow \boxed{t = 2} \\ \quad\;\; x = 2 \Rightarrow \boxed{t = 4} \end{array}\right]$ — ⑤

from ① ② ③ & ④ & ⑤ :-

$$I = \int_2^4 \left(\frac{1}{t/2} - \frac{1}{\cancel{2}\cdot t^2/\cancel{4}_2}\right) e^t \cdot dt/2 = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t \cdot dt$$

$$I = \frac{1}{\cancel{2}} \times \cancel{2} \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t \cdot dt \Rightarrow \int_2^4 e^x \left[f(x) + f'(x)\right] \cdot dx = e^x \cdot f(x)$$

$\qquad\qquad f(x) \quad f'(x)$

$$\left(e^t \cdot \frac{1}{t}\right)_2^4 = \frac{e^4}{4} - \frac{e^2}{2} \text{ Ans}$$

Q. $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} \cdot dx = \int_{1/3}^{1} \frac{(x)^{1/3}\left[1-x^2\right]^{1/3}}{x^4} \cdot dx$

$\theta = \sin^{-1}(1/3)$

$\sin^{-1}\frac{1}{3} = \theta$

$\sin\theta = \frac{1}{3} = \frac{P}{H}$

$\boxed{B = 2\sqrt{2}}$

$\cot\theta = \frac{B}{P}$

$\cot\theta = \frac{2\sqrt{2}}{1}$

let $x = \sin\theta \Rightarrow dx = \cos\theta \cdot d\theta$

when $x = \frac{1}{3} \Rightarrow \frac{1}{3} = \sin\theta$

$x = 1 \Rightarrow 1 = \sin\theta = \theta = \pi/2$

$\Rightarrow I = \int_{\sin^{-1}(1/3)}^{\pi/2} \frac{(\sin\theta)^{1/3}\left[1-\sin^2\theta\right]^{1/3} \cdot \cos\theta \cdot d\theta}{\sin^4\theta}$

$I = \int_{\sin^{-1}(1/3)}^{\pi/2} \frac{(\cos^2\theta)^{1/3} \cos\theta \cdot d\theta}{(\sin\theta)^{-1/3} \cdot \sin^2\theta \cdot \sin^2\theta} \Rightarrow \int_{\sin^{-1}(1/3)}^{\pi/2} \frac{\cos^{5/3}\theta}{\sin^{5/3}\theta} \cdot \frac{d\theta}{\sin^2\theta} = \int \cot^{5/3}\theta \, \mathrm{cosec}^2\theta \, d\theta$

let $\cot\theta = t \longrightarrow \theta = \sin^{-1}(1/3) \Rightarrow t = \cot\left[\sin^{-1}\frac{1}{3}\right] = 2\sqrt{2}$

$\theta = \pi/2 \Rightarrow t = \cot\pi/2 = 0$

$\mathrm{cosec}^2\theta \cdot d\theta = -dt$

$I = \int_{2\sqrt{2}}^{0} t^{5/3} \cdot (-dt)$